Broadband Acoustic Power Radiation from a Finite Plate Excited by Random Forces in a Subsonic Flow Field

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Abstract

This paper presents a simplified analytical formulation for computing acoustic power radiation from a rectangular plate exposed to random forces such as turbulent boundary layer pressure fluctuations and arbitrary mechanical force in a subsonic flow field. The expression for the acoustic power is derived using modal expansion method and light fluid loading is assumed on the plate. In order to simplify the formulation for acoustic power due to combined excitations of mechanical forces and turbulent pressures, it is assumed that the structural damping of the plate is small and excitations are broadband random forces having frequency spectra above the convective coincidence. Under these assumptions, an approximate solution for the broadband acoustic power radiation from a plate excited by both turbulent pressures and arbitrary mechanical forces is obtained and evaluated considering the effect of modal coupling on the radiated acoustic power. An efficient method is also suggested to compute modal acoustic impedance in a moving fluid medium by using averaged Green function.

I. Introduction

Many researchers have focused their attention on the acoustic power radiation from finite plates excited by turbulent boundary layer (TBL) pressure only[1], or by random mechanical forces only in a stationary fluid medium[2], or by time-dependent broadband mechanical forces only in a moving fluid medium[3]. However there have been no attempts to formulate the acoustic radiation from a plate due to the excitations by both flow turbulence and mechanical forces including the effect of moving fluid and modal coupling. This paper presents a simplified analytical formulation for the acoustic power radiation from a rectangular plate in an infinite rigid baffle placed in a subsonic flow field excited by both turbulent boundary layer pressure fluctuations and random mechanical forces. This analysis gives the possibility of reducing the acoustic power level by applying auxiliary mechanical forces. In order to numerically predict the acoustic radiation, precise knowledge both about the force coupling effects between structural mode and the two forcing terms (flow turbulence and mechanical forces) and about the acoustic coupling effects between structural mode and acoustic fluid is very important.

The computation of modal acoustic impedance, which describes the interaction of a structural mode with acoustic medium, has received considerable attention. The analysis is mostly based on the use of an in vacuo modal expansion for the vibration field, acoustic pressure and forcing functions. Many authors[4-7] have studied modal acoustic impedance of a rectangular plate positioned in a stationary fluid medium. Chang and Leehey[8] calculated the modal acoustic impedance in the presence of a moving fluid, using a modified Chebyshev quadrature. In this work, an alternative approach is suggested to efficiently compute the modal coupling impedance in a moving fluid medium, by using averaged Green function in wavenumber domain. The coupling of a structural mode with TBL pressure fluctuations is also studied. Chase model[9,10] was used for the TBL pressure fluctuations. It is known that Chase model more accurately describes low-wavenumber spectrum than Corcos formulation[11] does, even it shows high peaks at the convective wavenumber (high-wavenumber) region. In this study, the formulation and evaluation of auto- and cross-modal coupling coefficients between structural mode and flow turbulence are presented. Cross-modal coupling between modal forcing terms plays an important role in the presence of high structural damping and structural modal degeneracy when fluid loading is light.

Numerical evaluation of the acoustic power is extremely time-consuming for broadband random excitations. In order to obtain an approximate solution, it is assumed that the plate structural damping is low and fluid loading is light. Under these assumptions, the approximate

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solution of the acoustic power can be easily obtained and evaluated by using the Chase model of turbulent wavenumber-frequency spectrum and the types of arbitrary mechanical forces in a moving fluid medium. This approach can be used to design auxiliary mechanical forces to reduce the acoustic power radiation from a plate due to turbulent pressure fluctuations.

II. Theoretical Analysis of Acoustic Power Radiation

2.1. Exact solution

Consider a thin rectangular plate of length l_x and width l_y in an infinite rigid baffle excited by TBL pressure fluctuations and mechanical forces in the presence of a uniform subsonic flow, as shown in Figure 1. The flow is in the region of $z > 0$ and moves toward the positive x-direction. It is assumed that neither the plate vibration nor the acoustic pressure field affect the forcing terms. For simplicity, the effects of the back reaction of the radiated pressure on the plate due to the medium such as air will be neglected and a constant structural loss factor is assumed.

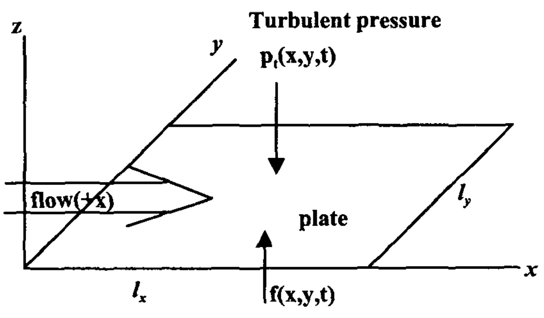


Figure 1. Schematic view of plate model.

The vibration equation for the plate velocity $v(x,y,\omega)$ under the actions of TBL pressure $p_t(x,y,\omega)$ and mechanical force $f(x,y,\omega)$ with the use of $e^{-i\omega t}$ time dependence is

$$D(1-i\eta)\nabla^4 v(x,y,\omega) - \rho_s\omega^2 v(x,y,\omega) = - i\omega[f(x,y,\omega) - p_t(x,y,\omega)] \quad (1)$$

where damping is modeled as a constant structural loss factor η. D is the flexural rigidity, ρ_s is the mass per unit area of the plate, and ω is the circular frequency. If viscous damping is assumed, the damping term will be $c_s v(x,y,\omega)$ in Eq. (1) where c_s is the viscous damping coefficient per unit area.

Expanding the plate velocity, mechanical force and turbulent pressure with the plate eigenfunctions $\varnothing_{mn}(x,y)$ gives

$$v(x,y,\omega) = \sum_{m,n} v_{mn}(\omega)\varnothing_{mn}(x,y,), \quad (2a)$$

$$f(x,y,\omega) = \sum_{m,n} f_{mn}(\omega)\varnothing_{mn}(x,y,), \quad (2b)$$

$$p_t(x,y,\omega) = \sum_{m,n} p^t_{mn}(\omega)\varnothing_{mn}(x,y,), \quad (2c)$$

where $v_{mn}(\omega)$ is the modal velocity amplitude of the (m,n) mode and so on. The orthogonal property of the eigenfunctions can be expressed as

$$\int_0^{l_x}\int_0^{l_y} \varnothing_{mn}(x,y,)\varnothing_{qr}(x,y)dxdy = \delta_{mq}\delta_{nr} \quad (3)$$

where δ_{mq} and δ_{nr} denote the Kronecker delta functions.

After substituting Eq. (2) into Eq. (1) and utilizing the orthonormal properties of the eigenfunctions, the modal equation for the plate velocity is easily obtained as

$$v_{mn}(\omega) = Y_{mn}(\omega)[f_{mn}(\omega) - p^t_{mn}(\omega)] \quad (4)$$

where $Y_{mn}(\omega)$ is the modal mobility function of the plate defined as

$Y_{mn}(\omega) = -i\omega/[\rho_s(\omega^2_{mn} - \omega^2 - i\eta\omega^2_{mn})]$

and ω_{mn} is the (m,n)th undamped natural frequency.

The wave equation for the acoustic pressure $p(x,y,z,t)$ in a uniform flow field is given as

$$\nabla^2 p(x,y,z,t) - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + u_\infty\frac{\partial}{\partial x}\right)^2 p(x,y,z,t) = 0 \quad (5)$$

where c is the sound speed in fluid and u_∞ is the free stream velocity in the x-direction. The acoustic pressure must satisfy the following boundary condition at the outer edge of the boundary layer:

$$\frac{\partial p(x,y,z=\delta,t)}{\partial z} = -\rho\left(\frac{\partial}{\partial t} + u_\infty\frac{\partial}{\partial x}\right)^2 w(x,y,t) \quad (6)$$

where $p(x,y,\delta,t)$ is the acoustic pressure at (x,y,δ), δ is the thickness of the boundary layer, ρ is the density of the acoustic medium and w(x,y,t) is the plate displacement. When the boundary layer is much thinner than one wavelength of the acoustic wave, i.e., $\delta << \lambda$, it can be assumed that $p(x, y, \delta, t) \approx p(x, y, 0, t)$ in Eq. (6). The modal acoustic pressure $p_{mn}(\omega)$ at the plate surface can be defined as

$$p_{mn}(\omega) = \int_0^{ly} \int_0^{lx} p(x, y, 0, \omega) \Phi_{mn}(x, y,) dx dy. \quad (7)$$

After some rearrangement, the relationships between the modal acoustic pressure and the modal velocity can be derived as

$$p_{mn}(\omega) = \rho c \sum_{q, r} z_{mnqr}(\omega) \nu_{qr}(\omega) \quad (8)$$

where $z_{mnqr}(\omega)$ is a modal acoustic impedance, connecting the (m,n) modal acoustic pressure and the (q,r) modal velocity, defined as

$$\begin{aligned} z_{mnqr}(\omega) &= R_{mnqr}(\omega) - iX_{mnqr}(\omega) \\ &= \frac{k}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a^2(k_x, k_y, \omega) \tilde{\Phi}^*_{mn}(k_x, k_y) \\ &\qquad \tilde{\Phi}_{qr}(k_x, k_y) dk_x dk_y. \end{aligned} \quad (9)$$

where $k = \omega / c$ is the acoustic wavenumber, $a(k_x) = 1 - Mk_x / k$, $M = u_\infty / c$ is the Mach number, and $\tilde{G}(k_x, k_y, \omega)$ is Green function[8] in a moving acoustic medium defined as

$$\tilde{G}(k_x, k_y, \omega) = [k^2 - (1 - M^2)k_x^2 - 2kMk_x - k_y^2]^{-1/2}$$

Also, in Eq. (9),

$$\tilde{\Phi}_{qr}(k_x, k_y) = \int_0^{l_y} \int_0^{l_x} \Phi_{qr}(x, y) e^{-i(k_x x + k_y y)} dx dy \quad (10)$$

is the shape function for the (q, r) mode $\tilde{\Phi}^*_{mn}(k_x, k_y)$ is the complex conjugate of $\tilde{\Phi}_{mn}(k_x, k_y)$

If the modal pressure is expressed in terms of the modal velocities, the acoustic power can be easily calculated. The acoustic power radiated from a plate under the action of stationary random forces is given by

$$\begin{aligned} W &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_0^{l_y} \int_0^{l_x} I(x, y, \omega) dx dy d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_0^{l_y} \int_0^{l_x} Re\Big\{ \lim_{T \to \infty} \frac{1}{T} p(x, y, \omega, T) \nu^* \\ &\qquad (x, y, \omega, T) \Big\} dx dy d\omega \end{aligned} \quad (11)$$

where $p(x, y, \omega, T)$ and $\nu^*(x, y, \omega, T)$ are the T-truncated ensembles of the surface acoustic pressure and velocity and T is the time interval of the stationary random process. Substituting Eqs. (2), (4) and (8) into Eq. (11) and utilizing the orthonormal properties of the eigenfunctions, the exact expression for the acoustic power is found to be

$$W = \sum_{m, n} \sum_{q, r} W_{mnqr} \quad , \quad (12)$$

$$\begin{aligned} W_{mnqr} &= \frac{\rho c}{2\pi} Re\Big\{ \int_0^{\infty} z_{mnqr}(\omega) T^*_{mn}(\omega) Y_{qr}(\omega) \times \\ &\qquad [F_{qrmn}(\omega) + T_{qrmn}(\omega) - C_{qrmn}(\omega) - C^*_{mnqr}(\omega)] d\omega \Big\} \end{aligned} \quad (13)$$

where $F_{qrmn}(\omega) - \lim_{T \to \infty} (2/T) f_{qr}(\omega, T) f^*_{mn}(\omega, T)$, $T_{qrmn}(\omega) - \lim_{T \to \infty} (2/T) p^t_{qr}(\omega, T) p^{t*}_{mn}(\omega, T)$, and $C_{qrmn}(\omega) - \lim_{T \to \infty} (2/T) f_{qr}(\omega, T) p^{t*}_{mn}(\omega, T)$. Here, $F_{qrmn}(\omega)$ is the one-sided modal spectral density function of the mechanical forces between (q,r) and (m,n) modes, $T_{qrmn}(\omega)$ is the modal spectral density function of turbulent pressure fields, and $C_{qrmn}(\omega)$ is the modal coupling forcing term between the mechanical force and the turbulent pressure. Notice that the integration range has been changed from 0 to positive infinity because of the definitions of one-sided spectra of the forcing functions.

It is assumed that the force fields are stationary and statistically homogeneous. Then, $F_{qrmn}(\omega)$ is easily obtained. $T_{qrmn}(\omega)$ can be calculated using the definitions of the wavenumber-frequency Fourier transformation, given in Eq. (10), as follows:

$$\begin{aligned} T_{qrmn}(\omega) &= \lim_{T \to \infty} \frac{2}{T} \int_A p_t(x, y, \omega, T) \Phi_{qr}(x, y,) dx dy \\ &\qquad \int_A p_t^*(x, y, \omega, T) \Phi_{mn}(x, t,) dx dy \\ &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{S}_p(k_x, k_y, \omega) \tilde{\Phi}^*_{qr}(k_x, k_y) \\ &\qquad \tilde{\Phi}_{mn}(k_x, k_y) dk_x dk_y \end{aligned} \quad (14)$$

where $\tilde{S}_p(k_x, k_y, \omega)$, is the wavenumber-frequency spectral density function of the TBL pressure field. However, it is difficult to evaluate the coupling forcing terms $C_{qrmn}(\omega)$, because the modal turbulent pressure $p^t_{mn}(\omega)$ can not be defined analytically and the wavenumber-frequency spectrum $\tilde{S}_p(k_x, k_y, \omega)$ of TBL pressure field can be obtained only from experimental observations.

In order to obtain the relationship between the spectra of the mechanical force and the turbulent pressure, only the acoustic response well above convective-coincidence frequency $(\omega_{cc}=u_c^2(\rho_s/D)^{1/2})$ for turbulent excitation is assumed to be of interest. Then the wavenumber spectrum of $\tilde{S}_p(k_x,k_y,\omega)$ becomes sufficiently smooth over the interested range of $\tilde{\Phi}_{qr}(k_x,k_y)$ and $\tilde{\Phi}_{mn}(k_x,k_y)$. In such cases (see Refs. 1 and 4), the cross terms are negligible and Eq. (14) can be reduced to

$$T_{qrmn}(\omega)=T_{mnmn}(\omega)\delta_{mq}\delta_{nr}\ . \tag{15}$$

One can assume that there is some relationship between the mechanical force and the turbulent force as follows:

$$\frac{f_{qr}}{p_{mn}^t}=Ge^{-i\Delta\Phi}\quad \text{with}\quad G=\left(\frac{F_{qrqr}}{T_{mnmn}}\right)^{1/2}$$

where $\Delta\Phi$ is the phase difference between the turbulent pressure and mechanical force. After multiplying the above equation by $p_{mn}^t p_{mn}^{t*}$ and taking the expected value, it then follows that the coupling forcing terms can be expressed as

$$C_{qrmn}=\left(\frac{F_{qrqr}}{T_{mnmn}}\right)^{1/2}\lim_{T\to\infty}\frac{2}{T}p_{mn}^t p_{mn}^{t*}e^{-i\Delta\Phi} = (F_{qrqr}T_{mnmn})^{1/2}e^{-i\Delta\Phi} \tag{16}$$

This relationship is very important to have a feedback control system in the fields of random vibration and acoustic radiation.

Numerical evaluation of the acoustic power, that is, the evaluation of W_{mnqr} in Eq. (13), is extremely time-consuming for broadband random excitations. In the following section, an approximate method to derive W_{mnqr} is developed. The idea is similar to the one used by Crandall[12] in his study on the problems of random vibration.

2.2. Approximate solution

In order to obtain an approximation of Eq. (13), the following assumptions, which are used in the asymptotic modal analysis (AMA)[13] and the statistical energy analysis (SEA)[14] of dynamic systems, are made in this work.

The first assumption is that the structural damping of the plate is low enough to separate the adjacent vibrational modes in some degree. This assumption does not always mean omitting of all terms in the fourfold modal summation except those for which $m=q$ and $n=r$. However, for a plate under light fluid loading (such as air) and resonant excitation, the effect of modal coupling on acoustic power radiation is not important, as already pointed out by Keltie and Peng[15]. The second assumption is the white noise-like excitation with frequency band $\Delta\omega=\omega_2-\omega_1$, which implies that the spectral density functions of the forcing functions, $F_{qrmn}(\omega)$, $T_{qrmn}(\omega)$ and $C_{qrmn}(\omega)$, are slowly varying with respect to frequency. And the modal acoustic impedance $z_{mnqr}(\omega)$ is slowly varying with respect to frequency related to rapidly varying modal mobility function $Y_{mn}(\omega)$ especially near $\omega=\omega_{mn}$.

With these assumptions, the diagonal terms of the inteasgral W_{mnqr}, where $q=m$ and $r=n$, can be written as

$$\begin{aligned} W_{mnmn}^a &\approx \frac{\rho c}{2\pi}\int_{\omega_1}^{\omega_2}|Y_{mn}(\omega)|^2 d\omega\times Re\{z_{mnmn}(\omega_{mn}) \\ &\quad [F_{mnmn}(\omega_{mn})+T_{mnmn}(\omega_{mn})-C_{mnmn}(\omega_{mn}) \\ &\quad -C_{mnmn}^*(\omega_{mn})]\} \\ &\approx \frac{\rho c R_{mnmn}(\omega_{mn})F_a(\omega_{mn})}{4\rho_s^2\eta\omega_{mn}} \end{aligned} \tag{17a}$$

where $R_{mnmn}(\omega_{mn})$ is the modal radiation resistance at resonant frequency ω_{mn}, $\Delta\Phi(\omega_{mn})$ is the phase difference between the two forcing functions, $F_a(\omega_{mn})$ is the combined force spectrum at frequency ω_{mn} defined as

$$F_a=F_{mnmn}(\omega_{mn})+T_{mnmn}(\omega_{mn}) -2\sqrt{F_{mnmn}(\omega_{mn})T_{mnmn}(\omega_{mn})}\cos\Delta\Phi(\omega_{mn})$$

and the result of

$$\int_{\omega_1}^{\omega_2}|Y_{mn}(\omega)|^2 d\omega\approx\int_0^\infty|Y_{mn}(\omega)|^2 d\omega=\pi/2\rho_s^2\eta\omega_{mn}$$

was used.

For a square plate with the same edge conditions along all edges, the modal degeneracy term W_{mnnm}, wher $m\neq n$, must also be considered due to the effect of modal degeneracy:

$$W_{mnnm}^b\approx\frac{\rho c}{4\rho_s^2\eta\omega_{mn}}Re\{z_{mnmn}(\omega_{mn})F_b(\omega_{mn})\} \tag{17b}$$

where

$$F_b=F_{mnnm}+T_{nmnm}-\sqrt{F_{nmnm}T_{mnmn}}\,e^{-i\Delta\Phi} -\sqrt{F_{mnmn}T_{nmnm}}\,e^{i\Delta\Phi} \tag{17b}$$

In this case, $\varnothing_{mn}$ and $\varnothing_{nm}(m \neq n)$ have the same resonant frequency ω_{mn} .

The modal coupling terms can be also obtained by repeating almost similar procedures as done with diagonal terms. The coupling terms of the integral W_{mnqr}, representing modal cross-correlation arising from the elastic modes of $\omega_{qr} \neq \omega_{mn}$, can be written as

$$\hat{W}^{c}_{mnqr} \approx \frac{\rho c}{2\pi} Re\Big\{ \int_{\omega_1}^{\omega_2} Y^{*}_{mn}(\omega) d\omega \cdot z_{mnqr}(\omega_{mn}) Y_{qr}(\omega_{mn}) [F_{qrmn} + T_{qrmn} - C_{qrmn} - C^{*}_{mnqr}]\Big\}$$
$$= \frac{\rho c}{2\pi \rho_s} Re\{ z_{mnqr}(\omega_{mn}) Y_{qr}(\omega_{mn}) F_c(\omega_{mn}) [I(\omega_2/\omega_{mn}, \eta) - I(\omega_1/\omega_{mn}, \eta)]\} \quad (17c)$$

where the integral factor $I(\omega/\omega_{mn}, \eta)$ is

$$I(\omega/\omega_{mn}, \eta) = \frac{1}{2}\tan\left\{ \frac{(\omega/\omega_{mn})^2 - 1}{\eta} \right\} - i\frac{1}{4}\ln\left\{ \frac{(\omega/\omega_{mn})^4 - (\omega/\omega_{mn})^2 + 1 + \eta^2}{\eta^2} \right\}$$

and $F_c = F_{qrmn} + T_{qrmn} - (F_{qrqr} T_{mnmn})^{1/2} e^{-i\Delta\varnothing} - (F_{mnmn} T_{qrqr})^{1/2} e^{i\Delta\varnothing}$

From Eqs. (17b) and (17c), it can be found that the imaginary parts of the acoustic impedance, X_{mnqr}, can contribute to the acoustic power radiation due to the effects of modal coupling. It is known that X_{mnqr} lead to virtual mass terms to be added to the plate mass. However, according to Keltie and Peng, for a plate under light fluid loading and resonant excitation, the coupling contribution caused by the interactions of elastic modes is negligibly small. Broadband random excitation means that many modes can be excited within the given frequency band. Therefore it can be shown that these coupling terms can produce little contribution to the radiated acoustic power

Substituting Eqs. (17) into Eq. (12), the approximate solution for the acoustic power including modal coupling terms becomes

$$\hat{W} = \hat{W}_1 + \hat{W}_2 + \hat{W}_3 = \sum_{m,n}^{\Delta\omega} \hat{W}^{a}_{mnmn} + \sum_{\substack{m,n \\ \omega_{qr} \neq \omega_{mn}}}^{\Delta\omega} \hat{W}^{b}_{mnmn} + \sum_{m,n}^{\Delta\omega} \sum_{q,r}^{\Delta\omega} \hat{W}^{c}_{mnqr} \quad (18)$$

where $\hat{W}_1$ is the acoustic power produced by the diagonal terms, $\hat{W}_2$ is the acoustic power produced by the degeneracy terms and $\hat{W}_3$ is the acoustic power produced by the coupling terms. The summation over the mode numbers m and n (or q and r) is confined within the range of excitation frequency, $\omega_1 < \omega_{mn} < \omega_2$. It is noted that the property of diagonal symmetry of the acoustic impedance, $z_{mnqr} = z_{qrmn}$, is used in evaluating the acoustic power.

For comparison, the approximate acoustic power obtained by using Rayleigh integral in a stationary fluid medium[2] is also calculated without considering the coupling terms as follows:

$$\hat{W} = \hat{W}_1 + \hat{W}_2 \quad (19)$$

where

$$\hat{W}_1 = \frac{\rho}{4\rho_s^2 \eta} \sum_{m,n}^{\Delta\omega} F_a(\omega_{mn}) \int_A \varnothing_{mn}(x, y) Re\{F^{-1}\{\tilde{\varnothing}_{mn}(k_x, k_y) \tilde{G}(k_x, k_y, \omega)\}\} dx dy,$$

$$\hat{W}_2 = \frac{\rho}{4\rho_s^2 \eta} \sum_{m,n}^{\Delta\omega} F_b(\omega_{mn}) \int_A \varnothing_{mn}(x, y) Re\{F^{-1}\{\tilde{\varnothing}_{mn}(k_x, k_y) \tilde{G}(k_x, k_y, \omega)\}\} dx dy.$$

Here F^{-1} denotes the inverse wavenumber Fourier transform. Green function in a stationary fluid medium is given as

$$\tilde{G}(k_x, k_y, \omega) = \begin{pmatrix} (\sqrt{k^2 - k_x^2 - k_y^2})^{-1}, k^2 \geq k_x^2 + k_y^2 \\ (i\sqrt{k_x^2 + k_y^2 - k^2})^{-1}, k^2 < k_x^2 + k_y^2 \end{pmatrix}$$

III. Modal Coupling due to Moving Fluid and Random Forces

3.1. Modal Coupling Impedance in a Moving Fluid Medium

The integral for the modal coupling impedance $z_{mnqr}(\omega)$ was expressed in Eq. (9). The analysis is limited to a simply supported rectangular plate but can be expanded to a plate with arbitrary boundary conditions. The natural frequencies and mode shapes for a simply supported rectangular plate are given as

$$\omega_{mn} = (D/\rho_s)^{1/2}(k_m^2 + k_n^2) = (D/\rho_s)^{1/2} k_{mn}^2$$
$$\varnothing_{mn}(x, y) = 2A^{-1/2} \sin k_m x \sin k_n y \quad (20)$$

where $k_m = m\pi/l_x$, $k_n = n\pi/l_y$ and $A = l_x l_y$. the modal wavenumber shape function for a simply supported rectangular plate is given as

$$\tilde{\varnothing}_{mn}(k_x, k_y) = 2k_m k_n A^{-1/2} \frac{[1-(-1)^m e^{-ik_x l_x}][1-(-1)^n e^{-ik_y l_y}]}{(k_m^2 - k_x^2)(k_n^2 - k_y^2)} \quad (21)$$

In a stationary fluid medium (M = 0), the constant frequency locus of the acoustic and structural wavenumber plots a circle with radius k and centered at the origin of the wavenumber space. But for a subsonic flow ($0<M<1$), the constant frequency locus plots an ellipse as shown in Figure 2. For subsonic flow, the radiation behavior grows from a circle at M = 0 to a ellipse in the regions of $0<M<1$. Thus some acoustically slow modes in the negative wavenumber region at M = 0 can be strongly radiant due to flow effect and some acoustically fast modes in the positive wavenumber region at M = 0 can, in some degree, be weekly radiant due to flow effect.

Previous papers[4-7] have studied the case of a stationary acoustic medium. Chang and Leehey[8] have developed the new integration scheme for the modal coupling impedance in a subsonic flow using a unique Gaussian quadrature with reusable abscissas.

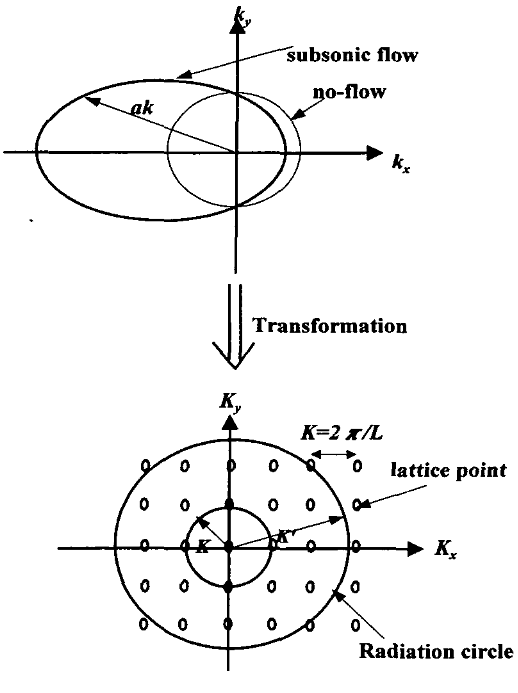


Figure 2. Constant frequency loci in original (k) and transformed (K) wavenumber space with lattice points.

In this paper, an efficient method is suggested to compute the modal coupling impedance in a moving fluid medium. The only singularity in the integrands of Eq. (9) is the square root singularity when $\tilde{G}^{-1}=(a^2k^2-k_x^2-k_y^2)^{1/2}=0$. An averaged Green function developed by Williams and Maynard[16] is used. To apply the above technique, a radiation ellipse must be transformed into a circle using the following transformation:

$$K=k/\sqrt{1-M^2};\ K_x=\sqrt{1-M^2}\,k_x+MK;\ K_y=k_y.$$

With some arrangements and using the averaged Green function, modal coupling impedance can be expressed as the following summation form:

$$z_{mnqr}(\omega)=\frac{K(\Delta K)^2}{\pi^2}\sum_{t=0}^{t_{max}}\sum_{s=-s_{max}}^{s_{max}} a^2(s)\tilde{\Phi}^*_{mn}(s,t)\,\tilde{\Phi}_{qr}(s,t)\,\tilde{G}(s,t) \qquad (22)$$

where $\tilde{\Phi}_{mn}(s,t)$ is mode sensitivity function at $(s\Delta K, t\Delta K)$ discretized in the transformed wavenumber space (K_x, K_y). Here

$$a(s)=1-\frac{M}{1-M^2}\left(\frac{s\Delta K}{K}-M\right) \quad \text{and}$$

$$\bar{\tilde{G}}(s,t)=\frac{2[(K^2-K_1^2)^{1/2}-(K^2-K_2^2)^{1/2}]}{K_2^2-K_1^2}$$

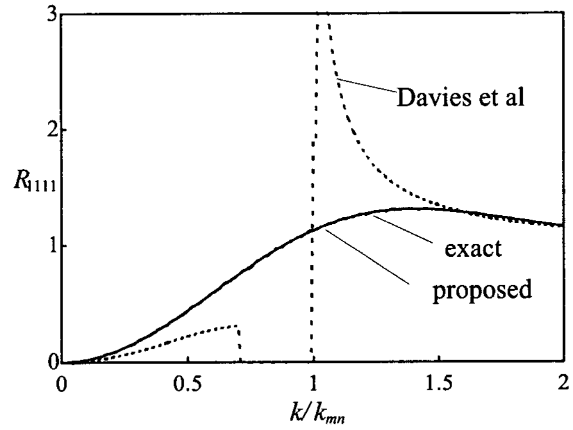
where

$$K_1=K_0-\Delta K/2,\ K_2=K_0+\Delta K/2,$$
$$K_0=\sqrt{s^2+t^2}\,\Delta K,\ \Delta K=2\pi/L$$

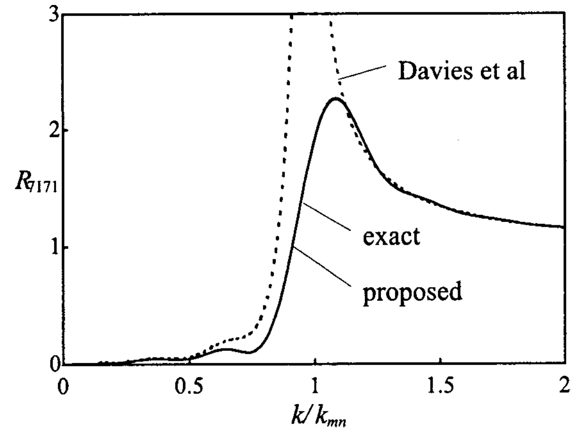
and L is the aperture length.

To obtain an accurate approximation for Eq. (22), the increment ΔK and the limit of summation, s_{max} and t_{max}, must be defined. When $K=\Delta K=2\pi/L$, only one lattice point in wavenumber space falls within the radiation circle (Figure 2). The accuracy will be improved by increasing the number of the lattice point within K, where K equals $N\Delta K$. Then the required aperture length will be $L=2\pi/\Delta K=2\pi Nc\sqrt{1-M^2}/\omega$ and that must be larger than the size of the plate. The limit of summation, $K_{max}=s_{max}\Delta K$, must be greater than the maximum of $K_{mn}(=[(k_m\sqrt{1-M^2}+Mk/\sqrt{1-M^2})^2+k_n^2]^{1/2})$, K_{qr} and K. $K_{max}=2\max(k_{mn}, K_{qr}, K)$ and N = 50 is recommended in this paper. Then the limit of summation, s_{max} or t_{max}, becomes $K_{max}/\Delta K$.

Figures 3 and 4 show the computed modal radiation impedance, z_{1111} and z_{7171}, for a simply supported square plate. As shown in Figure 3, the results for the modal radiation resistance computed by using the proposed method coincide with the exact values computed by Wallace[6] and those by Chang and Leehey[8]. Also it can be noticed from Figure 3 that the computations by Davies[4] and Leibowitz[5] using delta function approximations are very poor, except in the regions of very low and very high frequencies.



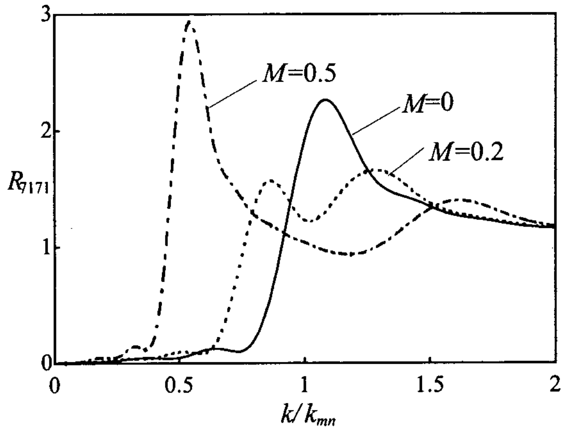
(a) (1,1) mode



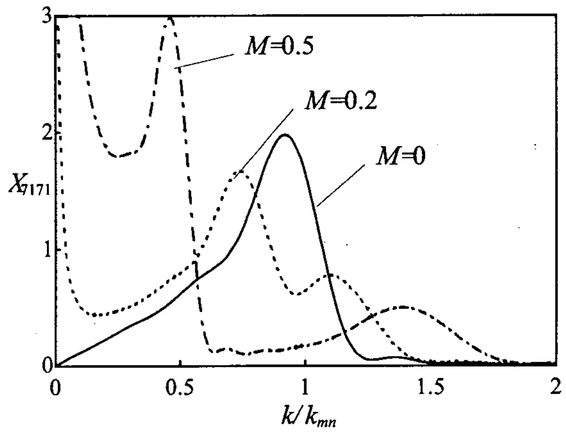
(b) (7,1) mode

Figure 3. The modal radiation resistance for a square plate at $M = 0$.

As shown in Figure 4, with increasing flow speed, the frequency showing the peaks of modal radiation impedance moves toward the low frequency while the peak values increase. This is due to the fact that some acoustically slow modes can become strongly radiant due to flow effect. At high frequency, the resistances approach unity and the reactances approach zero. Unlike the high-frequency results, the low-frequency data depend more upon Mach number. In contrast to the radiation resistances, the radiation reactances show a somewhat unexpected behavior at low-frequency region. Essentially the reactance becomes very large for nonzero Mach number. Chang and Leehey have also observed that phenomenon about the radiation reactances. They observed that subsonic flow has a greater effect on radiation reactance (e.g. added mass) than on radiation resistance (e.g. radiation damping). Although the effect of mode order number is not shown here, the effect of flow on radiation impedance increases as the order of the mode (m,n) increases.



(a) Radiation resistance



(b) Radiation reactance

Figure 4. The modal radiation impedance of (7,1) mode for a square plate at various Mach numbers.

3.2. Coupling of a Structural Mode with Turbulence

In order to evaluate the acoustic radiation in a moving fluid medium, one needs to know $T_{mnqr}(\omega)$, the coefficients of the modal coupling between a structural mode and the TBL pressure field. Modeling of a turbulent wall pressure spectrum acting on the plate as a forcing term has been a research subject for many years, but there is no overall agreement among these models regarding the spectrum in the low-wavenumber region. In this study, the Chase model[9,10] is chosen. The wavenumber-frequency spectrum suggested by Chase is

$$\hat{S}_p(k_x, k_y, \omega) = (2\pi)^3 \rho^2 v_*^3 [c_M k_x^2 K_M^{-5} + c_T (k_x^2 + k_y^2) K_T^{-5}] \tag{23}$$

where

$$K_i^2 = \left(\frac{\omega - u_c k_x}{h\nu_*}\right)^2 + k_x^2 + k_y^2 + (b_i \delta)^{-2},\ i = M,\ T,$$

$u_c = 0.6u_\infty$ is the convective velocity, $\nu_* = 0.035u_\infty$ is the friction, $\delta = 6\delta_*$ is the boundary layer layer thickness, δ_* is the displacement thickness (It is selected in this work as approximately 4.2mm in aeronautical environments when Mach number number is in the range of from 0.3 to 0.7), $h = 3$, $c_T h = 0.014$, $c_M h = 0.466$, $b_T = 0.378$ and $b_M = 0.756$. In figure 5. the typical TBL spectra of Chase model and Corcos modal are shown. Also the modao wavenumber sensitivity function $|\tilde{\Phi}_{mn}(k_x, k_y)|^2$ are superimposed on the plot of TBL spectrum $\tilde{S}_p(k_x, k_y, \omega)/S_p(\omega)$ where $S_p(\omega)$ is the point pressure spectrum per $\sqrt{Hz}$ given as $S_p(\omega) = 2\pi a_+ \rho^2 \nu_*^4 \omega^{-1}[r_M \alpha_M^{-3}(1+\mu^2\alpha_M^2) + r_T \alpha_T^{-3}(1+\alpha_T^2)]$

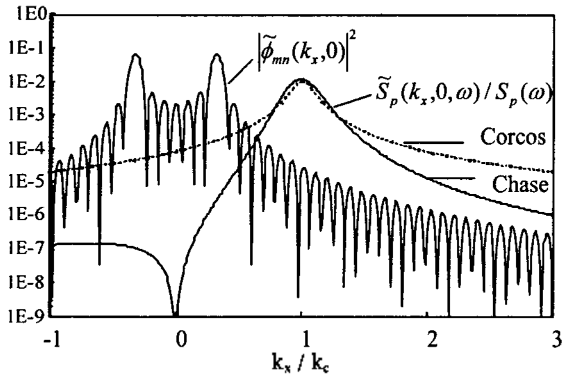


Figure 5. Typical characteristics of wavenumber sensitivity function $|\tilde{\Phi}_{mn}(k_x, 0)|^2$ as superimposed on the wavenumber spectrum $\tilde{S}_p(k_x, 0, \omega)/S_p(\omega)$ for (7, 1) mode of a square plate and St = $\omega_{mn}\delta_*/u_\infty$ = 0.2.

where $\alpha_i^2 = 1 + (k_c b_i \delta)^{-2}$, $i = M, T$, $k_c = \omega/u_c$ is the convective wavenumber, r_T=0.389, r_M=1-r_T, a_+=0.766. and μ=0.176. The mode sensitivity function of a plate has a high peak at modal wavenumber k_{mn} . Since the structural wave speed and the flow speed are usually of much closer orders of magnitude in most aeronautical environments, the full wavenumber spectrum of the TBL function can excite the structural mode. Therefore the coupling of a structural mode and TBL pressure field should be computed after considering the effect of full wavenumber region. Using Chase model, one can estimate the modal coupling coefficients $T_{qrmn}(\omega)$ using two-dimensional Gaussian quadrature. The normalized coupling coefficients, $J_{mnqr}(\omega)$, is defined as

$$J_{mnqr}(\omega) = \frac{T_{mnqr}(\omega)}{S_p(\omega)A^2} \qquad (24)$$

In Figure 6. the coupling coefficients for a simply supported rectangular plate at various mode order numbers (m,n) are shown using both Chase model and Corcos model, for comparison purpose. The comparison says that the contribution to the coupling by the low-wavenumber region becomes dominant in aeronautical environments as frequency increases and flow speed decreases. The coupling due to model degeneracy is not significant. This fact implies that the cross terms T_{qrmn}, where $q \neq m$ or $r \neq n$, can be neglected (see Eq.(15)).

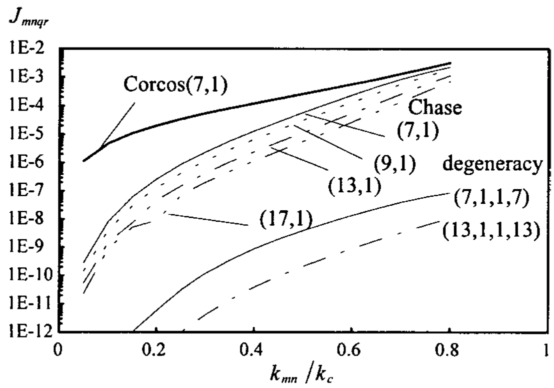


Figure 6. The coupling by TBL wavenumber spectrum for a square plate at various mode order numbers.

3.3. Coupling of Structural Mode with Mechanical Forces

Mechanical point forces are considered in this work. For stationary random multiple point forces, the modal force $f_{mn}(\omega)$ is given as

$$f_{mn}(\omega) = \int_A f(x, y, \omega)\Phi_{mn}(x, y)dxdy = \sum_{\alpha=1}^{N_f} F_\alpha(\omega)\Phi_{mn}(x_\alpha, y_\alpha) \qquad (25)$$

where $F_\alpha(\omega)$ is the α-th point force spectrum and N_f is the number of input point forces. Thus $F_{mnqr}(\omega)$ is found to be

$$F_{mnqr}(\omega) = \sum_{\alpha,\beta}\Phi_{mn}(x_\alpha, y_\alpha)\Phi_{qr}(x_\beta, y_\beta)S_{\alpha\beta}(\omega) \qquad (26)$$

where $S_{\alpha\beta}(\omega)$ is the one-sided spectral density function of the α-th and β-th point forces defined by $S_{\alpha\beta}(\omega) = \lim_{T\to\infty}\frac{2}{T}F_\alpha(\omega, T)F_\beta^*(\omega, T)$. In case of multiple point forces possessing identical bandlimited white noise spectra, the spectral density can be written as

$$S_{\alpha\alpha}(\omega)=\{\ S_f(\omega),\omega_1<\omega<\omega_2$$
$$0, otherwise$$
$$S_{\alpha\beta}(\omega)_{\alpha\neq\beta}=\{\ rS_f(\omega),\omega_1<\omega<\omega_2$$
$$0, otherwise$$

where r is the mechanical forces' correlation coefficient $(-1\leq r\leq 1)$. Using the same procedure as the case of the point forces, the modal force and the coupling coefficient of the multiple area forces can be easily obtained.

IV. Numerical Results of Acoustic Power Radiation

Several numerical problems were tested to illustrate the nature of the sound radiation from a plate excited by turbulence or mechanical forces. A square plate with dimensions of $l_x=l_y-0.8$m, thickness of $h=3$mm, structural loss factor $\eta=0.004$, mass density of 7700kg /m^3, poisson's ratio of ν-0.28, and Young's modulus of E-$1.95\times10^{11}N/m^2$, was prepared. The plate is simply supported and excited in air medium, that has density of ρ-1.21kg/m^3 and sound speed of c-343m/sec. The acoustic-coincidence frequency ω_{ac} is approximately 4kHz in a stationary fluid medium.

Table 1. Center frequencies and numbers of excited modes of one-third octave excitation band.

center frequency (Hz)	wavenumber ratio (k/k_b)	number of excited modes
500	0.349	7
630	0.391	8
800	0.439	11
1000	0.492	17
1250	0.553	20
1600	0.620	23
2000	0.696	32
2500	0.780	39
3150	0.876	49
4000	0.983	62

Approximate solutions for acoustic power integrated over one third octave band $\Delta\omega$ are presented. The center frequencies and the number of excited modes are summarized in Table 1. All the predictions were done in the frequencies above the convective-coincidence frequency ω_{cc} and below the acoustic-coincidence frequency ω_{cc}. When $k_c/k_b\approx1$, convective coincidence occurs at that frequency, where $k_b=(\rho_s\omega^2/D)^{1/4}$ is the structural free wavenumber at the center frequency of the one third octave band. The relationship between ω_{cc} and ω_{ac} is given as

$$\frac{\omega_{cc}}{\omega_{ac}}=(\frac{u_c}{c})^2\approx0.36M^2 \tag{28}$$

where M is the Mach number. When M=0.5 and $\omega_{ac}=4KHz$, ω_{cc} becomes 360Hz. In order to illustrate the effect of flow speed on the radiation characteristics, the results for radiated acoustic power and frequency-average radiation efficiency are presented at various flow speeds. Frequency-average radiation efficiency can be obtained as

$$\hat{\sigma}=\rho_s\hat{W}/\rho c\hat{E}_v \tag{29}$$

where the approximate solution for the vibration energy is given by

$$\hat{E}_v=\sum_{m,n}^{\Delta\omega}F_a(\omega_{mn})/\rho_s\eta\omega_{mn}\ .$$

For comparison, the average radiation efficiency obtained by using both Rayleigh integral and modal expansion technique in a stationary fluid medium (in that case $T_{qrmn}(\omega)=0$) are plotted in Figure 7. Both results are very much identical as expected.

In Figure 8, the acoustic powers and the average radiation efficiencies are plotted when a mechanical point force with white noise spectrum excites at the center point. The acoustic power caused by the interactions of elastic modes is negligibly small within the given band (coupling power/total power<10^{-5} in most cases), although it is not shown here. Figure 8 shows that moving fluid has a great effect on the characteristics of acoustic radiation. As flow speed increases, the acoustic-coincidence frequency decreases. When M=0.5, the acoustic-coincidence frequency occurs at about $k/k_b\approx0.5$, due to the effect of moving fluid. As expected, it is because the peaks of the radiation resistance move toward the lower frequency as flow speed increases.

In Figure 9, the acoustic powers as varying the flow speed are plotted when the plate is excited by turbulent pressure field. That is the case when there are no mechanical forces $(F_{qrmn}(\omega)=0)$. At frequencies well below acoustic coincidence, the acoustic power is nearly constant. At frequencies near acoustic coincidence, a resonant type peak is observed ($k/k_b \approx 0.5$ at $M = 0.5$ and $k/k_b \approx 0.6$ at $M = 0.4$). In contrast to the case of mechanical excitation, the effect of flow speed on the radiation characteristics is not significant at frequencies well above convective coincidence. This means that as flow speed increases the acoustic power uniformly increases independent of frequency. The reason is that some acoustically slow modes can become strongly radiant due to flow effect but the amplitude of the TBL wavenumber spectrum in low-wavenumber region is so small that the product of the radiation resistance R_{mnmn} and the turbulent forcing function T_{mnmn} does not much increase due to the flow effect, as compared to the case of mechanical excitation F_{mnmn} with white-noise spectrum.

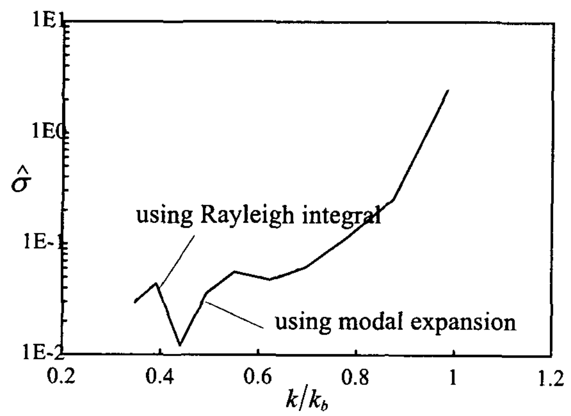
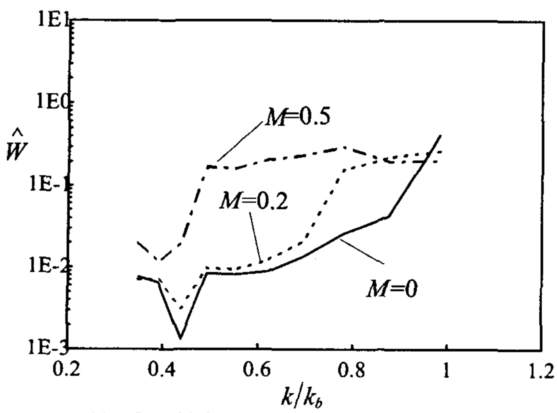
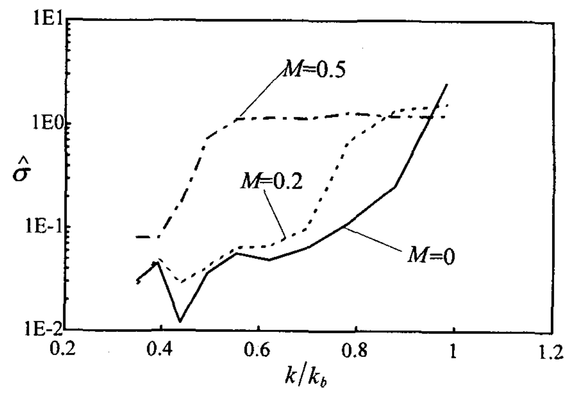


Figure 7. Comparison of radiation efficiencies obtained by using Rayleigh integral and using modal expansion at $M = 0$.



(a) One-third octave band acoustic powers



(b) Average radiation efficiencies

Figure 8. Acoustic powers and radiation efficiencies due to Mach number for point force excitation at the plate center ($T_{qrmn}(\omega) = 0$).

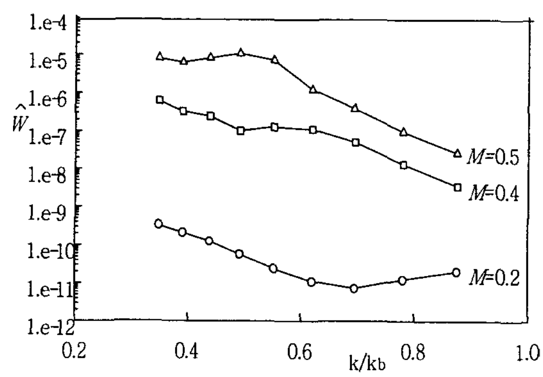


Figure 9. One-third octave band acoustic powers due to Mach number for turbulent excitation ($F_{qrmn}(\omega) = 0$).

V. Conclusion

A theoretical formulation for computing acoustic power radiation from a rectangular plate in an infinite rigid baffle exposed to flow turbulence and mechanical forces in a uniform subsonic flow has been presented. The approximate solution for the acoustic power radiated from a plate excited by random forces was obtained assuming light fluid loading on the plate and low structural damping. In order to numerically predict acoustic radiation, the modal acoustic impedance, $z_{mnqr}(\omega)$, between a structural mode and acoustic pressure and the modal coupling coefficients, $T_{qrmn}(\omega)$, between a structural mode and flow turbulence were evaluated in a subsonic flow field. An efficient method to evaluate the modal acoustic impedance was suggested by using averaged Green function.

Numerical predictions for the acoustic power radiation from a simply supported rectangular plate excited by turbulent pressure field and random mechanical forces were made in a subsonic flow field. It was shown that moving fluid gives more significant effect on radiation characteristics due to mechanical force excitation than due to turbulent excitation. For a broadband excitation which means that many modes are involved in plate response, the reasonable solution for the acoustic power radiated from a plate with random forces such as flow turbulence and mechanical forces in a subsonic flow field was obtained by considering the resonant modes only.

Further studies can be followed to reduce the acoustic power radiation from a plate due to turbulent pressure fluctuations. The simplified formulation can be used to design mechanical input forces to reduce the acoustic power radiation from a plate. In practical situation, it will be difficult to obtain the relationship between input forces and flow turbulence.

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