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Almost paracompactness and near paracompactness in L-smooth topological spaces

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ABSTRACT

We introduce in L-smooth topological spaces definitions of paracompactness, almost paracompactness and near paracompactness all of which turn out to be good extensions of their classical topological counterparts. These weak paracompactness are defined for arbitrary L-fuzzy sets and their properties studied.

1. Introduction

In 1985, Sostak[9] defined a fuzzy topology on a set X as a mapping $\tau: [0, 1]^X \rightarrow [0, 1]$ satisfying some natural axioms, where $[0, 1]^X$ denotes the family of all fuzzy subsets of X, and presented the fundamental concepts of such fuzzy topological spaces. In 1992, the same structure was rediscovered by Chattopadyay *et al.*[2]. They call the mapping $\tau: [0, 1]^X \rightarrow [0, 1]$ "a gradation of openness". In the same year, Ramadan [8] gave a similar definition of a fuzzy topology in Sostak's sense under the name of "smooth topological spaces", replacing [0, 1] by possibly more general lattices.

Paracompactness was studed in [0, 1]-fuzzy topological spaces by Malghan and Benchalli [7], Luo [6], and others. Bulbul and Warner [1] suggested good definitions of paracompactness, almost paracompactness and near paracompactness in [0, 1]-fuzzy topological spaces. Kudri [5] suggested a good definition of paracompactness in L-fuzzy topological spaces that is a good extension of classical paracompactness (given in, for example Willard [11]).

In this paper, based on the smooth compactness introduced in [4], we introduce in L-smooth topological spaces good definitions of paracompactness, almost paracompactness and near paracompactness. Defining these weaker forms of smooth paracompactness for arbitrary L-fuzzy sets we study their properties.

2. Preliminaries

Throughout this paper X and Y are non-empty ordinary sets and $L=L(\leq, \vee, \wedge, ')$ will denote a fuzzy lattice, i.e., a complete completely distributive lattice with a smallest element 0 and a largest element $1 (0 \neq 1)$ and with an order reversing involution $x \rightarrow x' (x \in L)$. We shall denotes by L^X the lattice of L-fuzzy subsets of X, by $P_r(L)$ the set of primes of L. For an ordinary subset A of X, we denote by χ_A the characteristic function of A.

A smooth L-fts [8,9] is an order pair (X, τ), where X is a non-empty set and $\tau: L^X \rightarrow L$ is a mapping satisfying the following conditions:

(O1) $\tau(\chi_\phi) = \tau(\chi_X) = 1$,

(O2) $\forall f, g \in L^X$, $\tau(f \wedge g) \geq \tau(f) \wedge \tau(g)$,

(O3) For every subfamily $\{f_i : i \in I\} \subseteq L^X$, $\tau(\vee_{i \in I} f_i) \geq \wedge_{i \in I} \tau(f_i)$.

Then the mapping $\tau: L^X \rightarrow L$ is called a smooth topology on X, $\tau(f)$ is called "the degree of openness of f" [8].

A mapping $\tau^*: L^X \rightarrow L$ is called a smooth cotopology [9] iff the following three conditions are satisfied:

(C1) $\tau^*(\chi_\phi) = \tau^*(\chi_X) = 1$,

(C2) $\forall f, g \in L^X$, $\tau^*(f \vee g) \geq \tau^*(f) \wedge \tau^*(g)$,

(C3) For every subfamily $\{f_i : i \in I\} \subseteq L^X$, $\tau^*(\wedge_{i \in I} f_i) \geq \wedge_{i \in I} \tau^*(f_i)$.

$\tau^*(f)$ is called the degree of closedness of f' and $\tau^*(f) = \tau(f')$[8].

Let (X, τ) be a smooth topological space and $f \in L^X$. Then, the τ-smooth closure (resp. τ-smooth interior) of f denoted by f^- (resp. f°) is defined by $f^- = \wedge \{g \in L^X : \tau^*(g) > 0, f \leq g\}$ (resp. $f^\circ = \vee \{g \in L^X : \tau(g) > 0, g \leq f\}$) [3]. And, for $f, g \in L^X$, we have the following [3]

(i) $f \leq g \Rightarrow f^\circ \leq g^\circ (f^- \leq g^-)$,

(ii) $\tau(f) > 0$ (resp. $\tau^*(f) > 0$) $\Rightarrow f = f^\circ$ (resp. $f = f^-$).

Let (X, τ_1) and (Y, τ_2) be two smooth topological spaces. A function $F: (X, \tau_1) \rightarrow (Y, \tau_2)$ is called smooth continuous (resp. smooth open) iff $\tau_1(F^{-1}(g)) \geq \tau_2(g)$ for every $g \in L^Y$ (resp. $\tau_1(h) \leq \tau_2(F(h))$ for every $h \in L^X$) [8]. And, if F is smooth continuous, then $(F^{-1}(g))^-$

$\leq F^{-1}(g^{-})$ for every $g\in L^Y$ [3].

Let (X, T) be an ordinary topological space, and $\alpha\in L$. A function $f:(X,T)\to L$, where L has its Scott topology (topology generated by the sets of the forms $\{t\in L: t\nleq p\}$ where $p\in P_r(L)$[10]) is said to be α-Scott continuous iff for every $p\in P_r(L)$ with $\alpha\nleq p$, $f^{-1}(\{t\in L; t\nleq p\})\in T$. It is clear that if f is Scott continuous then f is α-Scott continuous for every $\alpha\in L$. Moreover, f is 1-Scott continuous iff f is Scott continuous. Naturally, every function from (X, T) to L is 0-Scott continuous [4].

Halis Aygun *et al.* [4] proved that the mapping $W(T): L^X\to L$ defined by

$$W(T)(f) = \vee\{\alpha\in L; f \text{ is } \alpha\text{-Scott continuous}\}$$

for every $f\in L^X$, is a smooth L-fuzzy topology on X. Also the mapping $W(T_c): L^X\to L$ defined by

$$W(T_c)(f) = \vee\{\alpha\in L; f' \text{ is } \alpha\text{-Scott continuous}\}$$

for every $f\in L^X$, is a smooth cotopology on X.

Lemma 1[4] Let $(f_i)_{i\in I}$ be a family of functions from X to L. Then, for every $p\in L$

$$(\vee_{i\in I}f_i)^{-1}(\{t\in L: t\nleq p\})=\cup_{i\in I}f_i^{-1}(\{t\in L: t\nleq p\}).$$

Lemma 2. Let (X, T) be a topological space and $f\in L^X$. Considering the smooth L-fts $(X, W(T))$. Then,

(i) $(f^{-})^{-1}(\{t\in L: t\nleq p\})\subseteq(f^{-1}(\{t\in L: t\nleq p\}))^{-}$,

(ii) $(f^{\circ})^{-1}(\{t\in L: t\nleq p\})\subseteq(f^{-1}(\{t\in L: t\nleq p\}))^{\circ}$.

Proof. (i) We are going to prove that any closed set C in (X, T) with $f^{-1}(\{t\in L: t\nleq p\})\subseteq C$ satisfies $(f^{-})^{-1}(\{t\in L: t\nleq p\})\subseteq C$. Let $f^{-1}(\{t\in L: t\nleq p\})\subseteq C$, C is closed in (X, T) and let $g\in L^X$ defined by

$$g(x) = \begin{cases} 1 \text{ if } x\in C, \\ p \text{ otherwise.} \end{cases}$$

Since $\forall e\in L$, we have

$$g^{-1}(\{t\in L: t\geq e\}) = \begin{cases} X \text{ if } e\leq p, \\ C \text{ if } (e\nleq p). \end{cases}$$

We have $g^{-1}(\{t\in L: t\geq e\})$ is closed in (X, T), $\forall e\in L$, then g' is Scott continuous function from (X, T) to L with its Scott topology. Hence g' is 1-Scott continuous so $W(T)(g')=1$ and $W(T_c)(g)=1$. We also have $f\leq g$. Thus $f^{-}\leq g$. Then $C=g^{-1}(\{t\in L: t\nleq p\})\supseteq(f^{-})^{-1}(\{t\in L: t\nleq p\})$. Therefore, since $f^{-1}(\{t\in L: t\nleq p\})\subseteq(f^{-1}(\{t\in L: t\nleq p\}))^{-}$ and $(f^{-1}(\{t\in L: t\nleq p\}))^{-}$ is closed in (X, T) we have $(f^{-})^{-1}(\{t\in L: t\nleq p\})\subseteq(f^{-1}(\{t\in L: t\nleq p\}))^{-}$. (ii) It is obvious.

3. Definitions and its goodness

Definition 3.1 Let (X, τ) be a smooth L-fuzzy topological space and $g\in L^X$. A family $(f_i)_{i\in I}$ of L-fuzzy sets is said to be smooth locally finite in an L-fuzzy set g iff for every $p\in P_r(L)$ and each $x\in X$ with $g(x)\geq p'$, there are an L-fuzzy set r of X with $\tau(r)\nleq p$ and $r(x)\nleq p$ and a finite subset I_0 of I such that$(\forall z\in X)f_i(z)=0$ or $r(z)=0$, for every $i\in I-I_0$. When g is the whole space X, we shall directly say smooth locally finite, omitting "in an L-fuzzy set g".

Definition 3.2 Let (X, τ) be a smooth L-fts. A family $(f_i)_{i\in I}$ of L-fuzzy sets is said to be a refinement of the family $(g_j)_{j\in J}$ of L-fuzzy sets if and only if for each $i\in I$, there is $j\in J$ with $f_i\leq g_j$.

Definition 3.3 Let (X, τ) be a smooth L-fuzzy topological space and $g\in L^X$. The L-fuzzy set g is called:

(a) **smooth paracompact** iff for every $p\in P_r(L)$ and every collection $(f_i)_{i\in I}$ of L-fuzzy sets of X with $\tau(f_i)\nleq p$, $\forall i\in I$, such that

$$(\vee_{i\in I}f_i)(x)\nleq p,\ \forall x\in X \text{ with } g(x)\geq p'$$

there exists a family $(g_j)_{j\in J}$ of L-fuzzy sets such that $\tau(g_j)\nleq p$, $\forall j\in J$ that is a refinement of $(f_i)_{i\in I}$, smooth locally finite in g and

$$(\vee_{j\in J}g_j)(x)\nleq p,\ \forall x\in X \text{ with } g(x)\geq p'.$$

(b) **smooth almost paracompact** iff for every $p\in Pr(L)$ and every collection $(f_i)_{i\in I}$ of L-fuzzy sets of X with $\tau(f_i)\nleq p$, $\forall i\in I$ such that

$$(\vee_{i\in I}f_i)(x)\nleq p,\ \forall x\in X \text{ with } g(x)\geq p'$$

there exists a family $(g_j)_{j\in J}$ of L-fuzzy sets such that $\tau(g_j)\nleq p$, $\forall j\in J$ that is a refinement of $(f_i)_{i\in I}$, smooth locally finite in g and

$$(\vee_{j\in J}g_j^{-})(x)\nleq p,\ \forall x\in X \text{ with } g(x)\geq p'.$$

(c) **smooth lightly paracompact** iff for every $p\in P_r(L)$ and every countable collection $(f_i)_{i\in I}$ of L-fuzzy sets of X with $\tau(f_i)\nleq p$, $\forall i\in I$ such that

$$(\vee_{i\in I}f_i)(x)\nleq p,\ \forall x\in X \text{ with } g(x)\geq p'$$

there exists a family $(g_j)_{j\in J}$ of L-fuzzy sets such that $\tau(g_j)\nleq p$, $\forall j\in J$ that is a refinement of $(f_i)_{i\in I}$, smooth locally finite in g and

$$(\vee_{j\in J}g_j^{-})(x)\nleq p,\ \forall x\in X \text{ with } g(x)\geq p'.$$

(d) **smooth nearly paracompact** iff for every $p\in P_r(L)$ and every collection $(f_i)_{i\in I}$ of L-fuzzy sets of X with $\tau(f_i)\nleq p$, $\forall i\in I$ such that

$(\vee_{i\in I} f_i)(x) \not\leq p,\ \forall x\in X$ with $g(x)\geq p'$

there exists a family $(g_j)_{j\in J}$ of L-fuzzy sets such that $\tau(g_j)\not\leq p,\ \forall j\in J$ that is a refinement of $(f_i)_{i\in I}$, smooth locally finite in g and

$(\vee_{j\in J} g_j^{-o})(x)\not\leq p,\ \forall x\in X$ with $g(x)\geq p'$.

If the L-fuzzy set g is the whole space X, we say that the smooth L-fts (X, τ) is (a) smooth paracompact, (b) smooth almost paracompact, (c) smooth lightly paracompact, (d) smooth nearly paracompact.

The next theorem shows that smooth paracompactness is a good extension of the paracompactness property of general topological spaces.

Theorem 3.1 Let (X, T) be an ordinary topological space. Then (X, T) is paracompact, if and only if the smooth L-fts $(X, W(T))$ is smooth paracompact.

Proof. (**Necessity**) Let $p\in P_r(L)$ and $(f_i)_{i\in I}$ be a family of L-fuzzy sets in $(X, W(T))$ with $W(T)(f_i)\not\leq p$, $\forall i\in I$ and

$(\vee_{i\in I} f_i)(x)\not\leq p,\ \forall x\in X.$

From $W(T)(f_i)\not\leq p,\ \forall i\in I$, we have $f_i^{-1}(\{t\in L ; t\not\leq p\})\in T\ \forall i\in I$. Also from $(\vee_{i\in I} f_i)(x)\not\leq p\ \forall x\in X$, we have by Lemma(1) that $X\subseteq \cup_{i\in I} f_i^{-1}(\{t\in L : t\not\leq p\})$. From the paracompactness of (X, T), $\beta=(f_i^{-1}(\{t\in L : t\not\leq p\}))_{i\in I}$ has a locally finite open refinement Φ that covers X. For each $C\in\Phi$, take $f_{iC}\in(f_i)_{i\in I}$ such that $C\subset f_{iC}^{-1}(\{t\in L : t\not\leq p\})$ (Φ is a refinement of β). Now consider the family $\psi=(\chi_C\wedge f_{iC})_{C\in\Phi}$. Then, $W(T)(\chi_C\wedge f_{iC})_{C\in\Phi}\geq W(T)(\chi_C)\wedge W(T)(f_{iC})\not\leq p$ since the characteristic function of every open set is 1-Scott continuous. To show that ψ is smooth locally finite, take $x\in X$. Since Φ is locally finite, there exists an open neighborhood U of x in T such that $U\cap C=\phi$ for all but at most finitely many $C\in\Phi$. Then, for each $x\in X$ and $\forall p\in P_r(L)$, there exists an L-fuzzy set $r=\chi_U$ and $W(T)(r)=1\not\leq p$ with $r(x)=1\not\leq p$ such that $(\forall z\in X)h(z)=0$ or $r(z)=0$ for all but finitely many $h\in\psi$, because, $h(z)\neq 0$ and $r(z)\neq 0$ if and only if $z\in U\cap C$ and we have $U\cap C\neq\phi$ only for a finite number of $C\in\Phi$. Hence, ψ is a smooth locally finite. To show ψ is a refinement of $(f_i)_{i\in I}$, take $h=\chi_C\wedge f_{iC}\in\psi$, there is $g=f_{iC}\in(f_i)_{i\in I}$ such that $h\leq g$. Finally,

$(\vee_{h\in\psi} h)(x)\not\leq p,\ \forall x\in X$

because, for every $x\in X$, there is $C^*\in\Phi$ such that $x\in C^*$ which implies that $\chi_{C^*}(x)=1\not\leq p$ and $f_{iC^*}(x)\not\leq p$, so $(\chi_{C^*}\wedge f_{iC^*})(x)\not\leq p$. Hence $(\vee_{h\in\psi} h)(x)\not\leq p,\ \forall x\in X$ and $(X, W(T))$ is smooth paracompact.

(**Sufficiency**) Let $(A_i)_{i\in I}$ be an open cover of (X, T). Then $(\chi_{A_i})_{i\in I}$ is a family of L-fuzzy sets in $(X, W(T))$ such that $W(T)(\chi_{A_i})_{i\in I}=1\not\leq p$ and

$(\vee_{i\in I}\chi_{A_i})(x)\not\leq p,\ \forall x\in X$ and $\forall p\in P_r(L)$.

From the smooth paracompactness of $(X, W(T))$, there exists a smooth locally finite refinement Φ of $(\chi_{A_i})_{i\in I}$ such that $W(T)(f)\not\leq p$, $f\in\Phi$ with

$(\vee_{f\in\Phi} f)(x)\not\leq p,\ \forall x\in X$ and $\forall p\in P_r(L)$.

Then, $\beta=(f^{-1}(\{t\in L : t\not\leq p\}))_{f\in\Phi}$ is an open cover of (X, T). To show that β is a locally finite, take $x\in X$. Since Φ is smooth locally finite, there exists $r\in L^X$ with $W(T)(r)\not\leq p$ and $r(x)\not\leq p$ such that $(\forall z\in X)f(z)=0$ or $r(z)=0$ for all but finitely many $f\in\Phi$, say $f_1,\cdots,f_m$. Let $U=r^{-1}(\{t\in L : t\not\leq p\})$. Then $U\in T$ and $x\in U$. We also have that $f^{-1}(\{t\in L : t\not\leq p\})\cap U\neq\phi$ for at most finitely many $f\in\Phi$, because if $y\in f^{-1}(\{t\in L : t\not\leq p\})\cap U$ then $r(y)\not\leq p$ and $f(y)\not\leq p$ which implies that $f\in\{f_1,\cdots,f_m\}$. Therefore β is locally finite. Finally β is a refinement of $(A_i)_{i\in I}$, because for each $f\in\Phi$ there is $i_o\in I$ such that $f\leq\chi A_{i_o}$, so $f^{-1}(\{t\in L : t\not\leq p\})\subseteq(\chi_{A_{i_o}})^{-1}(\{t\in L : t\not\leq p\})=\chi_{A_{i_o}}$. Thus, (X, T) is paracompact.

The Proof provides the model, and the notation, for the following theorems. We indicate only the details where difference occur.

Theorem 3.2 Let (X, T) be an ordinary topological space. If $(X, W(T))$ is smooth almost paracompact, then (X, T) is almost paracompact.

Proof. Assuming $(X, W(T))$ to be smooth almost paracompact we obtain a smooth locally finite refinement Φ of $(\chi_{A_i})_{i\in I}$ such that

$(\vee_{f\in\Phi} f^{-})(x)\not\leq p,\ \forall x\in X.$

Again $\beta=(f^{-1}(\{t\in L : t\not\leq p\}))_{f\in\Phi}$ is an open locally finite refinement of $(A_i)_{i\in I}$. And $X\subseteq\cup_{f\in\Phi}(f^{-1}(\{t\in L : t\not\leq p\}))^{-}$. To see this, let $x\in X$. Then since $(\vee_{f\in\Phi} f^{-})(x)\not\leq p$, there exists $f_o\in\Phi$ such that $f_o^{-}(x)\not\leq p$, so $x\in(f_o^{-})^{-1}(\{t\in L : t\not\leq p\})\subseteq(f_o^{-1}(\{t\in L : t\not\leq p\}))^{-}$ (by Lemma(2)).

This complete the proof.

Theorem 3.3 Let (X, T) be an ordinary topological space. If $(X, W(T))$ is smooth lightly paracompact, then (X, T) is lightly paracompact.

Proof. This follows in the same way as in Theorem 3.2, considering the index set I to be countable.

Theorem 3.4 Let (X, T) be an ordinary topological space. If $(X, W(T))$ is smooth nearly paracompact, then (X, T) is nearly paracompact.

Proof. Assuming $(X, W(T))$ to be smooth nearly paracompact we obtain a smooth locally finite refinement Φ of $(\chi_{A_i})_{i\in I}$ such that

$(\vee_{f\in\Phi} f^{-o})(x) \not\leq p, \ \forall x\in X.$

Again $\beta = (f^{-1}(\{t\in L : t\not\leq p\}))_{f\in\Phi}$ is an open locally finite refinement of $(A_i)_{i\in I}$. And $X\subseteq(\cup_{f\in\Phi} f^{-1}(\{t\in L : t\not\leq p\}))^{-o}$. To see this, let $x\in X$. Then since $(\vee_{f\in\Phi} f^{-o})(x) \not\leq p$ there exists $f_o\in\Phi$ such that $f_o^{-o}(x) \not\leq p$, so $x\in (f_o^{-o})^{-1}(\{t\in L : t\not\leq p\}) \subseteq (f_o^{-1}(\{t\in L : t\not\leq p\}))^{-o}$ (by Lemma(2)).

This complete the proof.

4. Relations between the types of paracompactness

Proposition 4.1 Smooth paracompactness $\Rightarrow$ smooth near paracompactness $\Rightarrow$ smooth almost paracompactness.

Proof. Let (X, τ) be a smooth paracompact. Then, for $p\in P_r(L)$ and every collection $(f_i)_{i\in I}$ of L-fuzzy sets of X with $\tau(f_i) \not\leq p, \ \forall i\in I$ such that

$(\vee_{i\in I} f_i)(x) \not\leq p, \ \forall x\in X$

there exists a family $(g_j)_{j\in J}$ of L-fuzzy sets such that $\tau(g_j) \not\leq p, \ \forall j\in J$ that is a refinement of $(f_i)_{i\in I}$, smooth locally finite in X and

$(\vee_{j\in J} g_j)(x) \not\leq p, \ \forall x\in X.$

Now, since $\tau(g_j) \not\leq p, \ \forall j\in J$, we have $g_j=g_j^o, \ \forall j\in J$. Thus, $g_j=g_j^o \leq g_j^{-o}, \ \forall j\in J$. Therefore $(\vee_{j\in J} g_j)(x)\leq (\vee_{j\in J} g_j^{-o})(x) \not\leq p$ and hence the smooth L-fts (X, τ) is smooth nearly paracompact.

For the second implication, assuming (X, τ) to be smooth nearly paracompact we obtain a smooth locally finite refinement $(g_j)_{j\in J}$ of $(f_i)_{i\in I}$ such that $\tau(g_j) \not\leq p \ \forall j\in J$ and

$(\vee_{j\in J} g_j^{-o})(x) \not\leq p, \ \forall x\in X.$

Since, $g_j=g_j^o \leq (g_j)^{-o} \leq g_j^{-}, \ \forall j\in J$, it is obvious that

$(\vee_{j\in J} g_j^{-})(x) \not\leq p, \ \forall x\in X.$

Hence the smooth L-fts (X, τ) is smooth almost paracompact.

In fuzzy topological spaces, the converses of these two implications are not valid for paracompactness, near paracompactnesss and almost paracompactness which are nothing but special cases of smooth paracompactness, smooth near paracompactness and smooth almost paracompactness, respectively. Thus, the converse implications in Proposition 4.1, are not true, in general.

Definition 4.1 A smooth L-fts (X, τ) is smooth regular iff for every $p\in P_r(L)$ and each $f\in L^X$ satisfying $\tau(f) \not\leq p$ can be written as

$f=\vee\{g\in L^X: \ \tau(g)\geq\tau(f), \ g^{-}\leq f\}.$

Theorem 4.1 A smooth almost paracompact and smooth regular space (X, τ) is smooth paracompact.

Proof. Suppose that (X, τ) is smooth almost paracompact. Let $p\in P_r(L)$ and $(f_i)_{i\in I}$ be a collection of L-fuzzy sets of X with $\tau(f_i) \not\leq p, \ \forall i\in I$ such that

$(\vee_{i\in I} f_i)(x) \not\leq p, \ \forall x\in X$

there exists a family $(g_j)_{j\in J}$ of L-fuzzy sets such that $\tau(g_j) \not\leq p, \ \forall j\in J$, that is a refinement of $(f_i)_{i\in I}$, smooth locally finite in X and

$(\vee_{j\in J} g_j^{-})(x) \not\leq p, \ \forall x\in X.$

From smooth regularity of (X, τ), it follows:

$g_j=\vee\{h_j\in L^X : \ \tau(h_j)\geq\tau(g_j), \ h_j^{-}\leq g_j\}.$

Then, $g_j^{-} = g_j, \ \forall j\in J$ and so,

$(\vee_{j\in J} g_j)(x) \not\leq p, \ \forall x\in X.$

Hence, (X, τ) is smooth paracompact.

Theorem 4.2 A smooth nearly paracompact and smooth regular space (X, τ) is smooth paracompact.

Proof. The proof is quite similar to the proof of Theorem 4.1, taking into account that the smooth interior of a fuzzy set remains always smaller than the fuzzy set itself.

Theorem 4.3 Let $F : (X, \tau_1)\rightarrow(Y, \tau_2)$ be a smooth continuous, smooth open mapping and let g be a smooth almost paracompact L-fuzzy subset of (X, τ_1). Then $F(g)$ is a smooth almost paracompact of (Y, τ_2).

Proof. Let $p\in P_r(L)$ and $(f_i)_{i\in I}$ be a collection of L-fuzzy sets of Y with $\tau_2(f_i) \not\leq p, \ \forall i\in I$ such that

$(\vee_{i\in I} f_i)(y) \not\leq p, \ \forall y\in Y$ with $F(g)(y)\geq p'$

Then from the smooth continuity of F, $(F^{-1}(f_i))_{i\in I}$ is a family of L-fuzzy sets of X with $\tau_2(f_i)\leq \tau_1(F^{-1}(f_i)) \not\leq p, \ \forall i\in I$. We also have

$(\vee_{i\in I} F^{-1}(f_i))(x) \not\leq p, \ \forall x\in X$ with $g(x)\geq p'$

because, if $g(x)\geq p'$, then $F(g)(F(x))\geq p'$, so

$(\vee_{i\in I} F^{-1}(f_i))(x) = (\vee_{i\in I} f_i)(F(x)) \not\leq p.$

From smooth almost paracompactness of g in (X, τ_1), there exists a family $(h_j)_{j\in J}$ of L-fuzzy sets in (X, τ_1) such that $\tau_1(h_j) \not\leq p, \ \forall j\in J$, that is refinement of

$(F^{-1}(f_i))_{i\in I}$, smooth locally finite in g and

$$(\vee_{j\in J} h_j^-)(x) \not\leq p,\ \forall x\in X \text{ with } g(x)\geq p'.$$

Then, by the smooth openness of F, $(F(h_j))_{j\in J}$ is a family of L-fuzzy sets in (Y, τ_2), with $\tau_1(h_j)\leq \tau_2(F(h_j)) \not\leq p$, $\forall j\in J$, that is a refinement of $(f_i)_{i\in I}$ smooth locally finite in $F(g)$. We also have that

$$(\vee_{j\in J}(F(h_j))^-)(y) \not\leq p \quad \forall y\in Y \text{ with } F(g)(y)\geq p'.$$

In fact, if $F(g)(y)\geq p'$, then $\vee_{x\in F^{-1}(y)} g(x)\geq p'$ which implies that there is $x\in X$ with $g(x)\geq p'$ and $F(x)=y$. So,

$$\begin{aligned}(\vee_{j\in J}(F(h_j))^-)(y) &= (\vee_{j\in J}(F(h_j))^-)(F(x))\\ &= (\vee_{j\in J} F^{-1}(F(h_j))^-))(x)\\ &\geq (\vee_{j\in J}(F^{-1}F(h_j))^-)(x)\\ &\qquad \text{(by smooth continuity)}\\ &\geq (\vee_{j\in J} h_j^-)(x) \not\leq p.\end{aligned}$$

Thus, $F(g)$ is smooth almost paracompact.

Theorem 4.4 Let $F : (X, \tau_1)\rightarrow(Y, \tau_2)$ be a smooth continuous, smooth open mapping and let g be a smooth nearly paracompact L-fuzzy subset of (X, τ_1). Then $F(g)$ is a smooth almost paracompact of (Y, τ_2).

Proof. Similar to the proof of Theorem 4.3.

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