Output Feedback Fuzzy H^∞ Control of Nonlinear Systems with Time-Varying Delayed State

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Abstract: This paper presents an output feedback fuzzy H^∞ control problem for a class of nonlinear systems with time-varying delayed state. The Takagi-Sugeno fuzzy model is employed to represent a nonlinear systems with time-varying delayed state. Using a single quadratic Lyapunov function, the globally exponential stability and disturance attenuation of the closed-loop fuzzy control system are discussed. Sufficient conditions for the existence of fuzzy H^∞ controllers are given in terms of matrix inequalities. Constructive algorithm for design of fuzzy H^∞ controller is also developed. A simulation example is given to illustrate the performance of the proposed design method.

Keywords: Nonlinear systems, time delay systems, output feedback, fuzzy H^∞ control, linear matrix inequality

I. Introduction

There have been significant research efforts on the stability analysis and systematic design for fuzzy control systems [1]-[7]. These methods are conceptually simple and straightforward. The nonlinear system is represented by a Takagi-Sugeno(T-S)-type fuzzy model. And then, the control design is carried out on the basis of the fuzzy model via the so-called parallel distributed compensation scheme. Since uncertainties are frequently a source of instability, Tanaka *et al.* [5], [6] presented stability analysis for a class of uncertain nonlinear systems and method for designing robust fuzzy controllers to stabilize the uncertain nonlinear systems. However, all the above design method has to predetermine the state feedback gains before checking the stability condition of the closed-loop system. In real control problems, all of the states are not available, thus it is necessary to design output feedback controller. Ma *et al.* [8] presented the analysis and design of the fuzzy controller and fuzzy observer on the basis of T-S fuzzy model using separation property. Tanaka *et al.* [9] also presented systematic design method of the fuzzy regulator and fuzzy observer on the basis of T-S fuzzy model.

The H^∞ control approach is concerned with the design of controller which stabilizes a system while satisfying an H^∞-norm bound constraint on disturbance attenuation [10]-[12]. Over past a few years, H^∞ control of T-S fuzzy model are paid a lot of attention in fuzzy control [13]-[15]. Han and Feng [13] and Hong and Langari [14] presented H^∞ controller design for fuzzy dynamic systems with state feedback. Chen *et al.* [15] presented the design method of an observer-based fuzzy H^∞ controller via an LMI approach. Since time delay is frequently a source of instability and encountered in various engineering systems, the H^∞ control problem for delayed systems has received considerable attention over the last few decades [16]-[18].

However for a fuzzy control system, there are few publications on H^∞ control design for delayed systems.

In this paper, we design an output feedback fuzzy H^∞ controller for fuzzy dynamic system with time-varying delayed state. The controller design is carried out on the basis of the T-S fuzzy model and the resulting controller is tuned on-line based on fuzzy operations. A sufficient condition is derived such that the closed-loop system is globally exponentially stable and L^2 gain of the input-output map is bounded. Based on the derivation, constructive algorithm of LMI-based fuzzy H^∞ controller are presented.

II. Problem formulation

The continuous fuzzy dynamic model, proposed by Takagi and Sugeno, is described by fuzzy IF-THEN rules which represented local linear input-output relations of nonlinear system. Consider a nonlinear system with time varying delay that can be described by the following T-S fuzzy model with time-varying delay:

Plant Rule i:

$$\begin{aligned}&\textbf{IF } z_1(t) \text{ is } M_{i1} \text{ and } \cdots \text{ and } z_g(t) \text{ is } M_{ig}\\ &\textbf{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{A}_{d_i}\mathbf{x}(t-d(t)) + \mathbf{B}_{1_i}\mathbf{w}_1(t) \\ &\qquad + \mathbf{B}_{2_i}\mathbf{u}(t) \qquad (1)\\ &\qquad \mathbf{e}_1(t) = \mathbf{C}_i\mathbf{x}(t), \qquad \mathbf{e}_2(t) = \mathbf{u}(t)\\ &\qquad \mathbf{y}(t) = \mathbf{C}_{y_i}\mathbf{x}(t) + \mathbf{w}_2(t), \quad i = 1, 2, \cdots, r\\ &\qquad \mathbf{x}(t) = 0, \quad t \le 0,\end{aligned}$$

where M_{ij} is the fuzzy set, $\mathbf{x}(t) \in \mathbf{R}^n$ is the state vector, $\mathbf{u}(t) \in \mathbf{R}^{k_1}$ is the control input vector, $[\mathbf{w}_1^T(t)\ \mathbf{w}_2^T(t)]^T \in \mathbf{R}^p$ is the square-integrable disturbance input vector, $\mathbf{y}(t) \in \mathbf{R}^{k_2}$ is the measured output, $[\mathbf{e}_1^T(t)\ \mathbf{e}_2^T(t)]^T \in \mathbf{R}^q$ is the controlled output vector, r is the number of **IF-THEN** rules, $z_1 \sim z_g$ are some measurable system variables, i.e., the premise variables, and all matrices are constant matrices with appropriate dimensions, $d(t)$ are the time-varying delay with following assumptions:

$$0 \le d(t) < \infty, \ \dot{d}(t) \le \beta < 1. \qquad (2)$$

Let $\mu_i(\mathbf{z}(t))$ be the normalized membership function of the inferred fuzzy set $h_i(z(t))$, i.e.,

$$\mu_i(\mathbf{z}(t)) \;=\; h_i(\mathbf{z}(t))/\sum_{i=1}^{r} h_i(\mathbf{z}(t)), \qquad (3)$$

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where

$$h_i(\mathbf{z}(t)) = \prod_{j=1}^{g} M_{ij}(z_j(t))$$
$$\mathbf{z}(t) = [z_1(t)\, z_2(t) \cdots z_g(t)]^T. \quad (4)$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. It is assumed that

$$h_i(\mathbf{z}(t)) \geq 0, \quad i = 1, 2, \cdots, r$$
$$\sum_{i=1}^{r} h_i(\mathbf{z}(t)) > 0, \quad (5)$$

for all t. Then we can obtain the following conditions:

$$\mu_i(\mathbf{z}(t)) \geq 0, \quad i = 1, 2, \cdots, r$$
$$\sum_{i=1}^{r} \mu_i(\mathbf{z}(t)) = 1, \quad (6)$$

for all t. Let $\mathcal{P} \in \mathbf{R}^r$ be the set of membership function satisfying (6). Given a pair of $(\mathbf{y}(t), \mathbf{u}(t))$, by using a center average defuzzifer, product inference, and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{A}(\mu)\mathbf{x}(t) + \mathbf{A}_d(\mu)\mathbf{x}(t - d(t)) + \mathbf{B}_1(\mu)\mathbf{w}_1(t) \\
&\quad + \mathbf{B}_2(\mu)\mathbf{u}(t) \quad (7) \\
\mathbf{e}_1(t) &= \mathbf{C}(\mu)\mathbf{x}(t) \\
\mathbf{e}_2(t) &= \mathbf{u}(t) \\
\mathbf{y}(t) &= \mathbf{C}_y(\mu)\mathbf{x}(t) + \mathbf{w}_2(t),
\end{aligned}$$

where $\mu = [\mu_1, \mu_2, \cdots, \mu_r] \in \mathcal{P}$,

$$\begin{aligned}
\mathbf{A}(\mu) &= \sum_{i=1}^{r} \mu_i(z(t))\mathbf{A}_i, \quad \mathbf{A}_d(\mu) = \sum_{i=1}^{r} \mu_i(z(t))\mathbf{A}_{d_i}, \\
\mathbf{B}_1(\mu) &= \sum_{i=1}^{r} \mu_i(z(t))\mathbf{B}_{1_i}, \quad \mathbf{B}_2(\mu) = \sum_{i=1}^{r} \mu_i(z(t))\mathbf{B}_{2_i}, \\
\mathbf{C}(\mu) &= \sum_{i=1}^{r} \mu_i(z(t))\mathbf{C}_i, \quad \mathbf{C}_y(\mu) = \sum_{i=1}^{r} \mu_i(z(t))\mathbf{C}_{y_i}, \\
\mathbf{E}_x(\mu) &= \sum_{i=1}^{r} \mu_i(z(t))\mathbf{E}_{x_i}. \quad (8)
\end{aligned}$$

As a fuzzy H^∞ controller of the fuzzy system (1), we consider the following output feedback controller:

$$\begin{aligned}
\dot{\hat{\mathbf{x}}}(t) &= \mathbf{A}_k(\mu)\hat{\mathbf{x}}(t) + \mathbf{B}_k(\mu)\mathbf{y}(t); \quad \hat{\mathbf{x}}(0) = 0 \quad (9) \\
\mathbf{u}(t) &= \mathbf{C}_k(\mu)\hat{\mathbf{x}}(t),
\end{aligned}$$

where matrix functions $\mathbf{A}_k(\mu), \mathbf{B}_k(\mu)$, and $\mathbf{C}_k(\mu)$ are to be determined and tuned on-line based on fuzzy operation, i.e. $\mathbf{A}_k(\mu), \mathbf{B}_k(\mu)$, and $\mathbf{C}_k(\mu)$ are function of membership functions. From (7) and (9), we obtain the following closed-loop system

$$\begin{aligned}
\dot{\zeta} &= \hat{\mathbf{A}}(\mu)\zeta(t) + \hat{\mathbf{A}}_1(\mu)\mathbf{x}(t - d(t)) + \hat{\mathbf{B}}(\mu)\mathbf{w}(t) \\
\mathbf{e}(t) &= \hat{\mathbf{C}}(\mu)\zeta(t) \quad (10) \\
\zeta(t) &= 0, \quad t \leq 0,
\end{aligned}$$

where $(t) = [\mathbf{x}^T(t), \hat{\mathbf{x}}^T(t)]^T$, $\mathbf{w}(t) = [\mathbf{w}_1^T(t), \mathbf{w}_2^T(t)]^T$, and $\mathbf{e}(t) = [\mathbf{e}_1^T(t), \mathbf{e}_2^T(t)]^T$,

$$\begin{aligned}
\hat{\mathbf{A}}(\mu) &= \begin{bmatrix} \mathbf{A}(\mu) & \mathbf{B}_2(\mu)\mathbf{C}_k(\mu) \\ \mathbf{B}_k(\mu)\mathbf{C}_y(\mu) & \mathbf{A}_k(\mu) \end{bmatrix}, \\
\hat{\mathbf{A}}_1(\mu) &= \begin{bmatrix} \mathbf{A}_d^T(\mu) & 0 \end{bmatrix}^T, \\
\hat{\mathbf{B}}(\mu) &= \begin{bmatrix} \mathbf{B}_1(\mu) & 0 \\ 0 & \mathbf{B}_k(\mu) \end{bmatrix}, \quad (11) \\
\hat{\mathbf{C}}(\mu) &= \begin{bmatrix} \mathbf{C}(\mu) & 0 \\ 0 & \mathbf{C}_k(\mu) \end{bmatrix}.
\end{aligned}$$

We define a variable including all parameters of controller

$$\mathbf{K}(\mu) = \begin{bmatrix} 0 & \mathbf{C}_k(\mu) \\ \mathbf{B}_k(\mu) & \mathbf{A}_k(\mu) \end{bmatrix}, \quad (12)$$

and we introduce the abbreviations:

$$\begin{aligned}
\mathbf{A}_o(\mu) &= \begin{bmatrix} \mathbf{A}(\mu) & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{B}_{1o}(\mu) = \begin{bmatrix} \mathbf{B}_1(\mu) & 0 \\ 0 & 0 \end{bmatrix}, \\
\mathbf{B}_{00}(\mu) &= \begin{bmatrix} \mathbf{B}_2(\mu) & 0 \\ 0 & \mathbf{I} \end{bmatrix}, \quad \mathbf{C}_{00}(\mu) = \begin{bmatrix} \mathbf{C}_y(\mu) & 0 \\ 0 & \mathbf{I} \end{bmatrix}, \\
\mathbf{D}_{yo}(\mu) &= \begin{bmatrix} 0 & \mathbf{I} \\ 0 & 0 \end{bmatrix}, \quad \mathbf{C}_{1o}(\mu) = \begin{bmatrix} \mathbf{C}(\mu) & 0 \\ 0 & 0 \end{bmatrix}, \\
\mathbf{D}_{20}(\mu) &= \begin{bmatrix} 0 & 0 \\ \mathbf{I} & 0 \end{bmatrix}, \quad (13)
\end{aligned}$$

then the closed-loop matrices of (10) can be written as

$$\begin{aligned}
\hat{\mathbf{A}}(\mu) &= \mathbf{A}_o(\mu) + \mathbf{B}_{00}(\mu)\mathbf{K}(\mu)\mathbf{C}_{00}(\mu), \\
\hat{\mathbf{B}}(\mu) &= \mathbf{B}_{1o}(\mu) + \mathbf{B}_{00}(\mu)\mathbf{K}(\mu)\mathbf{D}_{yo}(\mu), \\
\hat{\mathbf{C}}(\mu) &= \mathbf{C}_{1o}(\mu) + \mathbf{D}_{20}(\mu)\mathbf{K}(\mu)\mathbf{C}_{00}(\mu). \quad (14)
\end{aligned}$$

Note that (13) involves only plant data and all matrices of (14) are affine form of the controller data $\mathbf{K}(\mu)$. For a given γ, we define L_2 gain γ-performance of the system (10) as the quantity

$$\int_0^T ||\mathbf{e}(t)||^2 dt \leq \gamma^2 [\int_0^T ||\mathbf{w}(t)||^2 dt], \quad (15)$$

for all $T > 0$ and all $\mathbf{w} \in \mathbf{L}_2[0\ \ T]$, where $||\cdot||$ denotes the Euclidean norm.

This paper addresses designing an output feedback fuzzy H^∞ controller (9) for the system (7) such that the closed-loop system is globally exponentially stable and achieves $\mathbf{L}_2$ gain γ-performance (**Globally ES-γ**).

III. Stability and L_2 norm analysis

Lemma 1 : Consider the unforced system of (10). If there exist matrices $\mathbf{P} > 0$ and $\mathbf{S}_{11} > 0$, and positive scalars λ and α satisfying the following inequalities

$$\begin{bmatrix} \hat{\mathbf{A}}(\mu)^T\mathbf{P} + \mathbf{P}\hat{\mathbf{A}}(\mu) + \hat{\mathbf{S}} + \hat{\mathbf{S}}_1 & \mathbf{P}\hat{\mathbf{A}}_1(\mu) \\ \hat{\mathbf{A}}_1^T(\mu)\mathbf{P} & -\hat{\mathbf{S}}_{11} \end{bmatrix} \leq 0, \quad (16)$$

for all $\mu \in \mathcal{P}$, where β is defined in (2) and

$$\begin{aligned}
\hat{\mathbf{S}} &= \begin{bmatrix} \mathbf{S}_{11} & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{\mathbf{S}}_1 = \begin{bmatrix} \alpha\mathbf{I} & 0 \\ 0 & 0 \end{bmatrix}, \quad (17) \\
\hat{\mathbf{S}}_{11} &= (1 - \beta)\mathbf{S}_{11},
\end{aligned}$$

then the equilibrium of the unforced system of (10) is globally exponentially stable.

Proof: Define a Lyapunov functional

$$V(\zeta,t) = \zeta^T(t)P\zeta(t) + \int_{t-d(t)}^{t} \mathbf{x}(\tau)^T\mathbf{S}_{11}\mathbf{x}(\tau)d\tau, \quad (18)$$

where $\mathbf{P} > 0$ and $\mathbf{S}_{11} > 0$. Then there exist scalars $\delta_1 > 0$ and $\delta_2 > 0$ such that $\delta_1||\zeta||^2 \leq V(\zeta,t) \leq \delta_2||\zeta||^2$. If there exists scalar $\alpha > 0$ such that $\dot{V}(\zeta,t) \leq -\alpha||\mathbf{x}||^2$, then the unforced system of (10) is globally exponentially stable [19]. From assumption (2)

$$\begin{aligned}\dot{V}(\zeta,t) \leq{}& \dot{\zeta}^T(t)\mathbf{P}\zeta(t) + \zeta^T(t)\mathbf{P}\dot{\zeta}(t) + \mathbf{x}(t)^T\mathbf{S}_{11}\mathbf{x}(t)\\ &- \mathbf{x}(t-d(t))^T\hat{\mathbf{S}}_{11}\mathbf{x}(t-d(t))\\ :={}& \dot{V}_a(\zeta,t). \quad (19)\end{aligned}$$

By using the Schur complement[20], the condition $\dot{\mathbf{V}}_a(\zeta,t) \leq -\alpha||\mathbf{x}||^2$ for all μ is equivalent to (16). ■

Lemma 2: Consider the system (10) and let $\gamma > 0$ be a given scalar. If there exist matrices $\mathbf{P} > 0$ and $\mathbf{S}_{11} > 0$, and positive scalar λ and α satisfying the following inequalities for all $\mu \in \mathcal{P}$

$$\begin{bmatrix} \hat{\mathbf{A}}(\mu)^T\mathbf{P} + \mathbf{P}\hat{\mathbf{A}}(\mu) + \hat{\mathbf{S}} + \hat{\mathbf{S}}_1 & \mathbf{P}\hat{\mathbf{A}}_1(\mu) & \mathbf{P}\hat{\mathbf{B}}(\mu) & \hat{\mathbf{C}}^T(\mu) \\ \hat{\mathbf{A}}_1^T(\mu)\mathbf{P} & -\hat{\mathbf{S}}_{11} & 0 & 0 \\ \hat{\mathbf{B}}^T(\mu)\mathbf{P} & 0 & -\gamma^2\mathbf{I} & 0 \\ \hat{\mathbf{C}}(\mu) & 0 & 0 & -\mathbf{I} \end{bmatrix} \leq 0, \quad (20)$$

where $\hat{\mathbf{S}}, \hat{\mathbf{S}}_1$, and $\hat{\mathbf{S}}_{11}$ are given by (17), then the corresponding closed-loop system (10) is globally exponentially stable and achieves L_2 gain γ-performance for all $\mathbf{w}(t)$.

Proof: The matrices $\mathbf{P} > 0$ and $\mathbf{S}_{11} > 0$, and positive scalars λ and α satisfying (20) also satisfy the inequalities (16). Using a Lyapunov functionals (18), (19) and following condition

$$J_a(t) := \dot{V}_a(\zeta,t) + \mathbf{e}^T(t)\mathbf{e}(t) - \gamma^2\mathbf{w}^T(t)\mathbf{w}(t) \leq 0, \quad (21)$$

inequality (20) is obtained. Since $V(\mathbf{x}(T)) > 0$, the condition (21) implies $\int_0^T ||\mathbf{e}(t)||^2dt \leq \gamma^2[\int_0^T ||\mathbf{w}(t)||^2dt]$. ■

IV. Fuzzy model-based robust H^∞ controller design

By applying the result of lemma 3.2, we present a sufficient condition of the existence of fuzzy H^∞ controllers for the T-S fuzzy model (7) and explain how to construct H^∞ controllers.

Definition 1: Given the system (7) and $\gamma > 0$. The **Globally ES-γ** problem is solvable if there exist a finite dimensional controller (9) and matrices $\mathbf{P} > 0$ and $\mathbf{S}_{11} > 0$, and positive scalars λ and α satisfying the inequality (20) for all $\mu \in \mathcal{P}$.

Using the notations of (12) and (13), define the left-hand side of inequality (20) as

$$\begin{aligned}\mathbf{G}(\mu) :={}& \mathbf{\Phi}(\mu) + \mathbf{\Sigma\Pi}(\mu)\mathbf{K}(\mu)\mathbf{\Theta}^T(\mu)\\ &+\mathbf{\Theta}(\mu)\mathbf{K}^T(\mu)\mathbf{\Pi}^T(\mu)\mathbf{\Sigma}^T, \quad (22)\end{aligned}$$

where

$$\begin{aligned}\mathbf{\Sigma} &= Diag(\mathbf{P},\mathbf{I},\mathbf{I},\mathbf{I}),\\ \mathbf{\Pi}(\mu) &= [\mathbf{B}_{00}^T(\mu)\ 0\ 0\ \mathbf{D}_{20}^T(\mu)]^T,\\ \mathbf{\Theta}(\mu) &= [\mathbf{C}_{00}(\mu)\ 0\ \mathbf{D}_{yo}(\mu)\ 0]^T, \quad (23)\end{aligned}$$

and

$$\mathbf{\Phi}(\mu) = \begin{bmatrix} \Phi_11(\mu) & \mathbf{P}\hat{\mathbf{A}}_1(\mu) & \mathbf{PB}_{1o}(\mu) & \mathbf{C}_{1o}^T(\mu) \\ \hat{\mathbf{A}}_1^T(\mu)\mathbf{P} & -\hat{\mathbf{S}}_{11} & 0 & 0 \\ \mathbf{B}_{1o}^T(\mu)\mathbf{P} & 0 & -\gamma^2\mathbf{I} & 0 \\ \mathbf{C}_{1o}(\mu) & 0 & 0 & -\mathbf{I} \end{bmatrix},$$

where

$$\Phi_{11}(\mu) = \mathbf{A}_o^T(\mu)\mathbf{P} + \mathbf{PA}_o(\mu) + \hat{\mathbf{S}} + \hat{\mathbf{S}}_1. \quad (24)$$

Now, define matrices $\mathbf{\Pi}_\perp(\mu)$ and $\mathbf{\Theta}_\perp(\mu)$ such that for all μ, $\mathbf{\Pi}_\perp^T(\mu)\mathbf{\Pi}(\mu) = 0$, $\mathbf{\Theta}_\perp^T(\mu)\mathbf{\Theta}(\mu) = 0$, and $[\mathbf{\Pi}(\mu), \mathbf{\Pi}_\perp(\mu)]$, $[\mathbf{\Theta}(\mu), \mathbf{\Theta}_\perp(\mu)]$ are full column rank. Then

$$\mathbf{\Pi}_\perp^T(\mu) = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 & 0 & -\mathbf{B}_2(\mu) \\ 0 & 0 & \mathbf{I} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{I} & 0 \end{bmatrix},$$

$$\mathbf{\Theta}_\perp^T(\mu) = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & -\mathbf{C}_y^T(\mu) & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix} \quad (25)$$

Since both $\mathbf{\Pi}_\perp(\mu)$ and $\mathbf{\Theta}_\perp(\mu)$ are full column rank for all $\mu \in \mathcal{P}$, it is clear that if $\mathbf{G}(\mu) < 0$, then $\mathbf{\Theta}_\perp^T(\mu)\mathbf{G}(\mu)\mathbf{\Theta}_\perp(\mu) < 0$ and $\mathbf{\Pi}_\perp^T(\mu)\mathbf{\Sigma}^{-1}\mathbf{G}(\mu)\mathbf{\Sigma}^{-1}\Pi_\perp(\mu) < 0$, which is equivalent to

$$\mathbf{\Pi}_\perp^T(\mu)\mathbf{\Sigma}^{-1}\mathbf{\Phi}(\mu)\mathbf{\Sigma}^{-1}\mathbf{\Pi}_\perp(\mu) < 0, \quad (26)$$

$$\mathbf{\Theta}_\perp^T(\mu)\mathbf{\Phi}(\mu)\mathbf{\Theta}_\perp(\mu) < 0, \quad (27)$$

for all $\mu \in \mathcal{P}$.

Theorem 1: Given the system (7) and $\gamma > 0$, **Globally ES-γ Problem** is solvable if and only if there exist matrices $\mathbf{X}(\in \mathbf{R}^n) > 0$ and $\mathbf{Y}(\in \mathbf{R}^n) > 0$ satisfying following inequalities for all $\mu \in \mathcal{P}$,

$$\begin{bmatrix} \Lambda(\mu) & \mathbf{A}_d(\mu) & \mathbf{B}_1(\mu) & \mathbf{YC}^T(\mu) & \mathbf{Y} \\ \mathbf{A}_d^T(\mu) & -\hat{\mathbf{S}}_{11} & 0 & 0 & 0 \\ \mathbf{B}_1^T(\mu) & 0 & -\gamma^2\mathbf{I} & 0 & 0 \\ \mathbf{C}(\mu)\mathbf{Y} & 0 & 0 & -\mathbf{I} & 0 \\ \mathbf{Y} & 0 & 0 & 0 & -(\mathbf{S}_{11}+\alpha\mathbf{I})^{-1} \end{bmatrix} < 0, \quad (28)$$

$$\begin{bmatrix} \mathbf{\Gamma}(\mu) & \mathbf{XA}_d(\mu) & \mathbf{XB}_1(\mu) & \mathbf{C}^T(\mu) \\ \mathbf{A}_d^T(\mu)\mathbf{X} & -\hat{\mathbf{S}}_{11} & 0 & 0 \\ \mathbf{B}_1^T(\mu)\mathbf{X} & 0 & -\gamma^2\mathbf{I} & 0 \\ \mathbf{C}(\mu) & 0 & 0 & -\mathbf{I} \end{bmatrix} < 0, \quad (29)$$

$$\begin{bmatrix} \mathbf{X} & \mathbf{I} \\ \mathbf{I} & \mathbf{Y} \end{bmatrix} > 0, \quad (30)$$

for some martix $\mathbf{S}_{11} > 0$, and positive scalars λ and α, where

$$\Lambda(\mu) = \mathbf{Y}\mathbf{A}^T(\mu) + \mathbf{A}(\mu)\mathbf{Y} - \mathbf{B}_2(\mu)\mathbf{B}_2^T(\mu), \quad (31)$$

$$\Gamma(\mu) = \mathbf{A}(\mu)^T\mathbf{X} + \mathbf{X}\mathbf{A}(\mu) + \mathbf{S}_{11} + \alpha\mathbf{I} - \gamma^2\mathbf{C}_y^T(\mu)\mathbf{C}_y(\mu).$$

Furthermore, one n-dimensional, strictly proper controller that solves the feedback problem is defined as:

$$\begin{aligned}\mathbf{A}_k(\mu) := &\ \mathbf{A}(\mu) - \gamma^2\mathbf{Z}^{-1}\mathbf{C}_y^T(\mu)\mathbf{C}_y(\mu) - \mathbf{B}_2(\mu)\mathbf{B}_2^T(\mu)\mathbf{Y}^{-1} \\ &+\gamma^{-2}\mathbf{B}_1(\mu)\mathbf{B}_1(\mu)^T\mathbf{Y}^{-1} + \mathbf{A}_d(\mu)\hat{\mathbf{S}}_{11}^{-1}\mathbf{A}_d^T(\mu)\mathbf{Y}^{-1} \\ &-\mathbf{Z}^{-1}\mathcal{H}(\mu), \quad (32)\end{aligned}$$

$$\mathbf{B}_k(\mu) := \gamma^2\mathbf{Z}^{-1}\mathbf{C}_y^T(\mu), \quad \mathbf{C}_k(\mu) := -\mathbf{B}_2^T(\mu)\mathbf{Y}^{-1},$$

where $\mathbf{Z} := \mathbf{X} - \mathbf{Y}^{-1}$ and

$$\begin{aligned}\mathcal{H}(\mu) = &-[\mathbf{Y}^{-1}\mathbf{A}(\mu) + \mathbf{A}^T(\mu)\mathbf{Y}^{-1} - \mathbf{Y}^{-1}\mathbf{B}_2(\mu)\mathbf{B}_2^T(\mu)\mathbf{Y}^{-1} \\ &+\mathbf{A}_d(\mu)\hat{\mathbf{S}}_{11}^{-1}\mathbf{A}_d^T(\mu))\mathbf{Y}^{-1} + \mathbf{C}^T(\mu)\mathbf{C}(\mu) + \mathbf{S}_{11} + \alpha\mathbf{I}] \\ &+\mathbf{Y}^{-1}(\gamma^{-2}\mathbf{B}_1(\mu)\mathbf{B}_1^T(\mu) + \lambda^{-1}\mathbf{H}\mathbf{H}^T. \quad (33)\end{aligned}$$

Proof: ($\Rightarrow$) Let $\mathbf{P} \in \mathbf{R}^{(n+m)\times(n+m)}$ be the positive definite matrix that satisfies the Lemma 3.2. Similarly to the proof procedure of [?], we define $\mathbf{Q} := \mathbf{P}^{-1}$ and partition $\mathbf{P}$ as

$$\mathbf{P} = \begin{bmatrix} \mathbf{X} & \mathbf{X}_2 \\ \mathbf{X}_2^T & \mathbf{X}_3 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \mathbf{Y} & \mathbf{Y}_2 \\ \mathbf{Y}_2^T & \mathbf{Y}_3 \end{bmatrix}, \quad (34)$$

where $\mathbf{X}, \mathbf{Y} \in \mathbf{R}^{n\times n}$ and $\mathbf{X}_3, \mathbf{Y}_3 \in \mathbf{R}^{m\times m}$. By the matrix inversion lemma, it follows that $\mathbf{X} - \mathbf{Y}^{-1} \geq 0$, which is LMI in equation (30). From (26) and (27), carrying out the algebraic manipulations, it can readily be seen by taking Schur complements of the resulting expressions in (26) and (27) that the conditions (28) and (29) are satisfied.

($\Leftarrow$) We verify that the controller given in (32) satisfies the **Globally ES-γ** problem using

$$\mathbf{P} := \begin{bmatrix} \mathbf{X} & -(\mathbf{X} - \mathbf{Y}^{-1}) \\ -(\mathbf{X} - \mathbf{Y}^{-1}) & \mathbf{X} - \mathbf{Y}^{-1} \end{bmatrix} > 0. \quad (35)$$

We redefine the left-hand side of inequality (20) as

$$\begin{aligned}\Upsilon(\mu) := &\ \hat{\mathbf{A}}^T(\mu)\mathbf{P} + \mathbf{P}\hat{\mathbf{A}}(\mu) + \hat{\mathbf{S}} + \hat{\mathbf{S}}_1 + \hat{\mathbf{C}}^T(\mu)\hat{\mathbf{C}}(\mu) \\ &+ \gamma^{-2}\mathbf{P}\hat{\mathbf{B}}(\mu)\hat{\mathbf{B}}^T(\mu)\mathbf{P} + \mathbf{P}\hat{\mathbf{A}}_1(\mu)\hat{\mathbf{S}}_{11}^{-1}\hat{\mathbf{A}}_1^T(\mu)\mathbf{P}, \quad (36)\end{aligned}$$

where the closed-loop matrices $\hat{\mathbf{A}}$, $\hat{\mathbf{A}}_1$, $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$ are defined in (11). Partition Υ into $n \times n$ blocks $\Upsilon_{11}, \Upsilon_{12}$, and Υ_{22}. Define a transformation

$$\mathbf{T} := \begin{bmatrix} \mathbf{I} & 0 \\ \mathbf{I} & \mathbf{I} \end{bmatrix}, \quad (37)$$

and transformed state-space data

$$\begin{aligned}\tilde{\mathbf{A}}(\mu) &:= \mathbf{T}^{-1}\hat{\mathbf{A}}(\mu)\mathbf{T}, \quad \tilde{\mathbf{B}}(\mu) := \mathbf{T}^{-1}\hat{\mathbf{B}}(\mu), \\ \tilde{\mathbf{C}}(\mu) &:= \hat{\mathbf{C}}(\mu)T, \quad \tilde{\mathbf{A}}_1(\mu) := \mathbf{T}^{-1}\hat{\mathbf{A}}_1(\mu), \quad (38)\end{aligned}$$

$$\tilde{\mathbf{P}}(\mu) := \mathbf{T}^T\mathbf{P}(\mu)\mathbf{T}, \quad \tilde{\mathbf{S}} := \mathbf{T}^T\hat{\mathbf{S}}\mathbf{T}, \quad \tilde{\mathbf{S}}_1 := \mathbf{T}^T\hat{\mathbf{S}}_1\mathbf{T}.$$

Then $\mathbf{T}^T\Upsilon(\mu)\mathbf{T} < 0$ can be written as

$$\begin{aligned}&\tilde{\mathbf{A}}^T(\mu)\tilde{\mathbf{P}} + \tilde{\mathbf{P}}\tilde{\mathbf{A}}(\mu) + \tilde{\mathbf{S}} + \tilde{\mathbf{S}}_1 + \tilde{\mathbf{C}}^T(\mu)\tilde{\mathbf{C}}(\mu) \\ &+\gamma^{-2}\tilde{\mathbf{P}}\tilde{\mathbf{B}}(\mu)\tilde{\mathbf{B}}^T(\mu)\tilde{\mathbf{P}} + \tilde{\mathbf{P}}\tilde{\mathbf{A}}_1(\mu)\hat{\mathbf{S}}_{11}^{-1}\tilde{\mathbf{A}}_1^T(\mu)\tilde{\mathbf{P}} < 0. \quad (39)\end{aligned}$$

Note $\Upsilon(\mu) < 0$ if and only if $\mathbf{T}^T\Upsilon(\mu)\mathbf{T} < 0$. Denote the left-hand side of (39) as $\tilde{\Upsilon}$ and partition it into blocks $\tilde{\Upsilon}_{11}, \tilde{\Upsilon}_{12}, \tilde{\Upsilon}_{22} \in \mathbf{R}^{n\times n}$. Using the controller (32), it can be shown that

$$\tilde{\Upsilon}(\mu) = \begin{bmatrix} -\mathcal{H}(\mu) & -\mathcal{H}(\mu) \\ -\mathcal{H}(\mu) & \Upsilon_{11} - \mathcal{H}(\mu) \end{bmatrix}. \quad (40)$$

Using the Schur complement, $\tilde{\Upsilon}(\mu) < 0$ is equivalent to (28) and (29). ■

Theorem 4.1 is existence condition of robust H^∞ control for the global fuzzy model. It is not easy to find the matrices $\mathbf{X}$ and $\mathbf{Y}$ from the global fuzzy model. From theorem 4.1, we derive the following condition, which can be expressed for each rule of fuzzy model.

Theorem 2: Consider the system (7) with assumption (2). Then the **Globally ES-γ** problem is solvable, if there exist common matrices $\tilde{\mathbf{X}} > 0$, $\tilde{\mathbf{Y}} > 0$, $\tilde{\mathbf{S}}_{11} > 0$ and positive scalars $\tilde{\gamma}$, $\tilde{\alpha}$, λ satisfying the following matrix inequalities:

$$\begin{aligned}&\Psi_{ii} < 0, \quad i = 1, 2, ..., r, \\ &\Psi_{ij} + \Psi_{ji} < 0, \quad i < j < r, \quad (41) \\ &\Omega_{ii} < 0, \quad i = 1, 2, ..., r, \\ &\Omega_{ij} + \Omega_{ji} < 0, \quad i < j < r, \quad (42) \\ &\begin{bmatrix} \tilde{\mathbf{X}} & \mathbf{I} \\ \mathbf{I} & \tilde{\mathbf{Y}} \end{bmatrix} \geq 0. \quad (43)\end{aligned}$$

In here

$$\Psi_{ij} = \begin{bmatrix} \Lambda_{ij} & \mathbf{B}_{1_i} & \tilde{\mathbf{Y}}\mathbf{C}_i^T & \tilde{\mathbf{Y}} & \tilde{\mathbf{Y}} \\ \mathbf{B}_{1_i}^T & -\tilde{\gamma}\mathbf{I} & 0 & 0 & 0 \\ \mathbf{C}_i\tilde{\mathbf{Y}} & 0 & -\lambda\mathbf{I} & 0 & 0 \\ \tilde{\mathbf{Y}} & 0 & 0 & -\tilde{\mathbf{S}}_{11}^{-1} & 0 \\ \tilde{\mathbf{Y}} & 0 & 0 & 0 & -\tilde{\alpha}^{-1}\mathbf{I} \end{bmatrix}, \quad (44)$$

$$\Omega_{ij} = \begin{bmatrix} \Gamma_{ij} & \mathbf{I} & \tilde{\mathbf{X}}\mathbf{A}_{d_i} & \tilde{\mathbf{X}}\mathbf{B}_{1_i} & \mathbf{C}_i^T \\ \mathbf{I} & -\tilde{\alpha}^{-1}\mathbf{I} & 0 & 0 & 0 \\ \mathbf{A}_{d_i}^T\tilde{\mathbf{X}} & 0 & -(1-\beta)\tilde{\mathbf{S}}_{11} & 0 & 0 \\ \mathbf{B}_{1_i}^T\tilde{\mathbf{X}} & 0 & 0 & -\tilde{\gamma}\mathbf{I} & 0 \\ \mathbf{C}_i & 0 & 0 & 0 & -\lambda\mathbf{I} \end{bmatrix}, \quad (45)$$

where

$$\begin{aligned}\Lambda_{ij} &= \tilde{\mathbf{Y}}\mathbf{A}_i^T + \mathbf{A}_i\tilde{\mathbf{Y}} - \lambda\mathbf{B}_{2_i}\mathbf{B}_{2_j}^T + (1-\beta)^{-1}\mathbf{A}_{d_i}\tilde{\mathbf{S}}_{11}^{-1}\mathbf{A}_{d_j}^T, \\ \Gamma_{ij} &= \tilde{\mathbf{X}}\mathbf{A}_i + \mathbf{A}_i^T\tilde{\mathbf{X}} + \tilde{\mathbf{S}}_{11} - \tilde{\gamma}\mathbf{C}_{y_i}^T\mathbf{C}_{y_j}. \quad (46)\end{aligned}$$

Furthermore, positive definite matrices $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{S}_{11}$ and positive scalars α, γ are given by

$$\begin{aligned}\mathbf{X} &= \lambda\tilde{\mathbf{X}}, \quad \mathbf{Y} = \lambda^{-1}\tilde{\mathbf{Y}}, \quad \mathbf{S}_{11} = \lambda\tilde{\mathbf{S}}_{11}, \\ \alpha &= \lambda\tilde{\alpha}, \quad \gamma^2 = \lambda\tilde{\gamma}, \quad (47)\end{aligned}$$

and a suitable controller is expressed in (32). ■

Proof: The inequality (28) and (29) of Theorem 4.1 are equivalent to

$$\begin{aligned}&\mathbf{Y}\mathbf{A}^T(\mu) + \mathbf{A}(\mu)\mathbf{Y} - \mathbf{B}_2(\mu)\mathbf{B}_2^T(\mu) \\ &+ (1-\beta)^{-1}\mathbf{A}_d(\mu)\mathbf{S}_{11}^{-1}\mathbf{A}_d^T(\mu) + \gamma^{-2}\mathbf{B}_1(\mu)\mathbf{B}_1^T(\mu) \\ &+ \mathbf{Y}\mathbf{C}^T(\mu)\mathbf{C}(\mu)\mathbf{Y} + \mathbf{Y}\mathbf{S}_{11}\mathbf{Y} + \alpha\mathbf{Y}\mathbf{Y} < 0, \quad (48) \\ &\mathbf{X}\mathbf{A}(\mu) + \mathbf{A}^T(\mu)\mathbf{X} + \mathbf{S}_{11} + \alpha\mathbf{I} - \gamma^2\mathbf{C}_y^T(\mu)\mathbf{C}_y(\mu) \\ &+ (1-\beta)^{-1}\mathbf{X}\mathbf{A}_d^T(\mu)\mathbf{S}_{11}^{-1}\mathbf{A}_d^T(\mu)\mathbf{X} \\ &+ \gamma^{-2}\mathbf{X}\mathbf{B}_1(\mu)\mathbf{B}_1^T(\mu)\mathbf{X} + \mathbf{C}^T(\mu)\mathbf{C}(\mu) < 0. \quad (49)\end{aligned}$$

First, multiply λ to (48) and λ^{-1} to (49). Next, let $\tilde{\mathbf{X}} = \lambda^{-1}\mathbf{X}$, $\tilde{\mathbf{Y}} = \lambda\mathbf{Y}$, $\tilde{\mathbf{S}}_{11} = \lambda^{-1}\mathbf{S}_{11}$, $\tilde{\alpha} = \lambda^{-1}\alpha$, and $\tilde{\gamma} = \lambda^{-1}\gamma^2$. Then, using Schur complement and notation (8), inequalities (48) and (49) are equivalent

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\mathbf{z}(t))\mu_j(\mathbf{z}(t))\mathbf{\Psi}_{ij} < 0,$$
$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\mathbf{z}(t))\mu_j(\mathbf{z}(t))\mathbf{\Omega}_{ij} < 0. \quad (50)$$

Inequalities (50) can be also written as

$$\sum_{i=1}^{r}\mu_i(\mathbf{z}(t))\mu_i(\mathbf{z}(t))\mathbf{\Psi}_{ii} + \sum_{i<j}^{r}\mu_i(\mathbf{z}(t))\mu_j(\mathbf{z}(t))(\mathbf{\Psi}_{ij}+\mathbf{\Psi}_{ji}) < 0,$$
$$\sum_{i=1}^{r}\mu_i(\mathbf{z}(t))\mu_i(\mathbf{z}(t))\mathbf{\Omega}_{ii} + \sum_{i<j}^{r}\mu_i(\mathbf{z}(t))\mu_j(\mathbf{z}(t))(\mathbf{\Omega}_{ij}+\mathbf{\Omega}_{ji}) < 0. \quad (51)$$

From (51), we get (41) and (42). ■

Note that the inequality (41) is an LMI for $\tilde{\mathbf{Y}}$, $\tilde{\mathbf{S}}_{11}^{-1}$ and $\tilde{\gamma}$, $\tilde{\alpha}$, λ and inequality (42) is an LMI for $\tilde{\mathbf{X}}, \tilde{\mathbf{S}}_{11}$ and $\tilde{\gamma}, \tilde{\alpha}, \lambda$. However (41) and (42) are not LMIs in terms of variable $\tilde{S}_{11}$ simultaneously. Thus we present a constructive algorithm for finding solution of the above nonconvex problems. The procedure is summarized as follows:

[Procedure]

Step 1: Find the regions

$$\bar{\mathbf{D}}_s = \{\tilde{\mathbf{S}}_{11} | \tilde{\mathbf{Y}} > 0,\ \tilde{\gamma} > 0,\ \tilde{\alpha} > 0,\ \lambda > 0,\ (41)\} \quad (52)$$

Step 2: Find the regions

$$\tilde{\mathbf{D}}_s = \{\tilde{\mathbf{S}}_{11} | \tilde{\mathbf{X}} > 0, \tilde{\gamma} > 0, \tilde{\alpha} > 0, \lambda > 0,\ (42)\} \quad (53)$$

Step 3: Obtain the intersection of $\tilde{\mathbf{D}}_s$ and $\bar{\mathbf{D}}_s$

$$\hat{\mathbf{D}}_s = \bar{\mathbf{D}}_s \cap \tilde{\mathbf{D}}_s \quad (54)$$

Step 4: Compute $\tilde{\mathbf{X}}$, $\tilde{\mathbf{Y}}$ and $\tilde{\alpha}$, $\tilde{\gamma}$, λ suct that

$$\min_{\tilde{\mathbf{S}}_{11}\in\hat{\mathbf{D}}_s} [\mathrm{trace}(\tilde{\gamma}) + \mathrm{trace}(\lambda)] \quad (55)$$
$$\text{subject to} \quad (41)-(43)$$

Step 5: Compute positive definite matrices $\mathbf{X}, \mathbf{Y}$ and positive scalars α, γ from (47)

Step 6. Construct finally controller from (32) with $\mathbf{X}, \mathbf{Y}, \alpha, \gamma$ and λ.

Remark 1: In step 3, the existence of the set $\hat{\mathbf{D}}_s$ does not imply that the matrix inequalities, (41)-(43), are solvable, but a necessary condition for the solvability of (41)-(43).

Remark 2: In step 4, the minimization of (55) is not convex problem in term of $\tilde{\mathbf{S}}_{11}$. However, it is not difficult to find the minimum $\tilde{\gamma} + \lambda$ because the computation can be executed within the searching region of $\tilde{\mathbf{S}}_{11}$ in step 3.

V. Design example

We will design a fuzzy H^∞ controller for the following nonlinear system;

$$\begin{aligned}\dot{x}_1(t) &= -0.1125x_1(t) - 0.0125x_1(t-d(t)) - 0.02x_2(t)\\ &\quad -0.67x_2^3(t) - 0.1x_2^3(t-d(t))\\ &\quad -0.005x_2(t-d(t)) + w(t) + u(t)\\ \dot{x}_2(t) &= x_1(t),\ \ y(t) = x_2(t)\\ e_1(t) &= x_2(t),\ \ e_2(t) = u(t),\end{aligned}$$

where time-varying delay is

$$d(t) = 4 + 0.5\cos(0.9t).$$

The purpose of control is to achieve closed loop stability and to attenuate the influence of the exogenous input disturbance $w(t)$ on the penalty variable $[e_1^T(t)\ e_2^T(t)]^T$. It is also assumed that $x_2(t)$ is measurable and

$$x_1(t) \in [-1.5\ 1.5], \quad x_2(t) \in [-1.5\ 1.5].$$

Using the same procedure as in [?], the nonlinear term can be represented as

$$-0.67x_2^3(t) = M_{11}\cdot 0\cdot x_2(t) - (1-M_{11})\cdot 1.5075x_2(t).$$
$$\begin{aligned}-0.1x_2^3(t-d(t)) &= M_{11}\cdot 0\cdot x_2(t-d(t))\\ &\quad -(1-M_{11})\cdot 0.225\cdot x_2(t-d(t)).\end{aligned}$$

By solving the equation, M_{11} is obtained as follows:

$$M_{11}(x_2(t)) = 1 - \frac{x_2^2(t)}{2.25},$$
$$M_{12}(x_2(t)) := 1 - M_{11}(x_2(t)) = \frac{x_2^2}{2.25}.$$

M_{11} and M_{12} can be interpreted as membership functions of fuzzy set. By using these fuzzy sets, the nonlinear system can be presented by the following uncertain T-S fuzzy model

Plant Rule 1:

IF $x_2(t)$ is M_{11} THEN
$$\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{A}_{d_1}\mathbf{x}(t-d(t)) + \mathbf{B}_{1_1}w(t) + B_{2_1}u(t)$$
$$y(t) = \mathbf{C}_{y_1}\mathbf{x}(t)$$
$$e_1(t) = \mathbf{C}_1\mathbf{x}(t),\ \ e_2(t) = u(t)$$

Plant Rule 2:

IF $x_2(t)$ is M_{12} THEN
$$\dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t) + \mathbf{A}_{d_2}\mathbf{x}(t-d(t)) + \mathbf{B}_{1_2}w(t) + \mathbf{B}_{2_2}u(t)$$
$$y(t) = \mathbf{C}_{y_2}\mathbf{x}(t)$$
$$e_1(t) = \mathbf{C}_2\mathbf{x}(t),\ \ e_2(t) = u(t),$$

where

$$\begin{aligned}\mathbf{x}(t) &= [x_1(t)\ x_2(t)]^T,\\ \mathbf{A}_1 &= \begin{bmatrix} -0.1125 & -0.02 \\ 1 & 0 \end{bmatrix},\\ \mathbf{A}_{d_1} &= \begin{bmatrix} -0.0125 & -0.005 \\ 0 & 0 \end{bmatrix},\end{aligned}$$

$$\mathbf{A}_2 = \begin{bmatrix} -0.1125 & -1.527 \\ 1 & 0 \end{bmatrix},$$
$$\mathbf{A}_{d_2} = \begin{bmatrix} -0.0125 & -0.23 \\ 0 & 0 \end{bmatrix},$$
$$\mathbf{B}_{1_1} = \mathbf{B}_{1_2} = [1\ 0]^T,\ \mathbf{B}_{2_1} = \mathbf{B}_{2_2} = [1\ 0]^T,$$
$$\mathbf{C}_{y_1} = \mathbf{C}_{y_2} = [0\ 1],\ \mathbf{C}_1 = \mathbf{C}_2 = [0\ 1].$$

Let $\tilde{\mathbf{S}}_{11} = k\mathbf{I}_2$ for simplicity. From step 1, step 2, and step 3 of procedure, we can obtain the set

$$\hat{\mathbf{D}}_s = \{\tilde{\mathbf{S}}_{11} | \tilde{\mathbf{S}}_{11} = k\mathbf{I}_2,\ 10^{-9} < k < 2.93 \cdot 10^8\}.$$

The minimization of step 4 is attained at $\tilde{\mathbf{S}}_{11} = 0.1926$ and the minimum value of γ is 4.66. The values of λ, α and X, Y, S_{11} are

$$\lambda = 4.3465, \quad \alpha = 0.2946 \cdot 10^{-6},$$
$$\mathbf{S}_{11} = \begin{bmatrix} 0.837 & 0 \\ 0 & 0.837 \end{bmatrix},$$
$$\mathbf{X} = \begin{bmatrix} 6.5588 & -3.7902 \\ -3.7902 & 11.7482 \end{bmatrix},$$
$$\mathbf{Y} = \begin{bmatrix} 0.5140 & -0.1811 \\ -0.1811 & 0.3146 \end{bmatrix}.$$

The simulation results for the nonlinear systems are shown in Fig. 1. For these simulations, the initial value of the states are $x_1(0) = -1$ and $x_2(0) = -1.2$. And The disturbance signal $w(t)$ is defined by

$$w(t) = \begin{cases} 0.3, & 2\text{ sec} \le t \le 3\text{ sec} \\ 0, & \text{otherwise.} \end{cases}$$

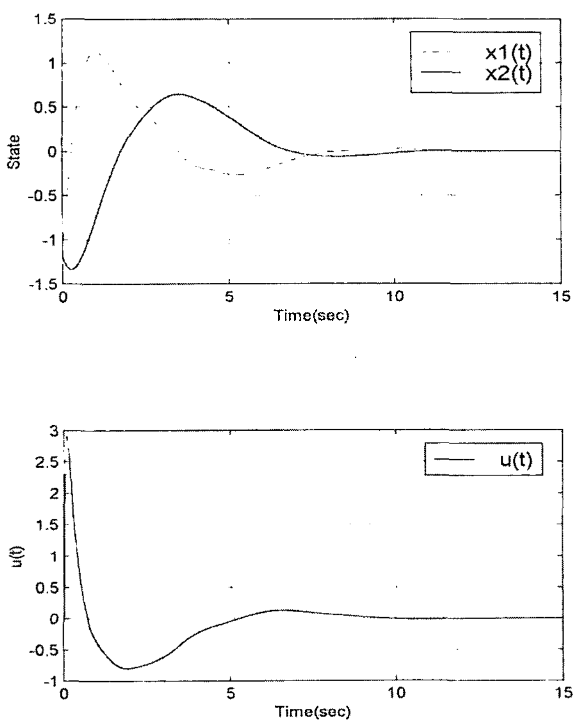

Fig. 1. The simulation results of nonlinear systems.

The designed fuzzy H^∞ controller stabilizes the the nonlinear system and attains disturbance attenuation effect.

VI. Conclusion

In this paper, we have developed output feedback fuzzy H^∞ controller design method for nonlinear systems with time-varying delayed state. We have obtained sufficient conditions for the existence of fuzzy H^∞ controller such that the closed-loop fuzzy system is globally exponentially stable and achieves a prescribed level of disturbance attenuation. The derived sufficient conditions are not LMI in all variables. Thus we presented a constructive algorithm for finding solution of the nonconvex problems. The controller is directly obtained from the derived LMI condition. The resulting controller is nonlinear and tuned on-line based on fuzzy operation. Through an example, the validity was demonstrated.

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