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Redundancy Optimization for the Mixed Reliability System

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Abstract

This paper deals with the problem of redundancy allocation for the mixed reliability system in an optimal way. Two kinds of the reliability system are considered for optimal allocation of parallel redundancy. The problem is approached as the optimization problems using the standard method of dynamic programming(DP). The algorithm for solving the optimal redundancy allocation is proposed and then the DP algorithm is applied to two numerical examples such as maximization of reliability subject to an allowable cost-constraint and minimization of the total cost subject to the specified minimum reliability-constraint. A consequence of this study is that the developed computer program package can be applied to the optimal redundancy allocation for the mixed reliability system.

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1. Introduction

Reliability is defined as the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered. The reliability of a system with $N-stages$ is the product of the individual stage reliabilities whenever the failure of any stage causes failure of the system, the failure of any stage is independent of the failure of any other stage, and the system is to be operated for a fixed period of time.

Kettelle[1] has presented an algorithm for allocating redundancy so as to achieve a specified reliability without exceeding the minimum cost. Specifically, we consider a system consisting of $N-stages$ and the system functions if and only if each stage functions.

In this paper, we are concerned with the redundancy optimization of a mixed series-parallel reliability system with $N-stages$. If stage j consists of X_j units (X_j -1 redundancies) in parallel, each of which has independent probability, $0< R_j<1$ of reliability at stage j, then the failure of stage j is $(1-R_j)^{X_j}$ and the whole system reliability, R_S is represented as $R_S = \prod_{j=1}^{N} [1-(1-R_j)^{X_j}]$.

In most reliability problems a fundamental method of improving the reliability of a system is to add redundant components in parallel or stand-by[2]. We will consider two kinds of reliability system problems. One problem is to find the redundancy at each stage j so as to maximize the whole system reliability with $N-stages$ in series where units are in parallel at each stage under the specified cost constraint. And another problem is to find the redundancy at stage j such that the total cost is to be minimized under the assumption of obtaining the minimum requirement of a system reliability, $R_{S,MIN}$. Then the optimal redundancy allocation could be found at a minimum cost satisfying $R_S \geq R_{S,MIN}$.

Sok[6] also has studied the application of dynamic programming to optimization of the system reliability for the case of maximization with and without a cost constraint in the previous study. In this paper, those mixed series-parallel system problems are solved by dynamic programming method for the case of maximization of the system reliability and minimization of the total cost under the constraints of specified cost and the minimum system reliability, respectively.

Two kinds of reliability system problems are introduced and then they are formulated as the optimal redundancy problems in section 2. And those problems can be approached as the optimization problems using standard method of dynamic programming in section 3. The dynamic programming algorithm will be applied to two typical numerical examples in section 4. And then computational results are discussed in section 5. Finally, concluding remarks will be given in the final section.

2. Formulation of the Problems

A reliability diagram for a nonredundant system can be shown in Figure 1. If stage j has probability of failure $1-R_j$, then the reliability of such a system is represented as $\prod_{j=1}^{N} [1-(1- R_j)]$.

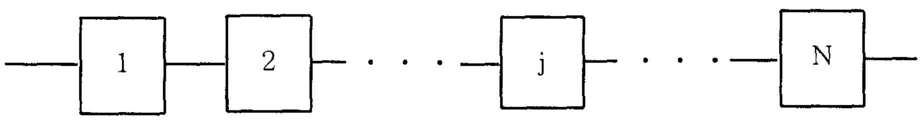

Figure 1. A Series Reliability System

To increase the reliability of the nonredundant system, redundancy allocations should be added as being shown in Figure 2. In a mixed series-parallel system with $N-$*stages*, at least one unit of type j must be used at each stage j. The whole system reliability with $N-$*stages* can be improved by adding the parallel redundancies at each stage. Here stage j is performed by X_j-1 redundancies in parallel[3]. If all units performing stage j have the probability of failure $1-R_j$ and the units fail independently, the whole system reliability is $\prod_{j=1}^{N} [1-(1- R_j)]$.

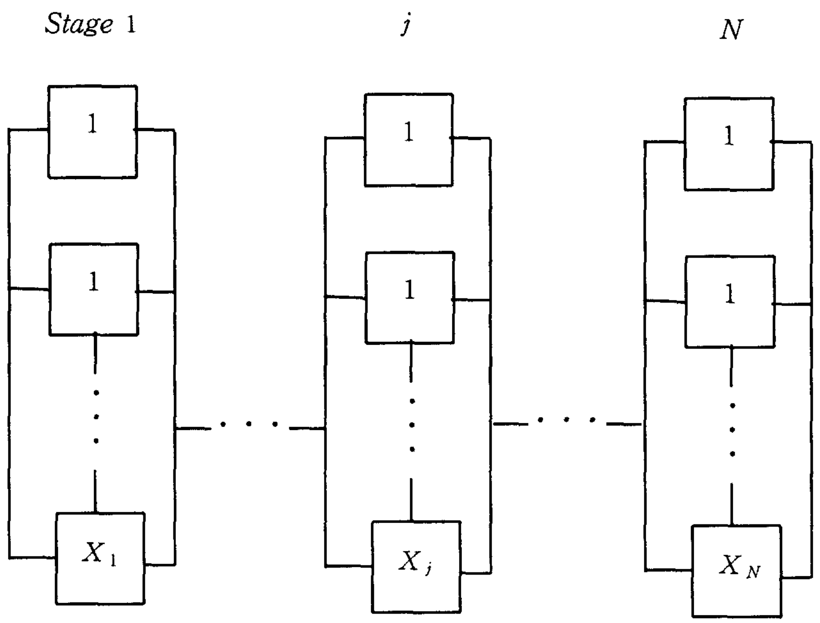

Figure 2. A Mixed Series-Parallel System with $N-stages$.

Two kinds of the mixed series-parallel reliability systems will be considered in this section. Consider the mixed series-parallel system with $N-stages$ shown in Figure 2 having (X_j -1) parallel redundancies at stage j. Suppose that the cost C_j is the unit cost for the redundant units at stage j. In this case, the whole system reliability, R_S is maximized subject to the allowable cost(A_c) constraint under the condition that R_S is greater than or equal to $R_{S,MIN}$. Hence the problem of the mixed series-parallel reliability system can be formulated as follows and then let us call this kind of problem as type-1 problem.

$$Max \quad R_S = \prod_{j=1}^{N} [\, 1-(1-R_j)^{X_j} \,] \qquad \cdots\cdots\cdots\cdots (1)$$

$$s.t. \quad g_1 = \sum_{j=1}^{N} C_j X_j \le A_c ,$$

$$R_{S,MIN} \le R_S \le 1$$

and $X_j \ge 0$ and are integers for j=1,⋯,N.

Suppose that the minimum requirement of the mixed series-parallel system reliability, $R_{S,MIN}$ should be obtained. Then the whole system cost, C_S is minimized subject to the fact that the whole system reliability, R_S is greater than or equal to $R_{S,MIN}$. Therefore, such problem can be formulated as follows and then let us call this kind of problem as type-2 problem.

$$Min \quad C_S = \sum_{j=1}^{N} C_j X_j \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (2)$$

$$s.t. \quad R_S \geq R_{S,MIN}$$

$$\text{and} \quad X_j \geq 0 \text{ and are integers for } j=1,\cdots,N$$

$$\text{where} \quad R_S = \prod_{j=1}^{N} [\, 1-(1-R_j)^{X_j} \,] \leq 1.$$

3. Dynamic Programming Formulation

In this section the mixed series-parallel system problems are formulated by the use of dynamic programming method for the case of maximization of the system reliability and minimization of the total cost under the specified cost and minimum system reliability constraints respectively.

The dynamic programming(DP) method is applicable to multi-stage decision or sequential decision problems. This method converts such a problem to a series of single-stage optimization problems.

Let D_j be the set of all possible decision alternatives available at stage j, its elements are denoted by $d_j \in D_j$, and let S_j be the set of all possible states at stage j, its elements are denoted by $s_j \in S_j$, j = 1, $\cdots$, N. Let $t_j(s_j, d_j)$ be a transformation function from s_j to s_{j-1} and $r_j(s_j, d_j)$ be a return function at stage j [6]. Then the general multi-stage decision problems may be converted to a series of single-stage decision problems. An $N-$*stages* decision process is displayed by Figure 3[4].

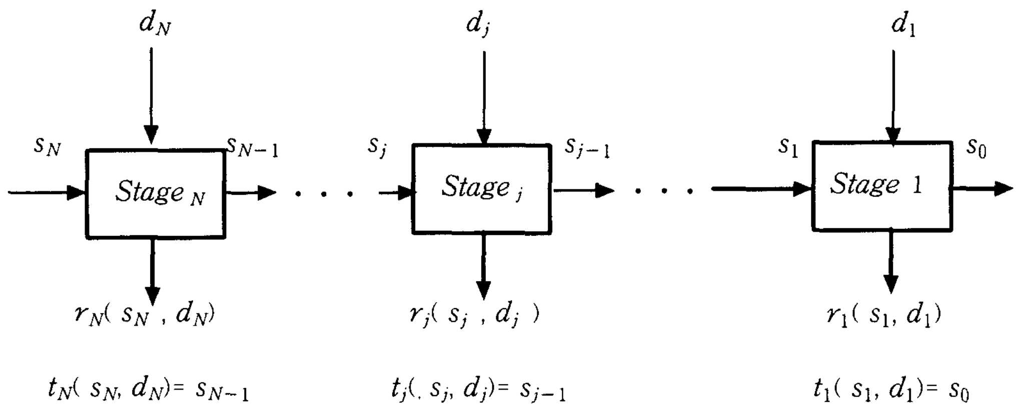

Figure 3. *N−stages* Dynamic Programming Representation.

Consider a mixed series-parallel reliability system model as type 1 problem in general. The dynamic programming formulation of the problem also can be derived as follows. The recursive equations of the problem are formulated as[7] :

$$f_1(A_c) = \max_{X_1^l < X_1 < X_1^u} [1 - (1-R_1)^{X_1}], \quad \cdots\cdots\cdots\cdots\cdots (3)$$

$$\vdots$$

$$f_j(A_c) = \max_{X_j^l < X_j < X_j^u} [(1 - (1-R_j)^{X_j}) f_{j-1}(A_c - g_{1j}(X_j))], \quad \cdots (4)$$

$$\vdots$$

$$f_N(A_c) = \max_{X_N^l < X_N < X_N^u} [(1 - (1-R_j)^{X_N}) f_{N-1}(A_c - g_{1N}(X_N))] \quad \cdots (5)$$

where X_j^l , j= 1,2,⋯, N is the minimum integer number used at each stage. Usually, X_j^l = 1 and X_j^u for j = 1,⋯, N are the maximum integer number used at each stage such that

$$\sum_{\substack{p=1 \\ p\neq j}}^{N} g_{1p}(X_j^l) + g_{1j}(X_j) \leq A_c \ .$$

4. Numerical Examples

Consider two numerical examples with four stages each where the whole system reliability, R_S associated with the final stage, the costs C_j , the reliabilities R_j of each of the units, the maximum allowable cost A_c and the minimum requirement of a system reliability $R_{S,MIN}$ are presented in Table 1.

Table 1. Two Numerical Examples

$Max\ R_S = \prod_{j=1}^{4} [1-(1-R_j)^{X_j}]$			$Min\ C_S = \sum_{j=1}^{4} C_jX_j$		
Stage	Cost	Reliability	Stage	Cost	Reliability
1	5.4	0.85	1	4.5	0.85
2	4.3	0.80	2	3.4	0.75
3	3.2	0.75	3	2.3	0.70
4	2.1	0.70	4	1.2	0.80
s.t. $\Sigma C_jX_j \le A_c$, $R_S \ge R_{S,MIN}$ and $X_j \ge 0$ and are integers for j=1,⋯,N where A_c = 84 (allowable cost) and $R_{S,MIN}$ = 0.999 as type-1.			s.t. $R_S \ge R_{S,MIN}$ and $X_j \ge 0$ and are integers for j=1,⋯,N where $R_{S,MIN}$ = 0.999, A_c = 70 and $R_S = \Pi [1-(1-R_j)^{X_j}]$ as type-2.		

We illustrate the application of the computational algorithm based on the DP method to two numerical examples in the followings. Consider the type-1 example in Table 1.

The computer program for type-1 problem has been developed in FORTRAN language. This computer program package can be used to find the optimal system reliability and the optimal redundant allocation for this kind of the problem in general. The flow diagram of the computer programming is shown in Figure 4 and the detailed computer program package can be accessed from the author.

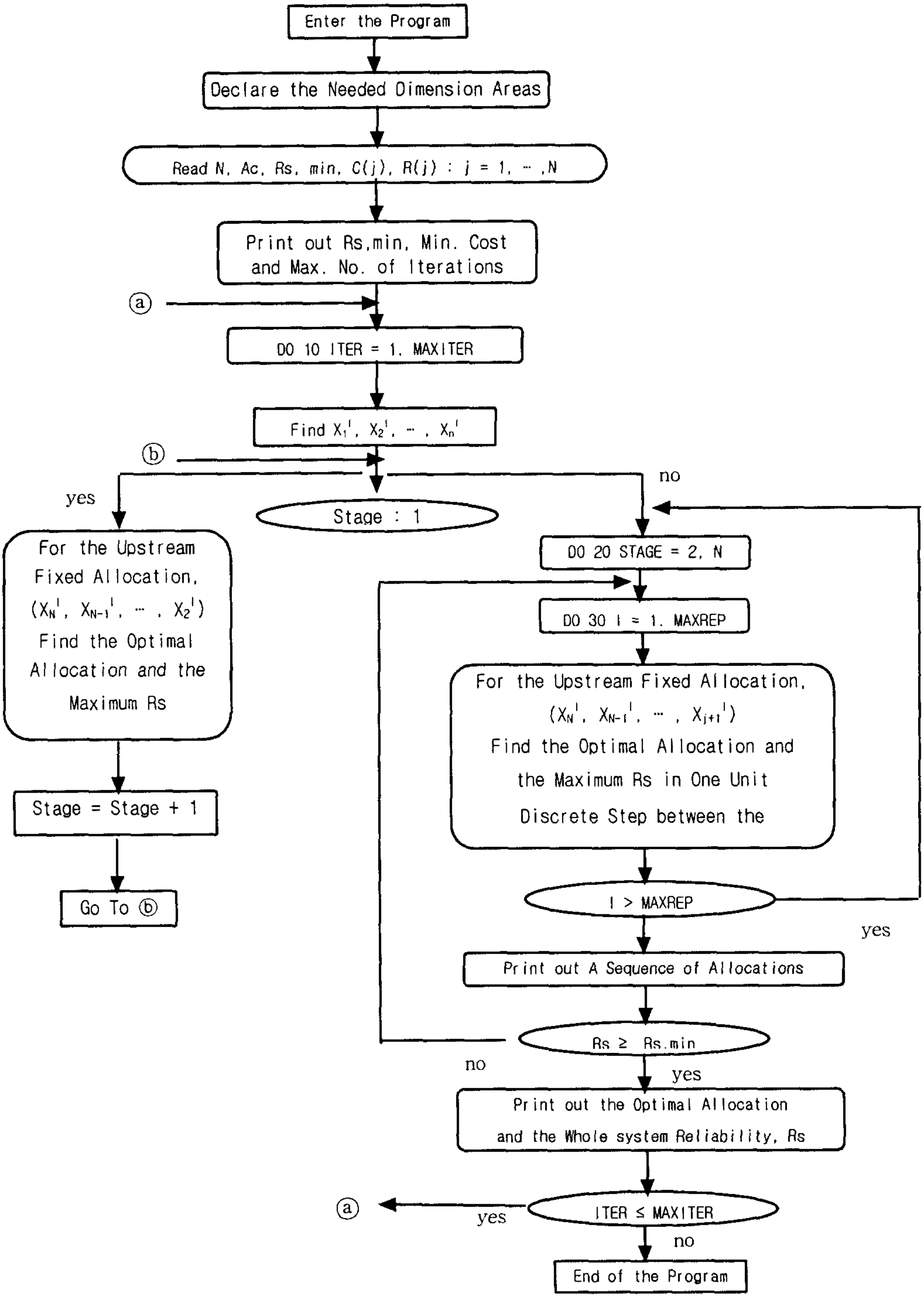

Figure 4. The Flow Diagram for Computer Programming

5. Computational Results and Discussion

To attain the minimum system reliability, $R_{S,MIN}$, let $R_j(X_j)$ = 1- $(1-R_j)^{X_j}$, then at stage 1, $R_1(4)=1-(1-0.85)^4=0.9995$ which is greater than 0.999. Hence, the minimum number of components at stage 1 is $X_1^l=4$. Similarly, at stage 2 $X_2^l=5$ with $R_2(5)=0.9997$, at stage 3 $X_3^l=5$ with $R_3(5)=0.9990$ and at stage 4 $X_4^l=6$ with $R_4(6)=0.9993$. Hence, $(X_1^l, X_2^l, X_3^l, X_4^l)=(4,5,5,6)$ respectively and the minimum cost should be 71.7 for this allocation initially.

For the first stage, the optimal allocation should be determined by the function as :

$f_1(A_c) = \max_{X_1^l \le X_1 \le X_1^u} [R_1(X_1)]$ where X_1^l = 4 and X_1^u is bounded by the allowable cost constraint. For a spectrum of A_c values, the optimization problem can be solved for the optimal X_1. Thus, for each value of A_c between 71.7, which is the minimum cost to attain Rs,min and 84.0, which is the maximum allowable cost, the optimal allocation will be found when all the upstream stage allocations are fixed by $(X_4^l, X_3^l, X_2^l)=(6,5,5)$. The optimal allocation for X_1 is shown in the following Table 2.

Table 2. The Optimal Allocations of for a spectrum of A_c values

A_c	X_4^l	X_3^l	X_2^l	X_1	R_S
71.70 - 77.10	6	5	5	4	0.99747
77.10 - 82.50	6	5	5	5	0.99790
82.50 - 83.70	6	5	5	6	0.99796

Since the upstream fixed allocation, $(X_4^l, X_3^l, X_2^l)=(6,5,5)$ and it's cost is correspond to 50.1 in the case of A_c value between 82.50 and 83.70, the rest of the cost can be used to allocate the optimal X_1 to be 6 and then the maximum system reliability, $f_1(A_c)=0.99796$. The optimal allocation and the maximum system reliability can be found for all possible A_c values between 72 and 84 in accordance with one unit discrete step under the minimum fixed allocations X_4^l, X_3^l and X_2^l. The sequences of allocations at stage 1 are shown in Table 3.

The next step is to determine the optimal allocation of stage 2 and stage 1 where $X_4^l=6$ and $X_3^l=5$ are fixed to the upstream allocation of stage 4 and stage 3 respectively. The maximization will be carried out for A_c values from 72 to 84 in the same manner. Also the optimal allocation of stage 3, stage 2 and stage 1 are searched for the maximum system reliability where the only upstream allocation of stage 4, $X_4^l=6$ is fixed.

Table 3. A Sequence of Allocations for each A_c at Stage 1

A_c	X_4^l	X_3	X_2	X_1	$f_2(A_c)$
72.0	6	5	5	4	0.99747
73.0	6	5	5	4	0.99747
74.0	6	5	5	4	0.99747
75.0	6	5	5	4	0.99747
76.0	6	5	5	4	0.99747
77.0	6	5	5	4	0.99747
78.0	6	5	5	4	0.99747
79.0	6	5	5	4	0.99747
80.0	6	5	5	4	0.99747
81.0	6	5	5	4	0.99747
82.0	6	5	5	5	0.99790
83.0	6	5	5	6	0.99796
84.0	6	5	5	6	0.99796

Table 4. A Sequence of Allocations for each A_c at stage 2 (and Stage 1)

A_c	X_4	X_3	X_2	X_1	$f_2(A_c)$
72.0	6	5	5	4	0.99747
73.0	6	5	5	4	0.99747
74.0	6	5	5	4	0.99747
75.0	6	5	5	4	0.99747
76.0	6	5	5	4	0.99747
77.0	6	5	6	4	0.99773
78.0	6	5	6	4	0.99773
79.0	6	5	6	4	0.99773
80.0	6	5	5	4	0.99773
81.0	6	5	6	4	0.99773
82.0	6	5	6	5	0.99816
83.0	6	5	6	5	0.99816
84.0	6	5	6	5	0.99816

Hence, the sequences of allocations for each allowable cost A_c at stage 2 (including stage 1), stage 3(including stage 2 and stage 1) and stage 4(including stage 3, stage 2 and stage 1) are presented in Table 4, 5 and 6 respectively.

Table 5. A Sequence of Allocations for each A_c at Stage 3 (and Stage 2 and 1)

A_c	X_4^l	X_3	X_2	X_1	$f_2(A_c)$
72.0	6	5	5	4	0.99747
73.0	6	5	5	4	0.99747
74.0	6	5	5	4	0.99747
75.0	6	5	5	4	0.99747
76.0	6	5	5	4	0.99747
77.0	6	6	5	4	0.99820
78.0	6	6	5	4	0.99820
79.0	6	6	5	4	0.99820
80.0	6	6	5	4	0.99820
81.0	6	6	5	4	0.99820
82.0	6	6	5	5	0.99863
83.0	6	7	6	4	0.99864
84.0	6	7	6	4	0.99864

Table 6. A Sequence of Allocations for each A_c at stage 4 (and Stage 3, 2,and 1)

A_c	X_4	X_3	X_2	X_1	$f_2(A_c)$
72.0	6	5	5	4	0.99747
73.0	6	5	5	4	0.99747
74.0	6	5	5	4	0.99747
75.0	6	5	5	4	0.99747
76.0	6	5	5	4	0.99747
77.0	7	6	5	4	0.99871
78.0	7	6	5	4	0.99871
79.0	7	6	5	4	0.99871
80.0	7	6	5	4	0.99871
81.0	7	6	5	4	0.99871
82.0	7	6	6	4	0.99863
83.0	7	6	5	5	0.99914
84.0	7	6	5	5	0.99914

Under the maximum allowable cost, A_c in type-1 problem, all possible allocations of stage 4 may be considered to cover the minimum system reliability, $R_{S,MIN}$. A sequence of allocations for each A_c at stage 4 will be summarized according to one unit discrete step in Table 6. Finally, the optimal allocation having the maximum system reliability will be seeked. The optimal redundancy allocation and the optimal system reliability at stage 4 is shown in the following Table 7. The computational results assert that the optimal allocation, (X_4, X_3, X_2, X_1) = (7, 6, 5, 5) gives the maximum system reliability, R_S = 0.99914 which is greater than the minimum system reliability, $R_{S,MIN}$ = 0.999 among all possible allocations. Therefore, the optimal redundancy allocation at stage j is the optimal allocation minus 1 i.e., $X_j - 1$.

Table 7. The Optimal Redundancy Allocation for the Type-1 Example

A_c	X_4-1	X_3-1	X_2-1	X_1-1	$f_4(A_c)$
83	6	5	4	4	0.99914

Consider the type-2 problem in Table 1. The primal minimization problem may be converted to the pseudo-dual maximization problem as though the primal problem is converted to the dual problem using the primal-dual relationship. And then the optimal solution is found by two-phase method. In phase 1, the optimal(minimum) cost solution will be searched in integer range. Let $R_j(X_j) = 1 - (1-R_j)^{X_j}$, then $X_1^l = 4$ with $R_1(4) = 0.9995$ at stage 1, $X_2^l = 5$ with $R_2(5) = 0.9990$ at stage 2, $X_3^l = 6$ with $R_3(6) = 0.9993$ at stage 3 and $X_4^l = 5$ with $R_4(5) = 0.9997$ at stage 4. Hence, $(X_1^l, X_2^l, X_3^l, X_4^l) = (4, 5, 6, 5)$ respectively and the minimum cost should be 54.8 for the fixed initial allocation.

Therefore, the maximization will be carried out for A_c values from 55 to 70 until not only the optimal allocation is obtained but also the corresponding system reliability, R_S should be greater than or equal to $R_{S,MIN} = 0.999$. Then the optimal redundancy allocation is shown in Table 8 and the computational output of iterations 8 and 9 are given in Appendix A-1.

Table 8. The Optimal Redundancy Allocation for the Type-2 Example

A_c	X_4-1	X_3-1	X_2-1	X_1-1	$f_4(A_c)$
63	6	6	5	3	0.99902

In phase 2, the optimal solution with decimal first place will be found in the same manner as in phase 1. Consequently, the optimal redundancy allocation and the optimal value are the same as the corresponding values in Table 8 but only the minimum cost solution may be 62.9 that is less than the integer solution. The computational output of iteration 9 will be given in Appendix A-2.

6. Concluding Remarks

The problem of redundancy allocation for the mixed series-parallel reliability system with $N-stages$ has been optimized in an optimal way. Two kinds of the reliability systems are considered for the parallel redundancy optimization in this paper. One is the maximization problem with a cost-constraint and another is the minimization problem subject to satisfying the requirement of the specified minimum reliability. They are approached as the optimization problems by the use of dynamic programming method.

The algorithm for solving the optimal redundancy allocation of the maximization problem has been proposed and then the developed computational program in FORTRAN language is applied to two typical numerical examples. The optimal solutions(redundancy optimization) can be shown as the computational results in the previous section.

The computational results assert that the performance of the algorithm works well and consequently the developed computer program package can be applied to the optimal redundancy allocation for the mixed reliability system. Hence, the program package developed by the dynamic programming method can be applied to this kind of the problems. And also this program may be extended to the models having $N-stages$ in general.

Appendix A-1 : The Edited Output for the Optimal Allocation and Value in Phase 1.

In the Case of $R_{S,MIN}$ = 0.999 and Min. of A_c = 55.0

Maximum No. of Iterations = 16,

★ ; Optimal Solution

X_1^l = 4 X_2^l = 5 X_3^l = 6 X_4^l = 5

A_c	X_4	X_3	X_2	X_1	R_s
54.80 - 59.30	5	6	5	4	.99747
59.30 - 62.80	5	6	5	5	.99790

*** Iter. No. = 8 at Stage 1 ***

A_c	X_4	X_3	X_2	X_1	R_s
55.00	5	6	5	4	.99747
60.00	5	6	5	5	.99790
62.00	5	6	5	5	.99790

*** Iter. No. = 8 at Stage 2 ***

A_c	X_4	X_3	X_2	X_1	R_s
55.00	5	6	5	4	.99747
60.00	5	6	6	4	.99820
62.00	5	6	7	4	.99838

*** Iter. No. = 8 at Stage 3 ***

A_c	X_4	X_3	X_2	X_1	R_s
55.00	5	6	5	4	.99747
60.00	5	6	6	4	.99820
62.00	5	7	6	4	.99871

*** Iter. No. = 8 at Stage 4 ***

A_c	X_4	X_3	X_2	X_1	R_s
55.00	5	6	5	4	.99747
60.00	6	6	6	4	.99846
62.00	6	7	6	4	.99897

X_1^l = 4 X_2^l = 5 X_3^l = 6 X_4^l = 5

A_c	X_4	X_3	X_2	X_1	R_s
54.80 - 59.30	5	6	5	4	.99747
59.30 - 63.80	5	6	5	5	.99790
63.80 - 63.80	5	6	5	6	.99796

*** Iter. No. = 9 at Stage 1 ***

A_c	X_4	X_3	X_2	X_1	R_s
55.00	5	6	5	4	.99747
60.00	5	6	5	5	.99790
63.00	5	6	5	5	.99790

*** Iter. No. = 9 at Stage 2 ***

A_c	X_4	X_3	X_2	X_1	R_s
55.00	5	6	5	4	.99747
60.00	5	6	6	4	.99820
63.00	5	6	6	5	.99863

*** Iter. No. = 9 at Stage 3 ***

A_c	X_4	X_3	X_2	X_1	R_s
55.00	5	6	5	4	.99747
60.00	5	6	6	4	.99820
63.00	5	8	6	4	.99886

*** Iter. No. = 9 at Stage 4 ***

A_c	X_4	X_3	X_2	X_1	R_s
55.00	5	6	5	4	.99747
60.00	6	6	6	4	.99846
63.00	**7**	**7**	**6**	**4**	**.99902** ★

Appendix A-2 : The Edited Output for the Optimal Allocation and Value in Phase 2.

In the Case of $R_{S,MIN}$ = 0.999 and Min. of A_c = 54.8

Maximum No. of Iterations = 11

★ : Optimal Solution

X_1^l = 4 X_2^l = 5 X_3^l = 6 X_4^l = 5

A_c	X_4	X_3	X_2	X_1	R_s
54.80 - 59.30	5	6	5	4	.99747
59.30 - 62.90	5	6	5	5	.99790

*** Iter. No. = 9 at Stage 1 ***

A_c	X_4	X_3	X_2	X_1	R_s
54.80	5	6	5	4	.99747
55.30	5	6	5	4	.99747
55.80	5	6	5	4	.99747
56.30	5	6	5	4	.99747
56.80	5	6	5	4	.99747
57.30	5	6	5	4	.99747
57.80	5	6	5	4	.99747
58.30	5	6	5	4	.99747
58.80	5	6	5	4	.99747
59.30	5	6	5	4	.99747
59.80	5	6	5	5	.99790
60.30	5	6	5	5	.99790
60.80	5	6	5	5	.99790
61.30	5	6	5	5	.99790
61.80	5	6	5	5	.99790
62.30	5	6	5	5	.99790
62.80	5	6	5	5	.99790
62.90	5	6	5	5	.99790

*** Iter. No. = 9 at Stage 2 ***

A_c	X_4	X_3	X_2	X_1	R_s
54.80	5	6	5	4	.99747
55.30	5	6	5	4	.99747
55.80	5	6	5	4	.99747
56.30	5	6	5	4	.99747
56.80	5	6	5	4	.99747
57.30	5	6	5	4	.99747
57.80	5	6	5	4	.99747
58.30	5	6	6	4	.99820
58.80	5	6	6	4	.99820
59.30	5	6	6	4	.99820
59.80	5	6	6	4	.99820
60.30	5	6	6	4	.99820
60.80	5	6	6	4	.99820
61.30	5	6	6	4	.99820
61.80	5	6	7	4	.99838
62.30	5	6	7	4	.99838
62.80	5	6	6	5	.99863
62.90	5	6	6	5	.99863

*** Iter. No. = 9 at Stage 3 ***

A_c	X_4	X_3	X_2	X_1	R_s
54.80	5	6	5	4	.99747
55.30	5	6	5	4	.99747
55.80	5	6	5	4	.99747
56.30	5	6	5	4	.99747
56.80	5	6	5	4	.99747
57.30	5	7	5	4	.99798
57.80	5	7	5	4	.99798
58.30	5	6	6	4	.99820
58.80	5	6	6	4	.99820
59.30	5	6	6	4	.99820
59.80	5	6	6	4	.99820
60.30	5	6	6	4	.99820
60.80	5	7	6	4	.99871
61.30	5	7	6	4	.99871
61.80	5	7	6	4	.99871
62.30	5	7	6	4	.99871
62.80	5	8	6	4	.99886
62.90	5	8	6	4	.99886

*** Iter. No. = 9 at Stage 4 ***

A_c	X_4	X_3	X_2	X_1	R_s	
54.80	5	6	5	4	.99747	
55.30	5	6	5	4	.99747	
55.80	5	6	5	4	.99747	
56.30	6	6	5	4	.99773	
56.80	6	6	5	4	.99773	
57.30	5	7	5	4	.99798	
57.80	5	7	5	4	.99798	
58.30	6	7	5	4	.99824	
58.80	6	7	5	4	.99824	
59.30	6	7	5	4	.99824	
59.80	6	6	6	4	.99846	
60.30	6	6	6	4	.99846	
60.80	5	7	6	4	.99871	
61.30	5	7	6	4	.99871	
61.80	6	7	6	4	.99897	
62.30	6	7	6	4	.99897	
62.80	6	7	6	4	.99897	
62.90	**7**	**7**	**6**	**4**	**.99902**	★

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