Effective Medium Theory for Particulate Composite Materials Carrying Elastic Waves

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Abstract

In this paper, two major theories of effective medium for composite materials are studied, which provide the average dynamic mechanical properties of particulate composites carrying elastic waves on the basis of the concept of self-consistent scattering of waves. These two theories are also compared in detail with each other to delineate the difference in the underlying ideas and their limitation of applicability. Furthermore, prospective directions for potential improvement of the theories are found. Numerical results for three particulate composites are given and discussed.

I. Introduction

There may be found a great number of various composite materials in engineering usage these days. Many of them are subjected to dynamic loading, where the composite materials behave dynamically, e.g. they undergo phenomena of waves and vibration. In general, when materials exhibit dynamic behaviors, their mechanical properties deviate from the corresponding static values. Therefore, the dynamic material properties of composites must be known for accurate analyses of problems involving dynamic behaviors.

There exists another obviously important case where the dynamic behavior of composite materials must be known: namely, ultrasonic nondestructive testing. As the use of composite materials in load-bearing applications increases, it becomes necessary to inspect the material to warrant its structural integrity. Ultrasonic testing is a particularly effective method for nondestructive evaluation of composite materials. However, the physical phenomena occurring when ultrasonic waves propagate in the specimen of a composite material are much more complicated than in homogeneous materials, due to the multiple scattering from the many randomly distributed reinforcements. In particular, waves scattered from reinforcements are superposed upon the waves scattered from the defects to be detected, which makes it difficult to extract useful information from the received signal. Therefore, it is necessary to know a priori the effects of the multiple scattering from reinforcements, so that they may be taken into account in identifying the characteristics related only to the target defects.

Motivated by the practical reasons discussed above, this paper is concerned with the theories that model the dynamic behavior of composite materials by considering wave phenomena occurring therein. Though these theories may be applied equally well to both fibrous and particulate composites, focus will be on the particulate composite materials in this paper, particularly for the numerical computation part. When a plane elastic wave is introduced into a composite material, there may form a coherent plane wave as a combined effect of multiple scattering from the randomly distributed reinforcements, under certain circumstances. Then, the real and imaginary parts of the (generally complex) wavenumber of this coherent wave are related to the wavespeed and attenuation, respectively, of the so-called effective or average medium. That is, the composite material consisting of many randomly distributed reinforcements behaves as a homogeneous medium from the perspective of this coherent wave. Mechanical properties of the effective medium are of course related to the complex wavenumber. This paper is concerned with the computation of the mechanical properties of the effective medium, rather than the coherent wavenumbers.

There are various theories to calculate the mechanical properties of the effective medium: see, for example, References [1-7]. Mal and Knopoff[1] computed the

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speeds of the statistically averaged waves in composites with spherical inclusions, using the Green's function approach, in the long wavelength limit. Varadan et al.[2] used the T-matrix approach for the multiple scattering of elastic waves in a fiber composite, and found the elastic properties of the corresponding effective medium, again in the limit of long wavelength. Similarly, Datta et al.[3] calculated the effective elastic properties of composites reinforced with transversely isotropic fibers. Also, Devaney[4] and Devaney and Levine[5] developed a self-consistent scheme for the multiple scattering phenomena, from which the elastic constants of the effective medium may be computed. Further, an approach combining the formulation by Mal and Knopoff[1] with Born approximation was developed by Berryman[6] for weak scatterer problems. Recently, Kim et al.[7] developed another formulation, using the coherent potential approximation, for the effective medium properties of composites by studying the average displacement field expressed in the integral representation. More recently, Jeong and Kim[8] provided numerical results, based on some of the above theories and some others, in the Rayleigh region, and compared them with experimental results.

All the approaches mentioned above differ from each other to various degrees. Our study in this paper is focused on the two approaches in References [4] and [7]. They will be referred to as Approach I and Approach II, respectively, hereafter. Also, numerical results for various particulate composites, as computed by Approach II, are presented, and the results are discussed from physical perspectives.

Ⅱ. Theory

Derivation of the two approaches focused upon in this paper is shown, along with the final resulting equations for the effective medium properties. By comparisons, differences in the derivation steps and in the final equations are observed, and are discussed from the physical perspectives.

2.1. Integral equations for elastic wave scattering

Both approaches start with the following governing equation for time-harmonic elastic wave displacement field in a homogeneous medium[9].

$$\mathbf{C}_{ijkl}\mathbf{u}_{k,jl} + \rho\omega^2\mathbf{u}_i + f_i = 0 \tag{1}$$

where $\mathbf{C}_{ijkl}$ and ρ denote the stiffness tensor and density of the medium, respectively; ω the angular frequency; and, f_i the body force. If an inhomogeneity (having stiffness $\mathbf{C}'_{ijkl}$ and density ρ') exists in the homogeneous medium, it scatteres waves. In Approach I, the Green's tensor g for scattering has been found to be

$$g_{mm'} = G_{mm'}(r-r') - \int_{\Omega} G_{mi}(r-r'')\cdot \left\{\left[\delta C_{ijkl}(r'')g_{km',l}(r'',r')\right]_{,j} + \omega^2\delta\rho(r'')g_{im'}(r'',r')\right\}dV'' \tag{2}$$

where $\delta\mathbf{C}_{ijkl} = \mathbf{C}'_{ijkl} - \mathbf{C}_{ijkl}$ and $\delta\rho = \rho' - \rho$; G denotes the free-space Green's tensor for the matrix medium; and, Ω is the volume of the scatterer.

On the other hand, in Approach II, the displacement field in the matrix medium has been shown to be[9]

$$u_i(\mathbf{r}) = u_i^I(\mathbf{r}) + \omega^2\delta\rho\int_{\Omega} G_{im}(\mathbf{r}-\mathbf{r}')u_m(\mathbf{r})dV' + \delta C_{jklm}\int_{\Omega} G_{ij,k}(\mathbf{r}-\mathbf{r}')u_{l,m}(\mathbf{r})dV' \tag{3}$$

where $u_i^I(\mathbf{r})$ is the i-th component of the incident wave displacement, and the two integrals represent the scattered wavefield. Recalling the relationship between the displacement field and the corresponding Green's function, it may be seen that (2) and (3) are equivalent in their physical implications.

2.2. Concept of self-consistent scattering

In Approach I, where the effects of multiple scattering are considered in a self-consistent manner, momentum operators and Dirac's notation often used in quantum mechanics are adopted for brevity of mathematical expressions. First of all, the scattering equation for the Green's tensor in (2) may be rewritten in operator form as

$$\hat{g}_{im} = \hat{G}_{im} + \hat{G}_{ii'}\Delta\hat{\Gamma}_{i'k}\hat{g}_{km}, \tag{4}$$

where all hatted letters denote the corresponding abstract operators, and $\Delta\hat{\Gamma}_{i'k}$ is an operator which depends on the properties of the matrix and the effective medium. On the other hand, note that due to its definition, the free-space Green tensor operator $\hat{G}$ satisfies

$$\hat{\Gamma}^e_{ik}\hat{G}_{km} = -\delta_{im}\hat{I}, \tag{5}$$

where $\hat{\Gamma}^e_{ik} = \hat{p}_j\hat{C}^e_{ijkl}\hat{p}_l - \omega^2\hat{\rho}^e\delta_{ik}$ which is the operator representing wave propagation in the effective medium; $\hat{I}$ is the identity operator; and, $\hat{p}_j = -i\partial/\partial x_j$ is the usual momentum operator from quantum mechanics. Now, the second term on the right-hand side of (4) may be rewritten by use of the concept of the transition operator $\hat{\tau}$. Then, taking an ensemble average on both sides of (4) results in the following expression for the average Green tensor of the scattered wavefield.

$$\langle\hat{\mathbf{g}}\rangle = \hat{\mathbf{G}} + \hat{\mathbf{G}}\cdot\langle\hat{\mathbf{T}}\rangle\cdot\hat{\mathbf{G}} \qquad (6)$$

Now, the concept of self-consistency is used: namely, the average transition operator must vanish, i.e. $\langle\hat{\mathbf{T}}\rangle = 0$. As a result, (6) becomes $\langle\hat{\mathbf{g}}\rangle = \hat{\mathbf{G}}$. Therefore, if $\hat{\mathbf{G}}$ in (5) is found, $\langle\hat{\mathbf{g}}\rangle$ becomes known.

In contrast, Approach II utilizes the coherent potential approximation, depicted in Fig. 1. The main idea of this approximation technique is that when an incident plane wave propagating in the unknown effective medium encounters a representative volume (shown as a circle) of the composite, the average scattered wave must vanish. This argument is similar to the vanishing of the average transition operator in Approach I. From the requirement of vanishing average scattered wave, the equations for the effective medium properties are derived. Particularly when the scatter's size in Fig. 1 becomes sufficiently small, the probability for the scatterer to be composed exclusively of either of the two constituents is equal to their respective volume fraction. If the volume fraction of the j-th constituent is denoted by $v^j (j = 1,2)$, and the single scattering operator for the scatterer of the j-th constituent is denoted by $\mathbf{t}^j$ the coherent potential approximation may expressed mathematically as

$$<\bar{\mathbf{t}}> = \sum_j v^j \bar{\mathbf{t}}^j = 0, \qquad (7)$$

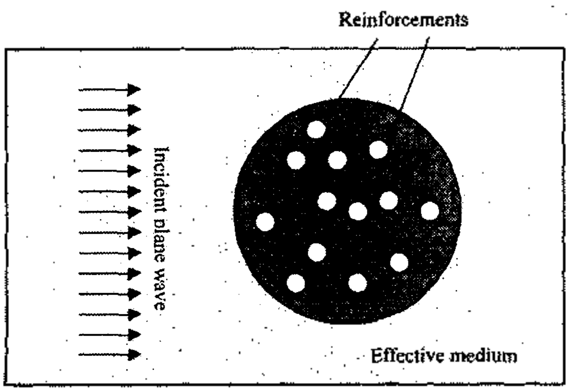

Figure 1. Schematic of coherent potential approximation approach.

where the overbar(¯) emphasizes the fact that the scattering occurs into the effective medium.

2.3. Equations for effective medium properties

The procedures by which equations for the effective medium properties are derived in the two approaches are depicted. In Approach I, first of all, the solution of (5) must be obtained. By adopting the quasicrystalline approximation[10], it may be shown that solving (5) is equivalent to solving the two following equations for $\hat{\mathbf{G}}$ and $\hat{\mathbf{Q}}$ simultaneously.

$$\hat{\mathbf{Q}}(\mathbf{R}) = \hat{\mathbf{T}}(\mathbf{R}) + \hat{\mathbf{T}}(\mathbf{R})\cdot\hat{\mathbf{G}}\cdot\int\gamma(|\mathbf{R}-\mathbf{R}'|)\hat{\mathbf{Q}}(\mathbf{R}')dV', \qquad (8)$$

$$[\hat{\Gamma}^0 + \bar{n}\int\hat{\mathbf{Q}}(\mathbf{R})dV]\cdot\hat{\mathbf{G}} = \hat{\Gamma}^e\cdot\hat{\mathbf{G}} = -\hat{\mathbf{I}}, \qquad (9)$$

where $\hat{\Gamma}^0$ is an operator defined similarly to $\hat{\Gamma}^e$ with all material properties replaced by those of the matrix; $\gamma(|R-R'|)$ is the so-called pair-correlation function for the locations of two scatterers; and, $\bar{n} = N/V$ denotes the average number of scatterers per unit volume. In the original derivation[4], instead of the transition operator $\hat{\mathbf{T}}$ in (8), its ensemble average over the scatterer's orientation is used. Such averaging is omitted here because the scatterers in this paper are assumed to be perfectly spherical. Now, note that an average plane wave ψ must propagate freely in the effective medium, so

Further, assume that the scatterers are perfectly

$$\hat{\Gamma}^e\cdot\psi = 0. \qquad (10)$$

randomly distributed, and that the concentration of the scatterers is sufficiently low so that the effect of $\gamma(|R-R'|)$ is negligible. Then, substituting $\hat{\Gamma}^e$ obtained from (9) into (10), the properties of the effective medium may be found. In doing so, the two cases, where the average plane wave in (10) is longitudinal and shear, are both considered. Utilizing the forward scattering theorem[9] results in the equations for the effective medium properties,

$$(\lambda^0 + 2\mu^0)p_1^2 - \omega^2\rho^0 - 4\pi\frac{\rho^e\omega^2}{p_1^2}\bar{n}\hat{\mathbf{a}}_1\cdot\hat{\mathbf{A}}_1 = 0, \qquad (11)$$

$$\mu^0 p_2^2 - \omega^2\rho^0 - 4\pi\frac{\rho^e\omega^2}{p_2^2}\bar{n}\hat{\mathbf{a}}_2\cdot\hat{\mathbf{A}}_2 = 0, \qquad (12)$$

where the parameters having the superscript "0" denote the properties of the matrix material ; p_1 and p_2 are the wavenumbers of the average longitudinal and shear waves, respectively, having amplitudes $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_2$ and, $\hat{\mathbf{A}}_1$ and $\hat{\mathbf{A}}_2$ are the forward scattering amplitudes in the two cases of the average longitudinal and shear waves, respectively.

Now, the procedure by which equations for the effective medium properties have been derived in Approach II is studied. First of all, it is assumed that the incident wave that causes wave scattering in (7) and in Fig. 1 is an average longitudinal wave propagating in the effective medium. Such a plane wave may be expressed as

$$\bar{\mathbf{u}} = \mathbf{u}_0 e^{i(\mathbf{k}_l^e \cdot \mathbf{r} - \omega t)}, \tag{13}$$

where $\mathbf{u}_0$ and $\mathbf{k}_l^e$ are the amplitude vector and the wavevector, respectively, of the average longitudinal wave. Taking the left and right dot products of (7) by $\mathbf{u}_0$ and using the forward scattering theorem[9] given by

$$\mathbf{u}_0 \cdot \mathbf{t} \cdot \mathbf{u}_0 = -\frac{\rho^e \omega^2}{2\pi k_l^{e^2}} \mathbf{u}_0 \cdot \mathbf{f}, \tag{14}$$

(7) may be rewritten in a convenient form,

$$\sum_j v^j \mathbf{u}_0 \cdot \mathbf{f}^j = 0. \tag{15}$$

In (14) and (15), $\mathbf{f}^j$ denotes the forward scattering amplitude of the longitudinal scattered wave produced by the scatterer, made of the j-th constituent, subjected to a longitudinal incident wave given by (13). In general, using the scattering equation (3) and the far-field asymptotic form of the free-space Green tensor G, the form functions may be expressed in the integral representation. Such an expression for $\mathbf{f}^j(k_l^e)$ in the present case, where the incident field in Fig. 1 is given by (13) and the scatterer is made of the j-th constituent, may be expressed as

$$\mathbf{f}^j(k_l^e) = \frac{k_l^{e^2}}{4\pi\rho^e\omega^2} \hat{\mathbf{r}} \int_\Omega [\delta\rho^j \omega^2 \hat{r}_p u_p^j + i k_l^e \hat{r}_q \delta C_{pqrs}^j \varepsilon_{rs}^j] e^{-ik_l^e \hat{\mathbf{r}} \cdot \mathbf{r}'} dV' \tag{16}$$

where $\hat{r}_k$ is the k-th component of the unit vector $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$, and ε_{rs} denotes the strain tensor. Note that because both the effective medium and the scatterer are homogeneous and isotropic, the stiffness tensor in (16) may be expressed in terms of the Lame's constants λ, μ. After some algebra, (16) may be rewritten as

$$\mathbf{f}^j(k_l^e) = \frac{i k_l^e}{4\pi\rho^e\omega^2} \hat{\mathbf{r}} \int_\Omega \{\delta\rho^j \omega^2 \bar{\mathbf{u}}^j \cdot \nabla + \delta\lambda^j k_l^{e2} \Delta^j - 2\delta\mu^j \mathbf{E}^j : \nabla\nabla\} e^{-i\mathbf{k}_l^e \cdot \mathbf{r}'} dV', \tag{17}$$

where $\delta\rho^j = \rho^j - \rho^e, \delta\lambda^j = \lambda^j - \lambda^e, \delta\mu^j = \mu^j - \mu^e$; Δ and $\mathbf{E}$ denote the dilatation and strain tensor, respectively; and : denotes the scalar product of second-order tensors. Now, substituting (17) into (15), a sufficient condition for (15) may be written as the three equations,

$$\sum_j v^j \delta\rho^j \omega^2 \int_\Omega \bar{\mathbf{u}}^j \cdot \nabla e^{-i\mathbf{k}_l^e \cdot \mathbf{r}'} dV' = 0, \tag{18}$$

$$\sum_j v^j \delta\lambda^j \int_\Omega \Delta^j e^{-i\mathbf{k}_l^e \cdot \mathbf{r}'} dV' = 0, \tag{19}$$

$$\sum_j v^j \delta\mu^j \int_\Omega \mathbf{E}^j : \nabla\nabla e^{-i\mathbf{k}_l^e \cdot \mathbf{r}'} dV' = 0. \tag{20}$$

In this paper, j=1 and j=2 denote the matrix and reinforcement, respectively.

2.4. Discussions and comparisons of two approaches

Having briefly outlined the two approaches considered in this study, they are compared with each other and their differences are discussed. The most significant difference between Approaches I and II may be seen by comparing the final results, (11) and (12), of Approach I with the final equations, (18) through (20), of Approach II. That is, (11) and (12) contain the characteristics of scattering from one constituent (e.g. reinforcements) only, whereas (18) through (20) have scattering characteristics of both scatterers, each made of one of the constituents. For this reason, Approach II is said to be symmetrical in the sense that the two constituents are treated on the same footing, and thus it is appropriate for cases where the two constituents are not clearly distinguished as the host or the reinforcement. The reason for this difference lies in the different underlying concepts used in the derivation. As explained above, Approach I adopts the concept of self-consistency in dealing with the multiple scattering phenomena, where the medium of the scatterers is designated. On the other hand, Approach II is based upon the coherent potential approximation, where the scatterer may comprise either of the two media.

Also, note that (18) through (20) are *sufficient* conditions for (15), i.e. there may be many cases where (15) holds without satisfying any of (18) through (20). In other words, (18) through (20) constitute an over-constraint on the effective medium properties. This is regarded as a serious point of arguments. Also, note that (11) and (12) have been obtained by separately considering the two cases of average longitudinal and shear waves, whereas (18) through (20) have been obtained by considering only one case of average longitudinal waves. Therefore, Approach II may be supplemented by considering average shear waves. In doing so, however, a natural question may arise: that is, already having three equations for three properties (λ^e, μ^e and ρ^e) of the effective medium, more equations would mean too many equations. This question may easily be answered when the first point in the present paragraph is recalled. Since (18) through (20) constitute a sufficient condition for (15), (18) through (20) should be replaced by new necessary and sufficient conditions for (15).

A similar concern may arise, as to the solution of the two equations, (11) and (12), in Approach I. That is, the two equations may not be sufficient to find the three properties of the effective medium because of the lack of equations. However, as Deveney and Levine solved the problem with the Rayleigh approximation[5] (when the frequency is sufficiently low), or with the Born approximation[4] (when the scatterer's properties differ only slightly from the matrix's), three equations for the three unknowns may be found from the two equations, (11) and (12). For example, when the Born approximation holds, (11) and (12) become [4]

$$[(\lambda^0+2\mu^0)+C(\delta\lambda+2\delta\mu)]p_1^2-\omega^2[\rho^0+C\delta\rho]=0, \quad (21)$$

$$[\mu^0+C\delta\mu]p_2^2-\omega^2[\rho^0+C\delta\rho]=0, \quad (22)$$

where $C=\bar{n}V_s$, the volume concentration of the scatterers, each having a volume V_s. Since the last terms of (21) and (22) are common, from the three different terms, the well-known rule of mixture follows, i.e. $\lambda^e=\lambda^0+C\delta\lambda$, $\mu^e=\mu^0+C\delta\mu$, $\rho^e=\rho^0+C\delta\rho$. In general, (21) and (22) may be solved for the three effective medium properties in a similar manner.

Finally, another difference between the two approaches is the capability of Approach I to take into account of the pair-correlation between two scatterers, as characterized by the γ function in (8), even though it has been neglected in the final stage of derivation. The effects of such pair-correlation become important as the scatterer concentration grows.Thus, in such cases, the potential capability to consider it may make a difference. Yet, there still is much controversy about the form of the pair-correlation function, so determination of the function is another topic of further research.

Ⅲ. Numerical results

In this section, several cases of particulate composites are considered, and their effective medium properties are computed using Approach II. Assuming that the scatterers are of spherical shape with identical size, the three-dimensional scattering problem in Fig. 1 with a spherical scatterer must be solved. (If fiber composites were considered, the problem would have become two-dimensional with a cylindrical scatterer.) Since the formulation of the problem of elastic wave scattering from a sphere is readily available in the literature (see, for example, References [7, 11, 12]), the reader is simply referred to Reference [13], where the formulation is given in detail with the errors detected in Reference [7] corrected. Once the problem has been formulated, (18) through (20) may be expressed in terms of the scattering coefficients that can be computed from the formulated problem. Further, the volume integral in (18) through (20) may be significantly simplified by making use of the orthogonality of Legendre functions[12, 14]. For example, (18) may be reduced as follows.

$$\begin{aligned}\int_\Omega \tilde{\mathbf{u}}^j\cdot\nabla e^{-i\mathbf{k}_l^e\cdot\mathbf{r}}dV &= \int_\Omega (u_r^j\mathbf{e}_r+u_\theta^j\mathbf{e}_\theta+u_\phi^j\mathbf{e}_\phi)\cdot \\ &\quad \left(\mathbf{e}_r\frac{\partial}{\partial r}+\mathbf{e}_\theta\frac{\partial}{\partial\theta}+\mathbf{e}_\phi\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\right)e^{-i\mathbf{k}_l^e\cdot\mathbf{r}}dV \\ &= \int\sum_{n=0}^{\infty}[C_n\{n(2n+1)j_n(k_l^jr)j_n(k_l^er)-nk_l^jrj_{n+1}(k_l^jr)j_n(k_l^er)\} \\ &\quad +D_n n(n+1)\{(2n+1)j_n(k_s^jr)j_{n+1}(k_l^er)-k_s^jrj_{n+1}(k_s^jr)j_n(k_l^er)\} \\ &\quad -C_n\{nk_l^erj_n(k_l^jr)j_{n+1}(k_l^er)-k_l^jk_l^er^2j_{n+1}(k_l^jr)j_{n+1}(k_l^er)\} \\ &\quad -D_n n(n+1)k_l^erj_n(k_s^jr)j_{n+1}(k_l^er)]dr,\end{aligned} \quad (23)$$

where C_n and D_n are the n-th scattering coefficients for the longitudinal and shear waves, respectively, transmitted into the scatterer; and, k_l^j and k_s^j denote the longitudinal and shear wavenumbers, respectively, of the j-th constituent. The computation of products of two Bessel functions in (23) may be computed conveniently by utilizing formulae in References [15, 16]. The three simultaneous (complex) equations such as (23), reduced from (18) through (20), are then solved numerically for the three (complex) properties, ρ^e, λ^e, μ^e, of the effective medium. Therefore, in this study, the Newton-Rhapson's method[17, 18] has been extended to complex problems and it is used to solve the equations.

Table 1 lists the constituent properties of the three particulate composites considered in this study: lead-epoxy, alumina-aluminum, and aggregate-mortar, with those following the hyphens being the matrix media. The first case of lead-epoxy is considered only for the purpose of comparison with the results in the literature. Several cases with different concentrations of lead particles have been considered, and all results have shown agreements

Table 1. Constituent properties of composites considered.

Property / Material	ρ [kg/m^3]	λ [GPa]	μ [GPa]
Lead	11,300	39.0	8.40
Epoxy	1,200	4.40	1.60
Alumina	3,700	144	144
Aluminum	2,600	41.0	25.0
Aggregate	2,770	19.2	22.5
Mortar	2,380	7.72	10.7

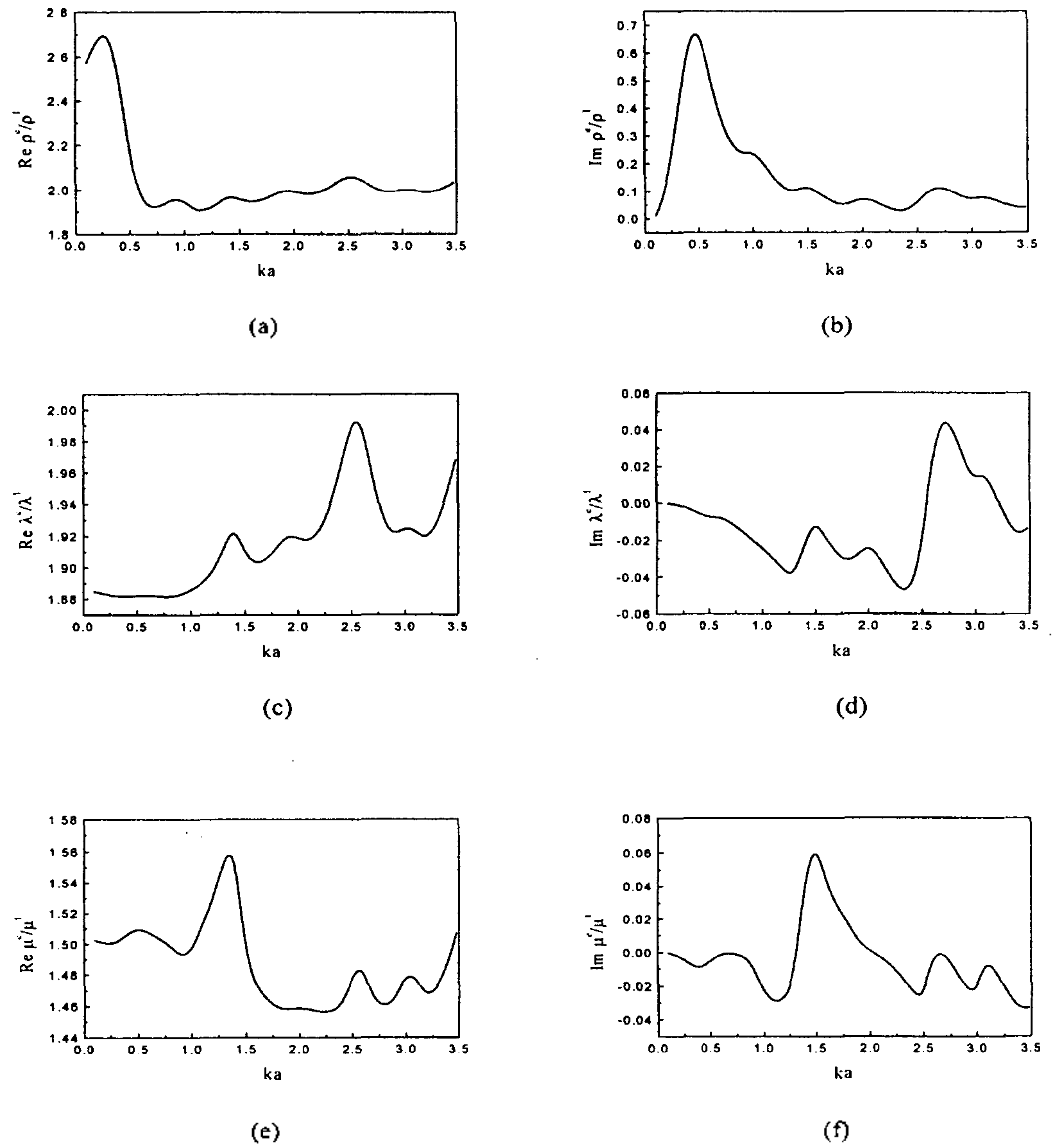

Figure 2. Effective medium properties of lead-epoxy with 10% volumetric concentration of lead particles versus normalized wavenumber *ka*; (a) real, and (b) imaginary parts of normalized effective density; (c) real, and (d) imaginary parts of normalized effective Lame constant λ; and (e) real, and (f) imaginary parts of normalized effective Lame constant μ.

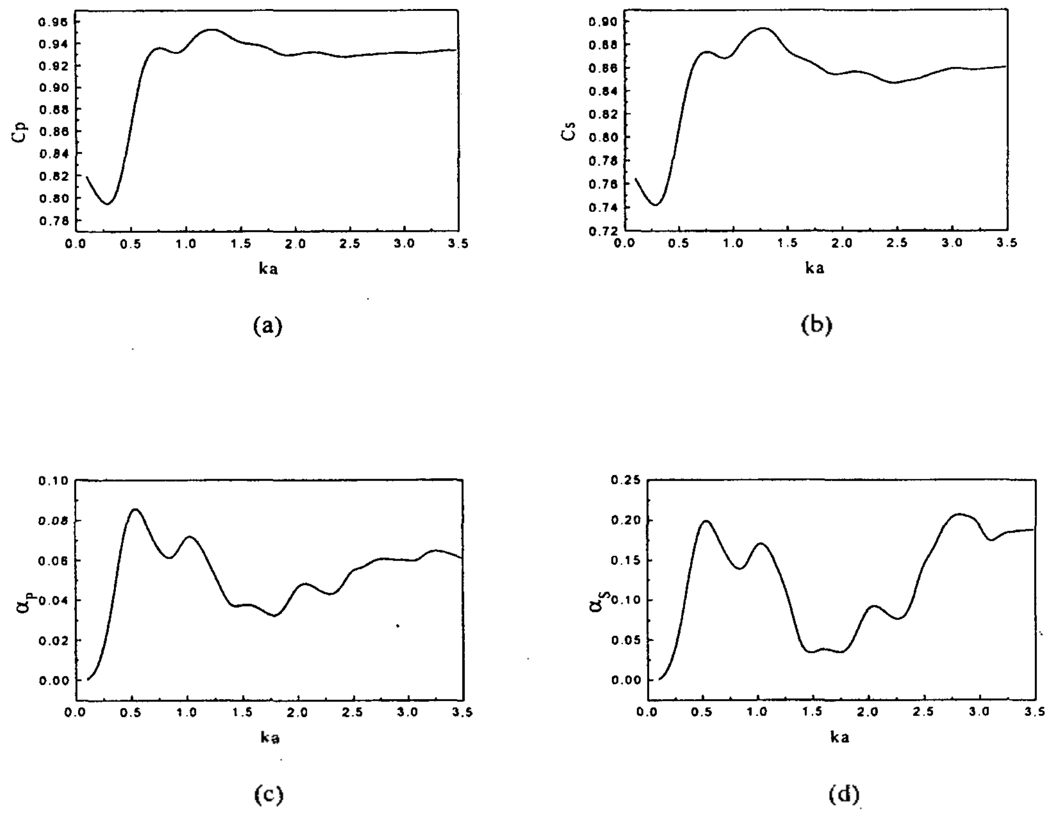

Figure 3. Wavespeeds and attenuations of effective medium for lead-epoxy with 10% volumetric concentration of lead particles versus normalized wavenumber *ka* ; (a) longitudinal wavespeed, (b) shear wavespeed, (c) longitudinal wave attenuation, and (d) shear wave attenuation.

with Reference [7]. Fig. 2 show the computed effective medium properties, in the order of real and imaginary parts of ρ^e, λ^e, μ^e, as functions of the normalized wavenumber *ka* (*k* meaning the longitudinal wavenumber in the epoxy matrix, and *a* the radius of the lead particle). All results in Fig. 2 are normalized by the value of the corresponding property of the epoxy matrix. It should be noted from Fig. 2 that the effective medium properties show noticeable variations with some peaks, which were explained by the argument of scatterer's resonant behaviors in Reference [7]. It is also noteworthy that in the limit of $ka \rightarrow 0$, the real parts of all properties in Fig. 2 approach the values given by the rule of mixtures. Fig. 3 shows the corresponding wavespeeds (c_p, c_s for longitudinal and shear waves, respectively) and attenuations (α_p, α_s for longitudinal and shear waves) of the effective medium, as computed from the results in Fig. 2. The peaks and valleys in the wavespeeds and attenuations shown in Fig. 3 are supposed to be related to the resonant behavior of the reinforcements, as they are computed from the material properties which exhibit such variations in Fig. 2 attributable to the same physical effects.

Numerical results for the alumina-aluminum composite (with 20% volumetric concentration of alumina particles) are shown in Fig. 4, in a manner similar to Fig. 2. It may be observed easily that the variations of the curves in Fig. 4 are remarkably smoother than those in Fig. 2, and also that the ranges of their variations are much smaller, even though the volumetric concentration of scatterers has doubled. This is because the difference in the acoustic impedance, ρc, between the two constituents in the case of alumina-aluminum is much smaller than that in the case of lead-epoxy. That is, the alumina particles act as much weaker scatterers in aluminum than the lead particles in epoxy, producing much less effects of multiple scattering. This phenomenon of less effects of multiple scattering due to weak scatterers is much more remarkable in the final case of aggregate-mortar, whose numerical results are similarly shown in Fig. 5. It may be seen that the variations of effective medium properties in Fig. 5 are much less and smoother than Fig. 4, even though the volumetric concentration of scatterers has increased further. This is again because of the even less difference in

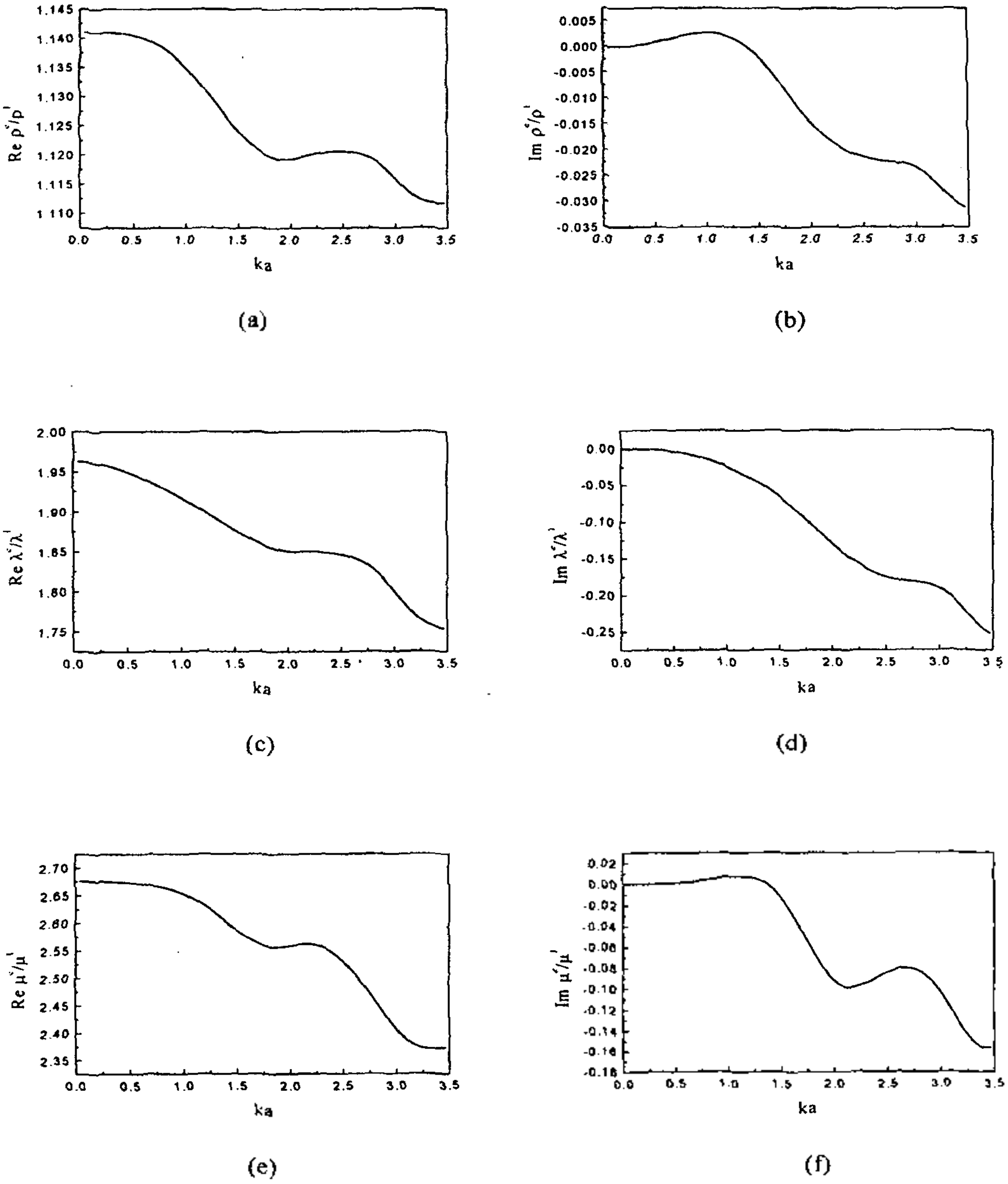

Figure 4. Effective medium properties of alumina-aluminum with 20% volumetric concentration of alumina particles versus normalized wavenumber *ka* ; (a) real, and (b) imaginary parts of normalized effective density; (c) real, and (d) imaginary parts of normalized effective Lame constant λ; and (e) real, and (f) imaginary parts of normalized effective Lame constant μ .

the acoustic impendance between the two constituents.

Ⅳ. Conclusions

Two major theories of effective medium for randomly inhomogeneous composites carrying elastic waves have been studied in detail, and compared with each other. It has been shown that the two theories are derived on the common basis of the self-consistent scattering concept. Yet, several differences have been found between their final equations, due to the different manners in which the concept is utilized in the derivation of the theories. By careful comparisons of the two theories, prospective ways to improve them have been found and proposed. (Such potential improvements are being attempted.) One way of the potential improvements is to consider the incidence of a plane shear wave in addition to the longitudinal wave incidence. Also, it has been noted that when the scatterer concentration becomes high, the interaction between scatterers will be more important, and it must be represented by the pair-correlation function.

In order to illustrate how the effective medium

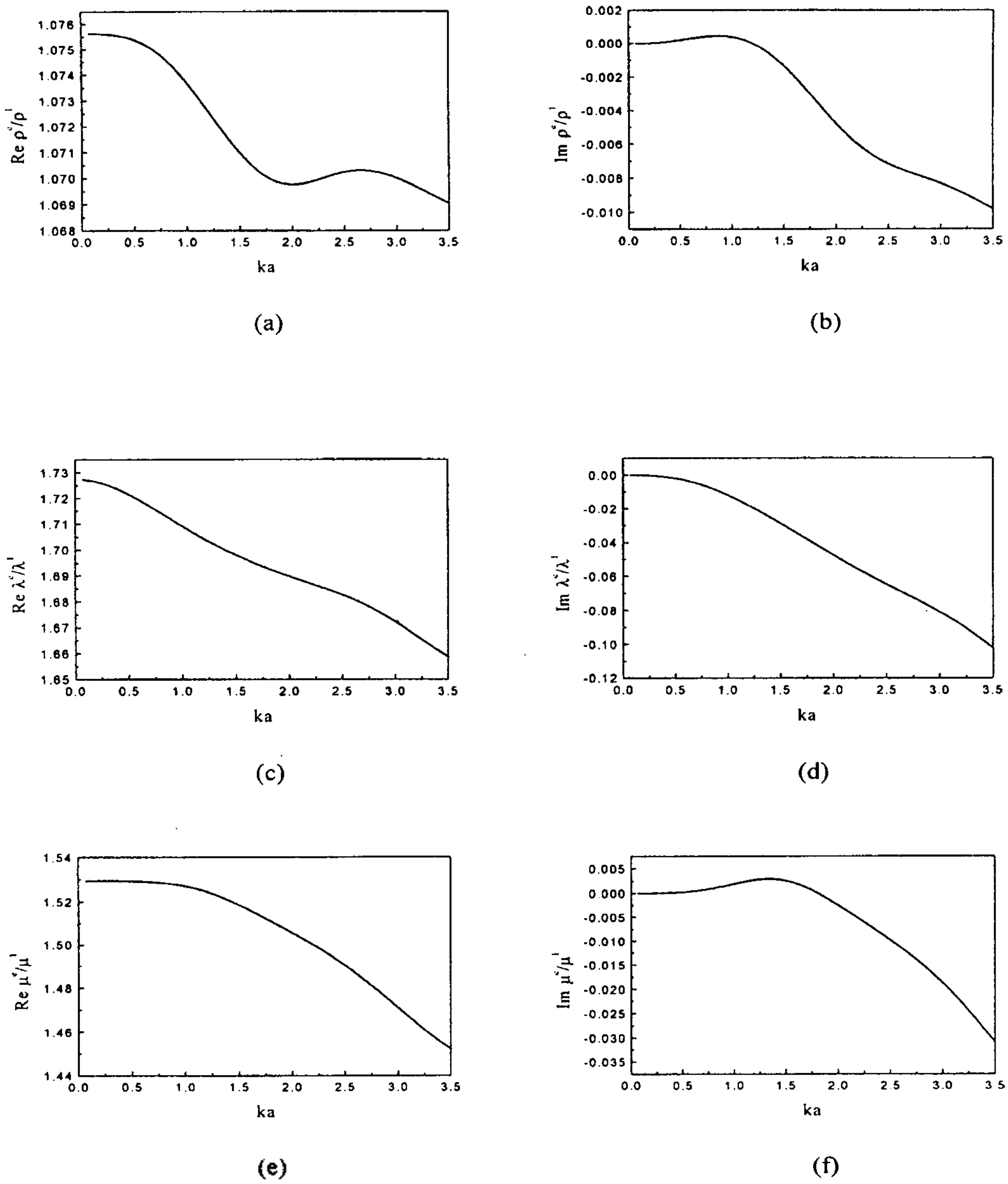

Figure 5. Effective medium properties of aggregate-mortar with 30% volumetric concentration of aggregates versus normalized wavenumber ka ; (a) real, and (b) imaginary parts of normalized effective density; (c) real, and (d) imaginary parts of normalized effective Lame constant λ ; and (e) real, and (f) imaginary parts of normalized effective Lame constant μ .

properties may be found, three particulate composites have been considered. The equations that the effective medium properties must satisfy have been solved numerically by the Newton-Rhapson method extended for complex variables. In the case of lead-epoxy composite considered repeatedly in previous studies, the numerical results have agreed with those in the literature. The other two cases considered in the study are alumina-aluminum composite and aggregate-mortar concrete. Numerical results for the effective medium properties in these two composites have shown much less variations, as the frequency varies, than those in the case of lead-epoxy composite. This may be explained by the relatively less difference in the acoustic impedance between the two constituents in the two latter cases.

As a result of this study, understanding of the effective medium theory for elastic waves has been enhanced, and prospective directions for future improvement of the theory have been found. Once such improvements have been completed, the theories will provide a reliable tool for accurate prediction of the dynamic behavior of engineering composite materials, and will also be useful for their ultrasonic nondestructive testing. Further, a theory for fiber composites may be developed, in a similar manner.

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