Estimation and Measurement of Forward Propagated Ultrasonic Fields in Layered Fluid Media

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Abstract

The forward propagated ultrasonic fields resulting from a circular plane or a concave transducer in layered fluid media as well as in homogeneous water are theoretically estimated by the angular spectrum method(ASM) combined with Rayleigh-Sommerfeld diffraction theory(RSDT), and measured by a precision 3-D scanning system with a needle-point hydrophone. To make the aliasing error negligible on the 2-D FFT in the theoretical estimation, the spatial discretization in the ASM are carefully considered for optimal selection of spatial sampling intervals and the size of discretization area. It is shown that the estimated fields agree reasonably with the measured ones.

I. Introduction

In ultrasonic field calculation, the ASM has been implemented by a number of investigators in various applications since the early of 1980s[1-3]. The method is able to calculate even complex near-fields with high computational efficiency using the 2-D FFT. Recently, Christopher and Parker[4] proposed a new algorithm for extending the ASM to layered media. Orofino and Pedersen[5,6] made guidelines for the efficient angular spectrum decomposition of acoustic sources, in which its efficiency was evaluated in terms of several angular parameters, such as angular range, angular resolution and spatial aliasing error. More recently, Wu *et al.*[7,8] considered the spatial aliasing errors related to angluar parameters in detail for more accurate acoustic field analysis.

Because the forward propagated acoustic fields have been directly calculated from the field on a transducer surface in many investigations so far, considerable aliasing errors due to the spatial sampling frequency limitation or the source size limitation are included for the forward propagated fields long distance away from the source. As one of the methods to reduce the errors in acoustic fields radiated by a circular plane transducer, we proposed the ASM combined with RSDT in a previous paper[9]. In the paper, the feasibility and computational efficiency of the proposed method was theoretically shown using virtual boundaries in homogeneous water.

In this paper, we have extensively applied the method not only to a circular plane transducer but also a concave one. To make the aliasing error negligible on the 2-D FFT in the theoretical estimation, the spatial discretization in the ASM are carefully considered for optimal selection of spatial sampling intervals and the size of discretization area. The ultrasonic fields in layered fluid media as well as in homogeneous water are theoretically estimated, and experimentally measured by a computer-controlled precision 3-D scanning system with a needle-point hydrophone for comparison.

II. ASM Combined with RSDT in Layered Fluid Media

A forward($+z$ direction) propagated 2-D particle velocity field $u(x,y;z)$ from a source $u_0(x_0,y_0;0)$ is analytically given by the RSDT in homogeneous media as follows[10]:

$$u(x,y;z)=\frac{1}{j\lambda}\int\int_S u_0(x_0,y_0;0)\frac{\exp(jkr)}{r}\cos\theta dS, \qquad (1)$$

where, λ is wavelength, r is propagation distance from a point on the source field of $u_0(x_0,y_0;0)$ to a point on the observation field of $u(x,y;z)$, S is the radiating area on the source, and θ is the angle that the

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position vector $\vec{r}$ makes relative to the outgoing surface vector $d\vec{S}$ which is normal to the surface of the transducer. In the range $z \gg x, y$ so that $r \fallingdotseq z$, equation (1) becomes

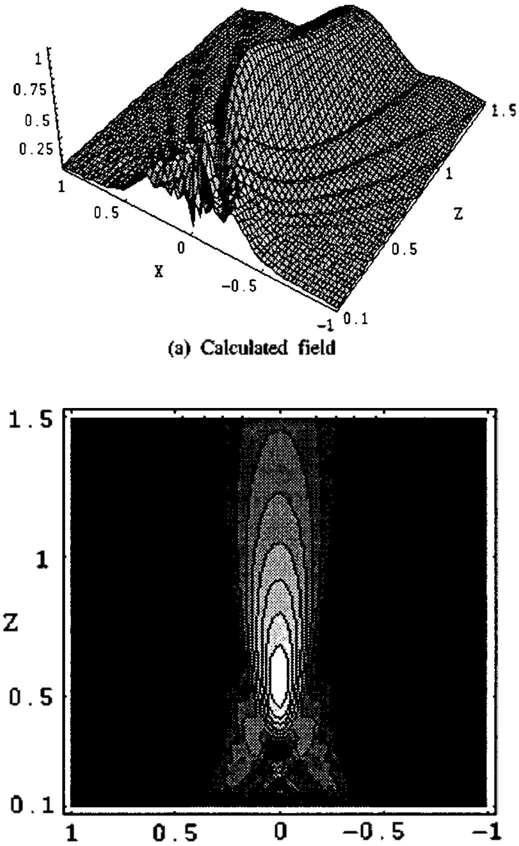
$$u(x,y;z) = \frac{2\pi}{j\lambda z} e^{jk[z+\frac{1}{2z}(x^2+y^2)]} \times \int_0^a e^{jk\frac{\rho^2}{2z}} J_0\left(-k\rho\frac{\sqrt{x^2+y^2}}{z}\right)\rho d\rho \quad (2)$$

for a circular plane transducer with radius a vibrating linearly with unit amplitude on $z=0$, where ρ is the distance from the center of the transducer and given by $\rho=(x_0^2+y_0^2)^{1/2}$. For a concave transducer, it is readily modified by

$$u(x,y;z) = \frac{2\pi}{j\lambda z} e^{jk[z+\frac{1}{2z}(x^2+y^2)]} \times \int_0^{\rho_m} e^{jk\rho^2\frac{R-z}{2z}} J_0\left(-k\rho\frac{\sqrt{x^2+y^2}}{z}\right)\rho d\rho, \quad (3)$$

where R is the radius of curvature and ρ_m is the distance from the convex of a transducer to its edge.

The 2-D fields for a circular plane transducer was already shown in our previous paper[9]. Figure 1 (a) and (b) show the field and its contour graph calculated by eq.(3) for a 2MHz concave transducer of which the radius of curvature is $R=10$cm and aperture width is $a_R=1.8$cm, respectively.



(a) Calculated field

(b) Contour plot

Figure 1. Calculated acoustic fields radiated by a 2MHz concave transducer in water. The radius of curvature of the transducer is $R=10$cm and the aperture width is $a_R=1.8$cm. The coordinates of X and Z are normalized by a_R and R, respectively.

From Fig.1 and the previous field plots for a circular plane transducer, it is shown that very accurate fields for any 2-D plane(x-y plane) in front of a transducer can be obtained by the RSDT in homogeneous medium. However, RSDT cannot be directly applied to any layered media. Considering the 1st medium is homogeneous, we calculate the field of $u(x,y;z)$ on the 1st boundary in layered fluid media by the RSDT and call it as a reference field. Thereafter the forward propagated field $u'(x,y;z')$ in layered fluid media can be obtained by the ASM using the reference field, and given as follows[4]:

$$u'(x,y;z') = F^{-1}[U(k_x, k_y;z)H_1(k_x,k_y;\Delta z_1)T_1(k_x,k_y;z_1) H_2(k_x,k_y;\Delta z_2)T_2(k_x,k_y;z_2)\cdots], \quad (4)$$

where, $U(k_x,k_y;z)$ is the spatial frequency of $u(x,y;z)$. The $H_i(k_x,k_y;\Delta z_i)$ is called as the transfer function of medium i of which thickness is Δz_i, and it is the angular spectrum of the field formed by a point source existing on the center of the prior plane. In the range of $k_x^2+k_y^2 \leq k_i^2$, where k_i is the medium wavenumber in the layer i, evanecent wave components do not exist and the expression of $H_i(k_x,k_y;\Delta z_i)$ is given as follows[2],

$$H_i(k_x,k_y;\Delta z_i) = \exp[j2\pi\Delta z_i\sqrt{k_i^2-k_x^2-k_y^2}]. \quad (5)$$

The 2-D spatial transmission function $T_i(k_x,k_y;z_i)$ on the boundary i is drived as follows:

$$T_i(k_x, k_y; z_i) = \frac{2}{1 + \frac{(\rho_i c_i)\sqrt{k_i^2 - (c_{i+1}/c_i)^2 k_x^2 - (c_{i+1}/c_i)^2 k_y^2}}{\rho_{i+1} c_{i+1} \sqrt{k_i^2 - k_x^2 - k_y^2}}}. \quad (6)$$

Because the reflection coefficients on boundaries in layered fluid media are very small, the multiple reflection effect can be ignored without any loss of generality in the above theorem.

III. Consideration for Aliasing Error Reduction

One of the problems that should be considered in the ASM application is how to reduce the aliasing error primarily caused by inadequate selection of sampling interval on the spatial discretization in the 2-D FFT. To avoid the aliasing error, the sampling interval δ should be choosen by

$$\delta \le \frac{1}{2 f_{max}} = \frac{\lambda_{min}}{2} \quad (7)$$

according to the spatial Nyquist theorem, where f_{max} is the highest spatial frequency and λ_{min} is its wave length. On the other hand, spatial frequency f is given by using the incident angle θ at a point on a receiver plane as follows:

$$f = \frac{\omega}{2\pi c} \sin\theta, \quad (8)$$

where, ω is the temporal angular frequency and c is the acoustic wave speed, and both of them could be constants in a given medium. It means that the spatial frequency f is directly related to the incident angle θ, and the f_{max} is limited by the θ_{max}. The θ_{max} can be determined through the geometrical consideration for the size of discretization area of a source and a receiver plane, and the distance between them.

In this study, we want to obtain the acoustic pressure field of the measurement area $d \times d$ which is arbitrarily determined within the discretization area on a receiver plane. If the discretization area on the receiver plane is $D \times D$, which is generally taken as same as the discretization area on a source or a reference plane for the convinence on FFT, and the distance from the source to the receiver plane is Δz, as shown in Fig. 2, then the θ_{max} is given by

$$\theta_{max} = \sin^{-1}\left[\frac{a + (D/2)}{\sqrt{\Delta z^2 + (a + D/2)^2}}\right]. \quad (9)$$

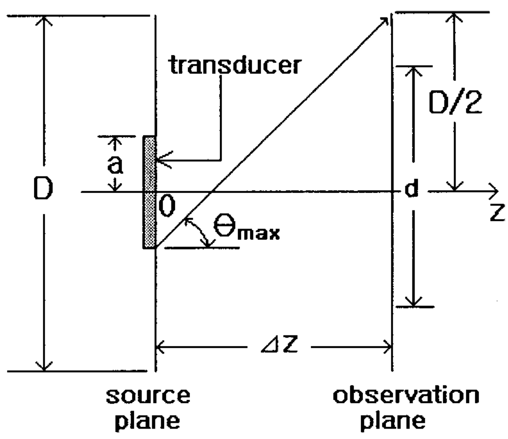


Figure 2. Geometry for the consideration of the spatial sampling intervals relating to the size of discretization area D and the maximum incident angle θ_{max}.

Because the size of discretization area D has a finite value, some errors in calculated fields are included near the edge of the receiver plane. If $D \gg a$ and $d \ll D$ is taken, the resultant field of measurement area becomes almost absolute one. According to the increment of the size of discretization area D, however, the sampling interval δ becomes small, and so the number of data and the calculation time for a same area are increased. In this study, the optimum ratio of d to D which makes the spatial aliasing error negligible is determined by numerical results with the experiment.

IV. Measurement System

A computer-controlled 3-D scanning system has been constructed for this study. Figure 3 shows the scheme with block-diagram of the system. The accuracy of the stepping-moter movement for each stage is 1㎛/step. A 2㎒ circular plane tranducer and a concave transducer of the same frequency are used as acoustic sources. The diameter of the circular plane tranducer is $a = 1.8$㎝. For the concave transducer, the aperture width is $a_R = 1.6$㎝ and its geometric radius of curvature is $R = 10$㎝, which are the same dimension as those of Fig. 2. A needle-point hydrophone(MH28-10) of which diameter is $1.0mm$ is used as a receiver for obtaining acoustic pressure fields.

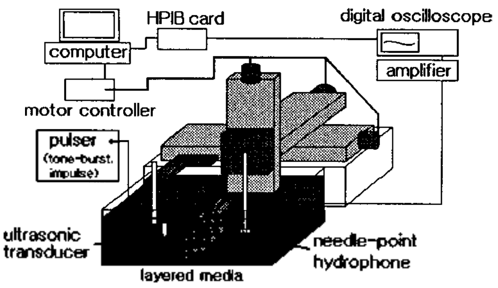


Figure 3. Scheme of the measurement system.

V. Results and Discussions

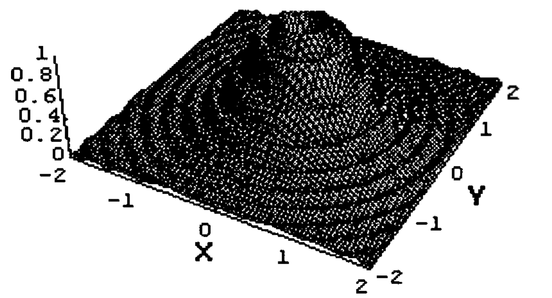


Figure 4. Reference field calculated by the RSDT for a circular plane transducer in homogeneous water. Normalized distance from the transducer is $Z=0.5$. The coordinates of X and Y are normalized by the radius of transducer a.

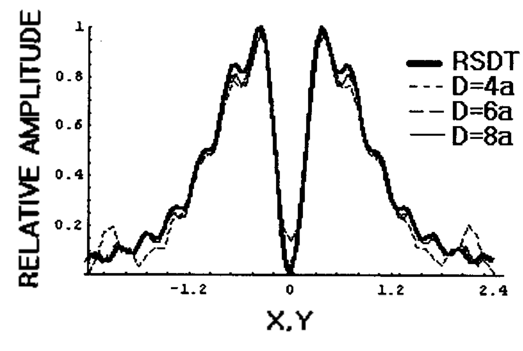
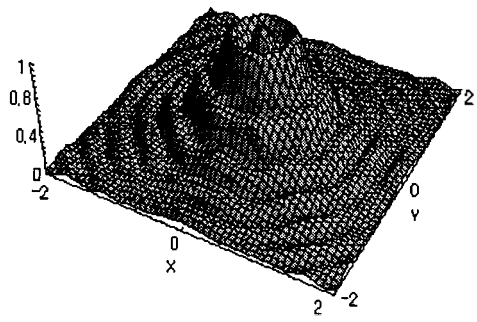


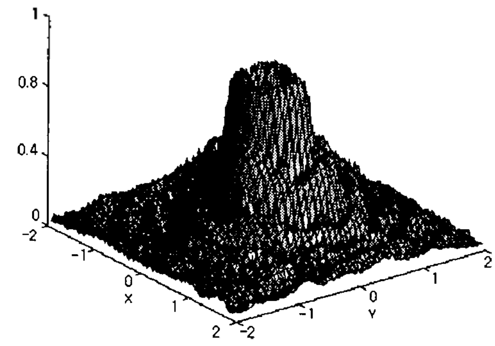
Figure 5. Field variation on the center lines in Fig. 4 according to the size of discretization area D. (The D=8a line '-' is included in the RSDT line '—'.)

Figure 4 shows a part of the reference field calculated by the RSDT on the normalized distance $Z=\lambda z/a^2=0.5$ for a circular plane transducer in homogeneous water. The coordinates of X and Y are normalized by the radius a of the transducer. In the calculation, the spatial sampling interval and the size of discretization area were selected by $\delta=1/2f_{max}$ and $D=8a$, respectively. The size of discretization area is equal to that of the whole reference field as well as twice that of the measurement area to be obtained. This value is determined because it is the minimum possible size to use and yet retain similar results to those by the RSDT, as shown in Fig. 5.

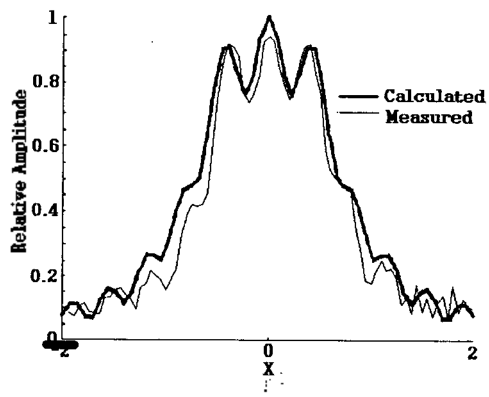
Figures 6(a) and (b) show the 2-D fields estimated theoretically from Fig. 4 and measured by the needle-point hydrophone on $Z=2/3$, respectively. Figure 6(c) shows the comparison of the variations on their center lines.



(a) Estimated



(b) Measured

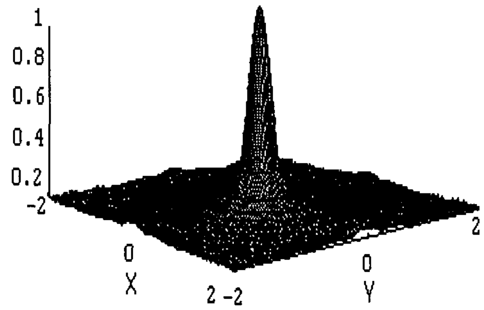


(c) Center lines

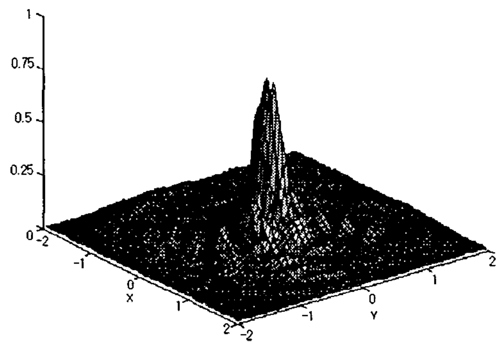
Figure 6. Estimated and measured fields on $Z=2/3$ for a circular palne transducer. The coordinates of X and Y are normalized by the radius of transducer a.

The results of a concave transducer are shown in Fig. 7. Figure 7(a) shows the 2-D field on $z=1.2R$ estimated from the field of focal plane($z=R=10$ cm) in homogeneous water and Fig. 7(b) is its measured one. Figure 7(c) shows the comparison of the variations on their center lines. From these results, it is clarified that the theoretical estimation results by the ASM combined the RSDT agree reasonably with the measured ones.

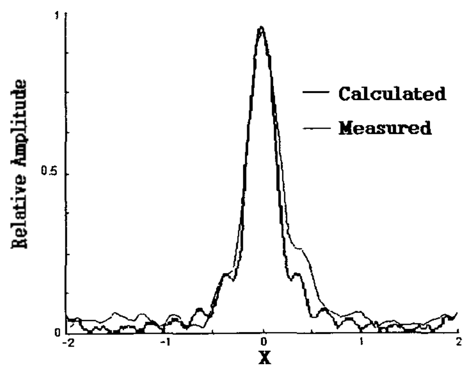
In the case of a concave transducer, the estimated and the measured 2-D fields in a simple layered fluid media are shown in Fig. 8(a) and Fig. 8(b), respectively. The layered media is stratified by water($10cm$)/ethanol($3cm$)/water ($0.2cm$). The ethanol contained in a rectangular box which made of thin vinyl film located at the focal plane. Figure 8(c) shows the comparison of the variations on their center lines.



(a) Estimated

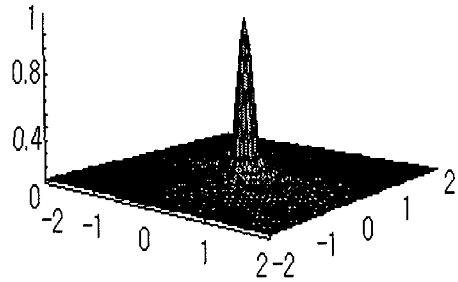


(b) Measured

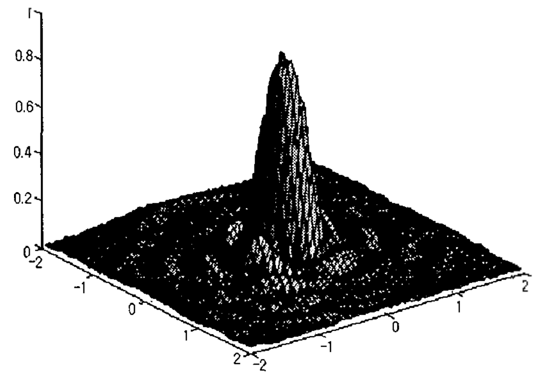


(c) Center lines

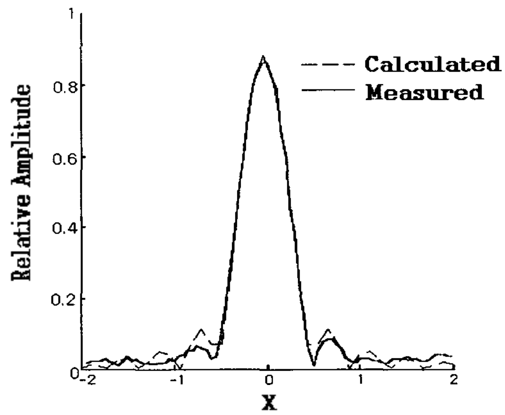
Figure 7. Estimated and measured fields on $z=1.2R$ for the concave transducer. The coordinates of X and Y are normalized by a_R.



(a) Estimated



(b) Measured



(c) Center lines

Figure 8. Estimated and measured fields in a simple layered fluid media of water(10cm)/ethanol(3cm) /water(0.2 cm). The coordinates of X and Y are normalized by a_R.

From all the above results, it is shown that the method of the ASM combined with RSDT gives such sufficient accuracy for forward propagated acoustic field estimation that it can be used practically in a layered fluid media.

VI. Conclusions

In this paper, the forward propagated ultrasonic fields from a circular plane or a concave transducer in homogeneous water and in layered fluid media are theoretically estimated using the ASM combined with RSDT, and the fields are compared with the measured ones. To reduce the aliasing errors on the 2-D FFT, the spatial discretization in the ASM are carefully treated for optimal selection of spatial sampling intervals and the size of discretization area. It is shown that the estimated fields agree reasonably with the measured ones. Conclusively, the ASM combined with RSDT has been confirmed as a useful method for the forward propagated ultrasonic field estimation in layered fluid media. In a future work, the method will be applied to biological tissues.

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