Prediction of Wall Shear Stresses in Transitional Boundary Layers Using Near-Wall Mean Velocity Profiles

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The local wall shear stress in transitional boundary layer was estimated from the near-wall mean velocity data using the principle of Computational Preston Tube Method (CPM). The previous DNS and experimental databases of transitional boundary layers were used to demonstrate the accuracy of the method and to provide the applicable range of wall unit y^+. The skin friction coefficients predicted by the CPM agreed well with those from previous studies. To re-examine the applicability of the CPM, near-wall hot-wire measurements were conducted in developing transitional boundary layers on a flat plate with different freestream turbulence intensities. The intermittency profiles across the transitional boundary layers were reasonably obtained from the conditional sampling technique. An empirical correlation between the representative intermittency near the wall and the free parameter K_1 of the extended wall function of CPM has been newly proposed using the present and other experimental data. The CPM has been verified as a useful tool to measure the wall shear stress in transitional boundary layer with reasonable accuracy.

Key Words : Wall Shear Stress, Transitional Boundary Layer, Intermittency

Nomenclature

A^+ : Van Driest constant
C_f : Skin friction coefficient
C_f' : Normalized skin friction coefficient
C_{f0} : Skin friction coefficient of DNS data
H : Shape factor
K_1, K_2, K_3 : Free parameters in the extended wall function
K_P : Pressure gradient parameter $(=\nu/U_\infty^2)(dU_\infty/dx))$
m : Slope of linear fitting curve between $K_1 \sim C_f'$
Re_x : Reynolds number based on streamwise distance
P : Pressure
TI : Turbulence intensity of freestream
u : Streamwise mean velocity
$\overline{u'}$: Rms value of velocity fluctuation
u_τ : Friction velocity
U_∞ : Freestream velocity
x : Coordinate of streamwise direction
y : Coordinate of normal direction
y_{lb} : Lower bound of y
y_{ub} : Upper bound of y
δ : Boundary layer thickness
δ^* : Displacement thickness
γ : Intermittency
γ_{max} : Maximum intermittency
Γ : Representative intermittency
χ : Von Karman constant
ν : Kinematic viscosity of air
ρ : Density of air
ξ : Nondimensional distance in transition

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region

Superscripts

+ : Wall unit

Subscripts

S : Start of transition

E : End of transition

1. Introduction

The local wall shear stress is a significant flow parameter in the transitional boundary layers, as this quantity represents one of boundary condition of wall shear layer and so closely related to transition process. Most boundary layer flows in turbomachinery blades are transitional. Hence, the profiles of wall skin friction on blades greatly affect hydraulic performance and efficiency of turbomachinery. The skin friction coefficient is the most sensitive quantity of other integral parameters indicating whether the flow is laminar, transitional or turbulent. However, it is difficult to accurately measure wall skin friction because it is highly sensitive and dependent upon experimental methods.

The Preston tube method (Preston, 1954) has been widely used because of its simple applicability. The method basically assumes that the mean velocity profile satisfies the near-wall similarity (i. e., the law of the wall in the vicinity of the surface of equilibrium turbulent boundary layer). Therefore, for more complex situations (e. g., transitional boundary layers), the method is not valid. To supplement the conventional Preston tube method, Nitsche et al. (1983) proposed the Computational Preston Tube Method (CPM) which could be easily applied to various wall shear layers with high accuracy. Escudier and Acharya (1987) suspected the validity of CPM because their measured velocities in a transition region did not coincide with any extended wall function obtained from the CPM. For this critique, Nitsche (1987) pointed out that an accurate measurement of velocities near the wall was required because their small error caused large error of wall shear stress in the CPM. Jeon and Kang (1995) measured the mean velocities near the wall with multi-sized Preston tubes in developing transitional boundary layers and applied these data to the CPM. They reported that the errors in CPM depended mostly on the displacement factor of Preston tubes. These studies showed that a more in-depth investigation of CPM was required with more databases in transitional boundary layers.

In the present study, previous DNS and experimental data are compiled to examine the validity of CPM in transitional boundary layers. Hot-wire measurements have also been performed in the developing transitional boundary layers on a flat plate to demonstrate the accuracy of CPM. Especially, intermittency profiles are estimated using the conditional sampling technique. Finally, a new empirical correlation between the representative intermittency near the wall and the free parameter K_1 of the extended wall function in CPM has been proposed.

2. Evaluation of wall shear stress

2.1 Principle of the Computational Preston Tube Method

The basic assumption in the Computational Preston Tube Method (CPM) is that similarity of mean velocity profiles near the wall exists even in non-equilibrium wall boundary layers like transitional boundary layers. Instead of the well-known wall function in the conventional Preston tube method, an extended wall function with several free parameters is adopted in the CPM.

Nitsche et al. (1983) used the following family of profiles with parameters (K_1, K_2, K_3) as an extended wall function, originally proposed by Szablewski (1969):

$$u^+=\int_0^{y^+}\frac{2(1+K_3y^+)\,dy^+}{1+[1+4(K_1y^+)^2(1+K_3y^+)\{1-\exp(-y^+\sqrt{1+K_3y^+}/K_2)\}^2]^{0.5}} \quad (1)$$

where u is the streamwise mean velocity and y is the wall-normal distance. K_1, K_2 and K_3 are parameters which correspond to the von Karman constant κ, van Driest constant A^+ and pressure gradient parameter $K_3 = p^+ = \nu/\rho u_\tau^3 (dp/dx)$, respectively, in the fully turbulent boundary layer. In recent studies related to the CPM applications, (e. g. shown in Weiser et al. (1989), Kreplin and Höhler (1992), Nitsche (1994) and Jeon and Kang (1995)), K_1 has been proven to be the superior iteration parameter in CPM process, whereas K_2 was set to be the van Driest standard value 26. K_3 is zero if the boundary layer develops under zero pressure gradient. Therefore, the two unknowns, K_1 and friction velocity u_τ, are determined by the least square method from the measured values of velocity near the wall. The detailed computational procedure of CPM is presented in Nitsche et al. (1987) and Jeon and Kang (1995). The correlation of displacement factor of each Preston tube is important for skin friction measurement because the errors in the CPM depend mostly on the uncertainty in the displacement factor (Jeon and Kang, 1995). In the present study, the traverse hot wire measurement of mean velocity is employed to determine the skin friction. Therefore, the present method for estimation of wall shear stress has smaller errors than the method using multi- sized Preston tubes because displacement factor is not needed and much data are provided near the wall. The applicable range of nondimensional wall normal distance, y^+, in Eq. (1) also plays an important role in the prediction of wall shear stress. The sensitivity of y^+ range in CPM is described in 2. 2.

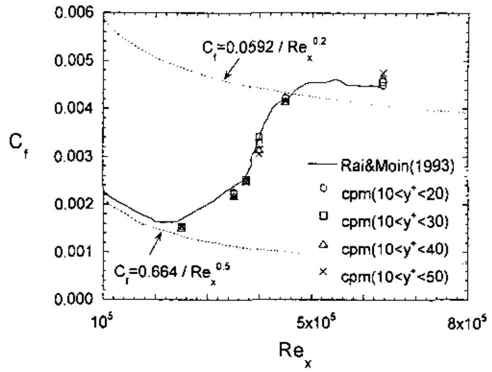


Fig. 1 Skin friction coefficients calculated by Rai and Moin (1993) and estimated from CPM in the transition region

2.2 Validation of the method using DNS data

Direct numerical simulation (DNS) has been considered as an effective method to study the details of turbulent shear flow. DNS databases provide various turbulent flow statistics which are generally in good agreement with experimental data. Recently, DNS of the transitional boundary layer has been also tried to investigate transition phenomena. Numerical data are generally more and smoother than the corresponding experimental results, especially near the wall. Thus, the present method to estimate the wall shear stress in a transition zone will be examined with the available DNS data. Rai and Moin (1993) conducted a DNS of spatially evolving boundary layer on a flat plate and presented many statistical data in the transition region. In the present study, their mean velocity data in the vicinity of the wall are used to evaluate wall shear stress with CPM. Figure 1 shows the skin friction coefficients determined from the CPM using the velocity data of Rai and Moin over different range of y^+ near the wall. Although many near-wall velocities are available in DNS databases, $y^+ = 10$ is selected as a lower bound in the present study because most of hot-wire measurements do not practically provide sufficient and accurate near-wall velocities below $y^+ = 10$. More accurate wall skin frictions are obtained using the data closer to the wall in CPM. Generally, the estimated wall skin friction is in good agreement with DNS results in the whole transition region as shown in Fig. 1. The error becomes greater with upper bounds y^+ increasing. Thus, the maximum error is approximately 10% for $10 < y^+ < 50$ at $Re_x = 4 \times 10^5$. The sensitivity of lower and upper bounds y^+ to the estimated skin friction coefficients is well depicted in Fig. 2, where C_{f0} is the skin friction coefficient of DNS result. According to the distribution of C_f vs. Re_x in Fig. 1, three Reynolds numbers in Fig. 2 are associated with the onset, middle and end position of transition, respectively. The skin friction coefficients obtained from the CPM are underestimated for the most part. However, in the range of $0 < y^+ < 20$, these values show good

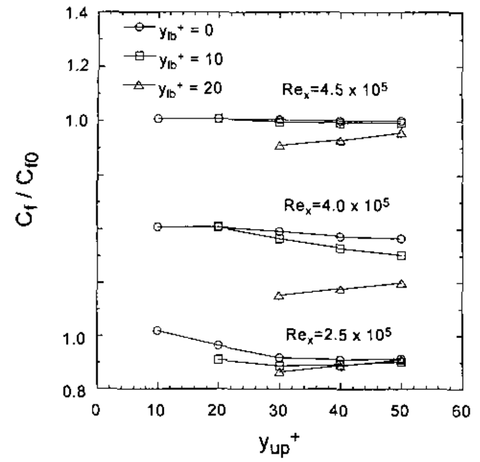


Fig. 2 Variation of estimated wall skin frictions with respect to the lower and upper bound of y^+ based on DNS data of Rai and Moin (1993)

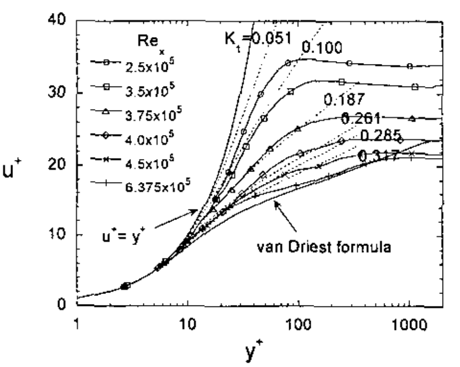


Fig. 3 Mean velocity profiles calculated by Rai and Moin (1993) (solid line) and extended wall functions (dot line) predicted from CPM in the transition region

agreement with DNS data. The error becomes smaller as the lower bound y_{lb}^+ comes closer to the wall for all three Reynolds numbers. In the cases of $y_{lb}^+=0$ and 10, the increases of upper bound y_{ub}^+ give rise to more deviation. However, in the case of $y_{lb}^+=20$, the error decreases as the upper bound increases even though the estimated values deviate from DNS data in this case.

Figure 3 presents the streamwise mean velocities for Grid B in Rai and Moin. As the Reynolds number increases, mean velocity profiles gradually change from the Blasius solution of laminar region to the wall function of turbulent region, which shows well the typical trend of a transitional boundary layer. The dashed lines are the extended wall functions, Eq. (1), obtained from the CPM with Rai&Moin's mean velocities of the range $10<y^+<30$. In this range, all extended wall functions in the transition region are similar to DNS results except that of early-transition point, $Re_x=2.5\times10^5$. The free parameter, K_1 gradually increases as the boundary layer becomes more transitional.

2.3 Validation of the method using previous experimental data

Some experimental databases of transitional boundary layer were also used to examine the present method. Kuan and Wang (1990), Sohn and Reshotko (1991) and Kim et al. (1994) performed the hot-wire measurements of transitional boundary layers on a flat plate under zero pressure gradient and various freestream turbulence intensity. Wang and Keller (1999) did the same experiment under three values of favorable pressure gradient. Sohn and Reshotko (1991) determined the wall skin friction from a momentum balance technique in a transition region and from a profile fitting technique with near-wall velocities in the laminar and turbulent regions. In the other three researches, C_f was acquired from near-wall velocity profile fitting in all boundary layers. Using the present method in Section 2.2, wall skin friction was evaluated with the experimental near-wall velocities in the range $0<y^+<30$ as shown in Fig. 4(a), (b), (c), (d). The lower bound y_{lb}^+, is set to zero because, in most experiments, the number of mean velocity data even within $y^+<30$ are not sufficient to accurately evaluate wall skin friction. All of the results of CPM shown in Fig. 4 are obtained via K_1 iteration with $K_2=26$, $K_3=0$. The predicted wall skin frictions agree well with the corresponding experimental data. The transition onset and end points as well as the path of C_f in the transition region coincide with the results of respective experimental data. Figures 5 and 6 exhibit the nondimensional mean velocity profiles measured by Sohn and Reshotko (1991) and Wang and Keller (1999), respectively and the extended wall func-

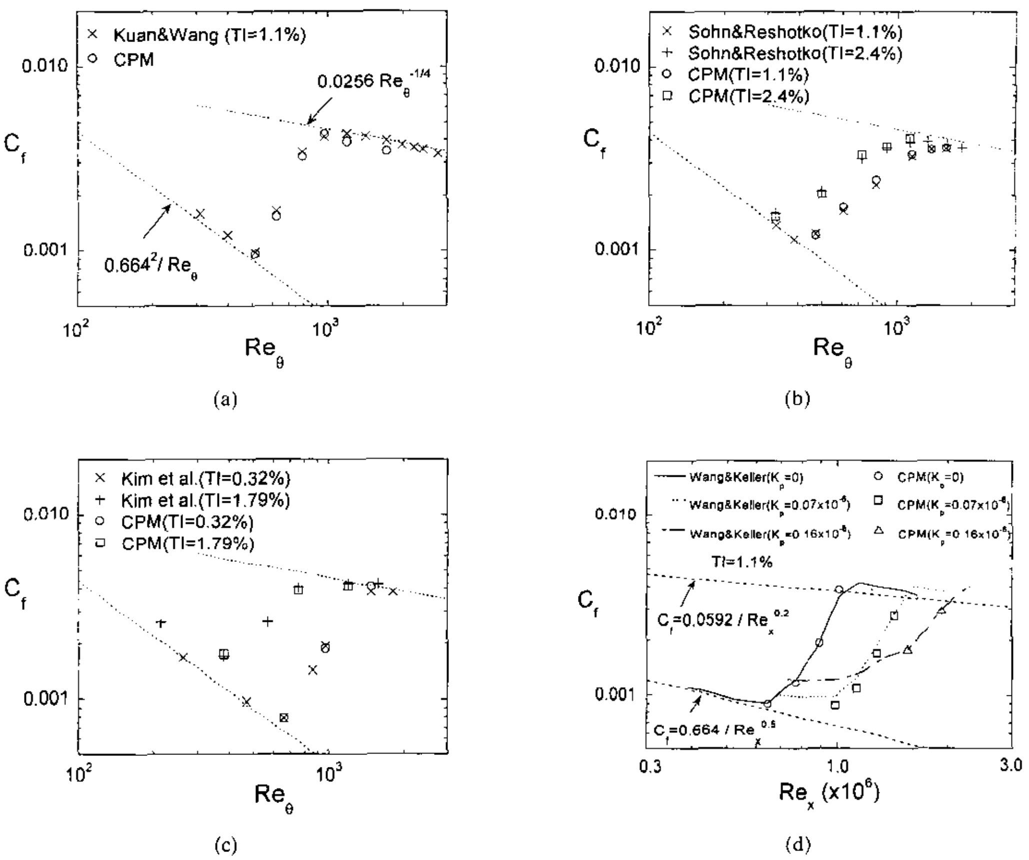


Fig. 4 Skin friction coefficients measured by (a) Kuan and Wang (1990), (b) Sohn and Reshotko (1991), (c) Kim, Simon and Kestoras (1994) and (d) Wang and Keller (1999) (where K_p is pressure gradient parameter) and obtained from CPM using respective near-wall velocity profiles

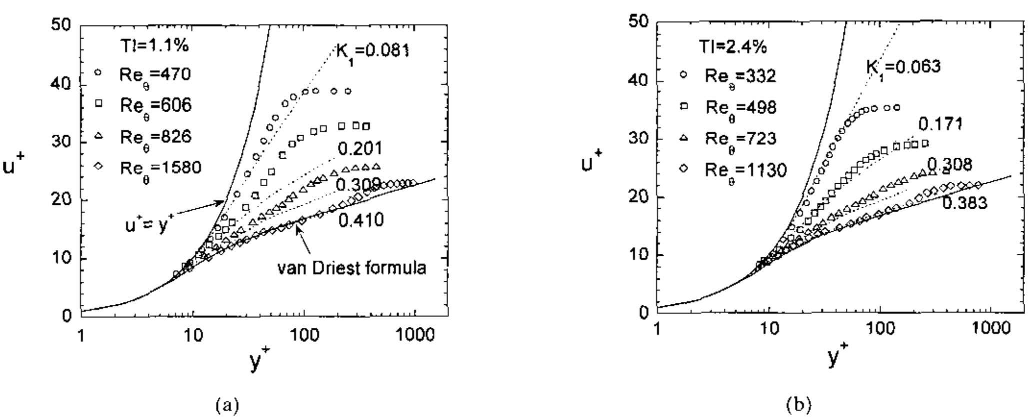


Fig. 5 Mean velocity profiles measured by Sohn and Reshotko (1991) (solid line) and extended wall functions (dot line) predicted from CPM in the transition region: (a) TI=1.1%, (b) TI=2.4%

tions, Eq. (1), gained from the CPM. In Fig. 6, the pressure gradient parameter, K_p, is defined as $(\nu/U_\infty^2)(dU_\infty/dx)$. All of the extended wall functions are in good agreement with the measured data in the applied range, i. e. $0<y^+<30$. The deviation from the experimental data at early

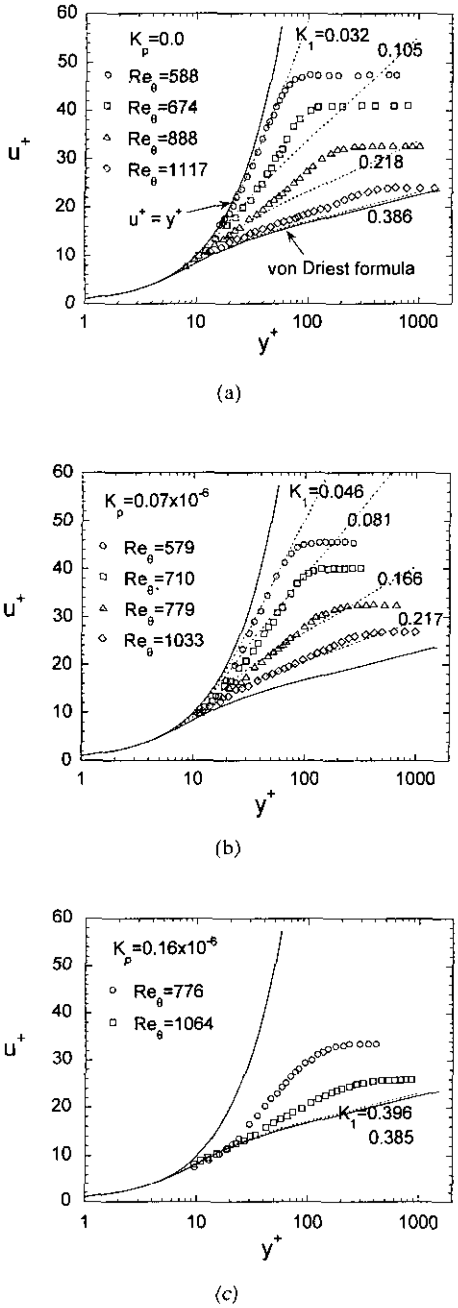


Fig. 6 Mean velocity profiles measured by Keller and Wang (1991) and extended wall functions predicted from CPM in the transition region: pressure gradient parameter (a) $K_p=0$ (b) $K_p=0.07\times10^{-6}$ (c) $K_p=0.16\times10^{-6}$

-transition point is greatly less than the CPM result using DNS data at the similar point in Section 2. 2.

The results in Section 2 suggest that the present method is a useful tool to estimate the wall shear stress in transitional boundary layers with reasonable accuracy. To re-examine the reliability of the method, near-wall hot-wire measurements have been also performed in the transitional boundary layer on a flat plate under different freestream turbulence intensities.

3. Experiment of Transitional Boundary Layers for the Validation of the Method

3.1 Experimental apparatus and method

An open-circuit suction type wind tunnel was specially built for the experiment as shown in Fig. 7. The flow was provided by a speed controlled axial fan sucking air at the downstream end. The air was cleaned through an air filter and then passed through a settling chamber with five damping screens. Downstream of the last screen, the flow passed through a 16:1 contraction whose shape was elliptic with 4.6 : 1 ratio. The test section was a rectangular of 0.4m wide, 0.3m high and 2.0m long. The sidewall could be moved outward to compensate for the blockage effect of the sidewall boundary layers. Consequently, zero pressure gradient boundary layer was developed on the flat plate which was set in the middle of test section. The 1.5m long rectangular duct was additionally connected to the end of test section to alleviate fan-generating swirl. The uniformity of streamwise mean velocity and turbulence intensity at the test section was less than 0.5% and 0. 1%, respectively, at 10 m/sec.

Boundary layer measurements were conducted on a smooth flat plate of 12mm thickness, 0.28m width and 1.5m length. The flat plate was vertically installed at the center of the test section to prevent deflection due to its weight. The static pressure was measured at forty pressure taps (1mm diameter) which were located along two different spanwise locations from the midspan and at x=0.02, 0.03, 0.04, 0.07, 0.10, 0.15, 0.20, 0. 30, … 1.50m from the leading edge. The leading edge was designed as an ogive to avoid local separation and it's length (0.12 m) was numeri-

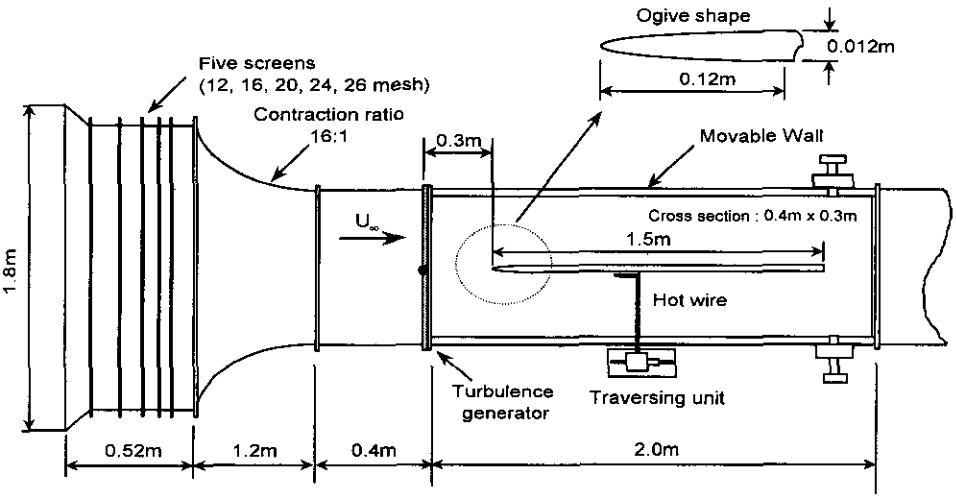


Fig. 7 Schematic of wind tunnel, test plate and equipment (overview)

cally determined to minimize local pressure gradient (Fig. 7). After $x=0.15$ m, all boundary layers were developed under zero pressure gradient in experimental freestream speed. The turbulence grid was made up of 3mm-diameter cylinder-bar with approximately 72% open area. The shape of the mesh was square and the mesh size was 20mm.

Streamwise mean velocities and velocity fluctuations were measured with a single hot-wire probe (DANTEC 55P15). A digital manometer (MKS 220D) was also used to measure the static pressure on a flat plate and to calibrate the hot-wire sensor. The probes were traversed manually in the streamwise direction (x) and automatically in the lateral direction (y) with a stepping motor and a linear head controlled by a personal computer. The signals from the manometer and the hot-wire anemometers were recorded by an 8 bit A/D converter (HP-35665A) by the personal computer for further processing. Data were sampled 16, 000 times in 10 sec.

3.2 Conditional sampling technique

In the transition region, intermittent behavior between laminar and turbulent flow is observed. The prediction of intermittency is important to understand the complex phenomenon of transition process because it's extent and distribution is closely related to generation and propagation of turbulent spots in the transitional boundary layers.

Conditional sampling based on intermittency allows segregation of the intermittent signals into turbulent and non-turbulent parts. A method similar to that of Sohn and Reshotko (1991) was applied to estimate intermittency in the present study. Squares of the first derivative of the streamwise instantaneous velocity were used as a detect function for discrimination between non-turbulent and turbulent flows. The differentiation of velocity signal emphasizes the high frequency component for turbulent flow. In the next step, smoothing the derivatives over a short period of time was applied to eliminate excessive zero-crossing of the first derivative signal. Smoothing time depends on the sampling rate of data acquisition and the probe resolution. Hedley and Keffer (1974) proposed 15-30 times the Kolmogorov time scale as a smoothing time in turbulent boundary layers, however, actually used four times data sampling rate (100μsec). In the transitional boundary layer, Kuan and Wang (1990) and Sohn and Reshotko (1991) set longer smoothing times such as three (400μsec) and ten times the sampling rate (200μsec), respectively, because the eddies in transition region were larger than those of turbulent boundary layer. The smoothing time of the present experiment was five times the sampling rate (160μsec). Based on the predetermined threshold values, a signal was classified as turbulent if the value of the smoothed detector function was greater than the threshold value. Otherwise, the signal was sorted as non-turbulent.

Although there were no physically logical methods to determine the threshold value, several approaches have been suggested for estimation of intermittency. A modified probability distribution approach, dual slope method, proposed by Kuan and Wang (1990), was adopted in the present study. After intermittency profile with the variation of threshold was previously acquired in a measuring point, the optimum threshold value was determined at the point where the curve of intermittency profile had its maximum curvature. This point could be taken at the intersection of dual tangential lines whose slopes were coincident with the maximum and minimum derivatives of the previous intermittency profile. Details of the dual slope method were given in Kuan and Wang (1990).

3.3 The result of present experiment

Figure 8 shows the variation of freestream streamwise turbulence intensity over the flat plate without (Grid0) and with (Grid1) the turbulence grid. The freestream velocity for Grid0 and Grid1 was 26 m/sec and 10 m/sec, respectively. The intrinsic turbulence intensity (Grid0) in the test section is uniformly distributed with a constant value of 0.1%. On the other hand, the turbulence intensity for Grid1 decreases exponentially from 3.8% at the leading edge ($x=0$) to approximately 1.0% at the end of the plate ($x=1.5$m). The turbulence level behind a uniform grid decays downstream as $u' \sim 1/x^n$, where the value of n is 0.5 - 0.7 (Hinze, 1975). The turbulence intensity profile for Grid1 is in good agreement with this relation if $n=0.63$. T3A Test Case, with a freestream turbulence profile similar to the present Grid1 case, was one set of the experimental databases that European Research Community on Flow Turbulence and Combustion (ERCOFTAC) special interest group (Savill, 1993) built up for research on transitional boundary layers. Under these conditions, the streamwise velocities of a developed boundary layer on a flat plate were measured with a single hot wire probe.

The shape factor obtained by integrating the measured mean velocity profiles, gradually changes from 2.59 in a laminar flow to 1.4 in a turbulent flow as shown in Fig. 9. The measured shape factors for Grid1 are in good agreement with those of T3A test case.

Using the same process as in Section 2.3, wall skin friction was evaluated with the measured mean velocities of range $0<y^+<30$. The estimated skin friction shows a typical distribution during the transition process as presented in Fig. 10. The value of C_f gradually decreases in the laminar region, rapidly increases during the transition, attains its maximum value at the end of transition and decreases gradually in the turbulent region. The predicted values for Grid0 coincide with the Blasius solution in the laminar region where pressure gradient is zero. Due to a

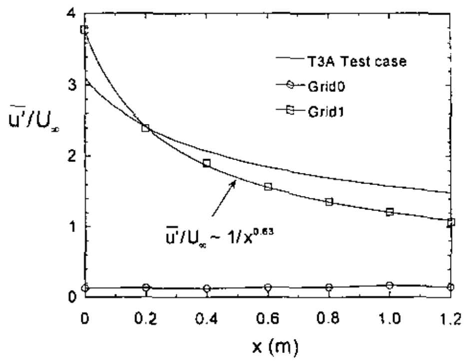


Fig. 8 Streamwise freestream turbulence intensity through the test section

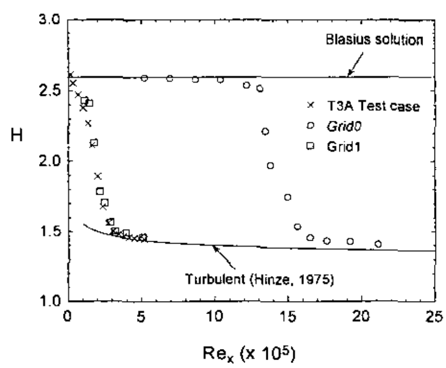


Fig. 9 Distribution of shape factor in streamwise direction

small local adverse gradient over the leading edge, however, the C_f for Grid1 in the laminar region is a little bit lower than the Blasius solution.

A numerical calculation of a laminar boundary layer was conducted with the measured pressure distribution and the simulation program of Cebeci and Smith (1974) to see the pressure gradient effect on the development of boundary layer over the leading edge. The measured skin friction agrees with the result of laminar calculation better than the Blasius solution. In the turbulent region, the skin friction for Grid0 and Grid1 coincide with the empirical correlation of Schlichting (1979). The variation of C_f for Grid1 is very similar to T3A test case where the skin friction is directly measured. Considering the affinity of freestream turbulence distribution between Grid1

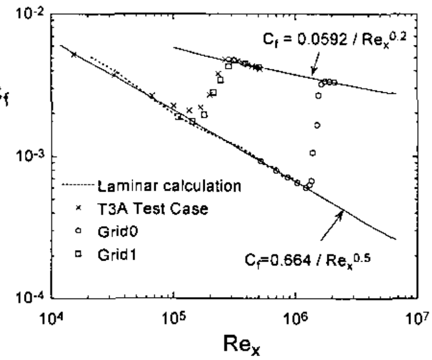
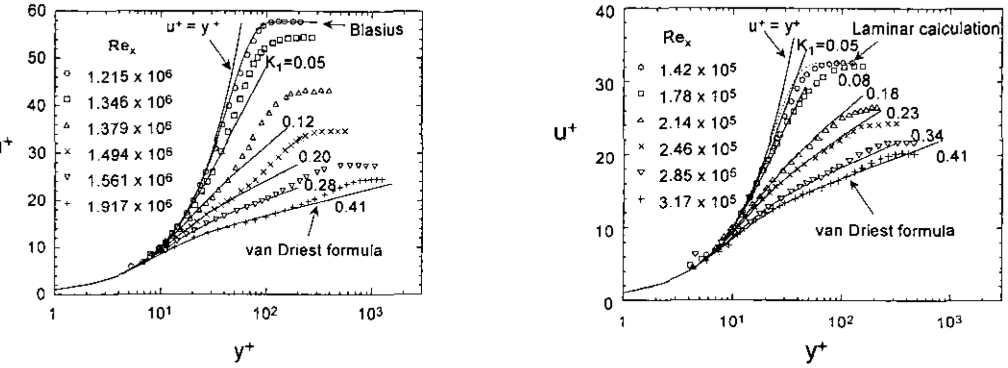


Fig. 10 Skin friction coefficients obtained from CPM using measured near-wall velocity profiles

and T3A test case in Fig. 8, the present method for the evaluation of skin friction is reasonable in transitional boundary layers.

The mean velocity profiles for Grid0 and Grid1 in the wall units are presented in Figs. 11 (a), (b). Before the onset of transition, the profile for Grid0 coincides well with the Blasius profile. However, the profile for Grid1 is in better agreement with the numerical calculation. From the onset of transition, the profiles of mean velocity for Grid0 and Grid1 approach the logarithmic profile and finally converge to it in turbulent region. Owing to the high freestream turbulence, the mean velocity profiles for Grid1 converges onto the turbulent profile more rapidly than those for Grid0. The free parameter K_1 of Eq. (1) is nearly zero in the laminar region and approaches the von Karman constant of 0. 41 in the turbulent region. All of the velocity profiles in transition region coincide well with the extended wall function up to $y^+=30$.

The distributions of the intermittency $\gamma(y)$ across the boundary layer at various streamwise locations are shown in Figs. 12 (a), (b). After the threshold value at each point was determined using the dual-slop method, intermittency was obtained from the segregation of the velocity signals into turbulent and non-turbulent parts. The solid curve represents a distribution of intermittency for the fully turbulent boundary layer suggested by Klebanoff (1955). The intermittency values in the early stages of transition increase from the wall and reach a peak around $y/\delta^*=1$



(a) Grid 0 (b) Grid 1

Fig. 11 Streamwise mean velocity profiles in wall units for (a) Grid0 (b) Grid1

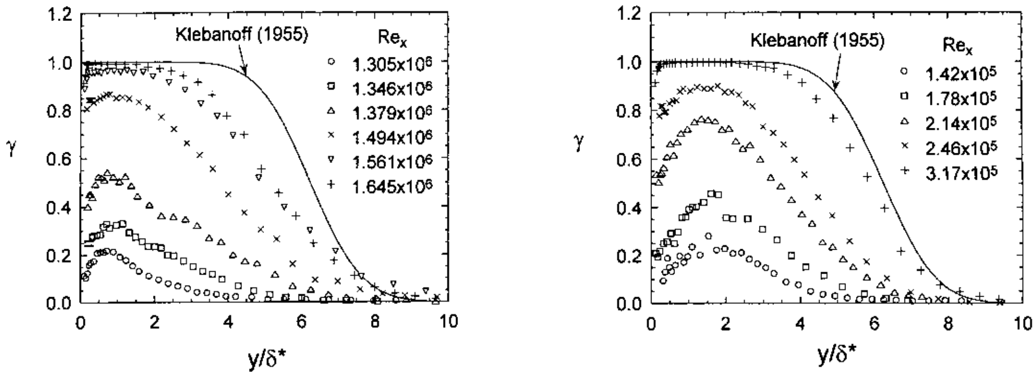


Fig. 12 Intermittency profiles across the boundary layer for (a) Grid0 (b) Grid1

and then gradually decrease to zero at the edge of the boundary layer. This near-wall profile of $\gamma(y)$ is closely related to the leading and trailing overhangs in the vertical shape of turbulent spot as observed by Cantwell et al. (1978). Similar results for the intermittency profile were reported by Kuan and Wang (1990) and Sohn and Reshotko (1991). In the late stages of transition, a plateau of constant intermittency following a peak is distributed near the wall and then intermittency values gradually decrease to zero in the outer boundary layer. The peak value of intermittency increases as the transition advances and finally the profile of γ access to the distribution of fully turbulent boundary layer. Thus, the distribution of maximum intermittency γ_{max} is directly related to the transition process of boundary layers. Dhawan and Narasimha (1958) proposed a following universal intermittency distribution under the hypothesis that laminar breakdowns of two-dimensional boundary layer were point- like and that the spots originated in a restricted region.

$$\Gamma(\xi) = 1 - \exp(-0.412\xi^2) \tag{2}$$

where Γ is a representative intermittency of boundary layer and $\xi = (x - x_S)/(x_{\gamma=0.75} - x_{\gamma=0.25})$ is a normalized streamwise coordinate in the transition zone. To determine the onset of transition x_S and characteristic length scale $x_{\gamma=0.75} - x_{\gamma=0.25}$ of the transition region, intermittency data are plotted on the $F(\gamma) = \sqrt{-\ln(1 - \Gamma(x))}$ as a function of x and the best straight line fit to

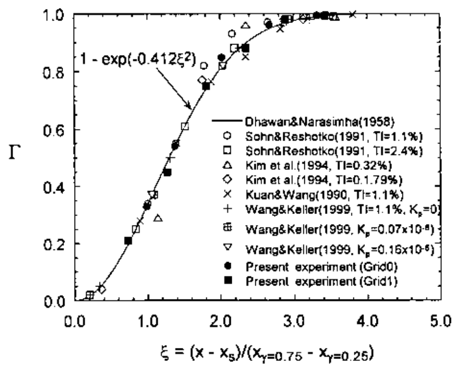


Fig. 13 Representative intermittency profiles along streamwise direction compared with Narasimha's universal intermittency distribution

the data between $0.25 < \Gamma < 0.75$. The intercept with x-axis is x_S and $x_{\gamma=0.75}$, $x_{\gamma=0.25}$ are obtained from the reverse function of the fitted straight line. This technique was introduced by Narasimha et al. (1984) and adopted by Keller and Wang (1995) and Gostelow et al. (1994). There were different previous results about the selection of representative intermittency in the transition zone. Mayle (1991) reported that most researchers using hot-wire determined the near-wall intermittency around $y/\delta = 0.2$. On the other hand, Keller and Wang (1995) proposed that the intermittency around $y/\delta = 0.1$ was more appropriate. To find the representative intermittency in the measured data of this study, intermittency profiles at several y locations were plotted and compared with Narasimha's formula, Eq. (2). The distribution of peak value, $\gamma_{max}(x)$ in the transition zone was found to be in the best agreement with Eq. (2), and, therefore, was selected as the representative intermittency.

Figure 13 exhibits the representative intermittency profiles $\Gamma(\xi)$ of the present data as well as other experimental data. Although each database is measured under different freestream turbulence intensity and pressure gradient, all intermittency profiles agree well with the universal intermittency distribution formula of Narasimha.

4. Correlation for Near-Wall Velocity Profiles in Transitional Boundary Layer

Many theoretical and experimental studies reported that the mean quantities of velocity profile, shape factor and wall skin friction were closely concerned with near-wall intermittency in the transitional boundary layer. Dhawan and Narasimha (1958) estimated mean quantities in the transition zone based on intermittency and regarded the non-turbulent part of transitional flow as laminar flow and the turbulent part as fully turbulent flow. However, others (e. g. Kuan and Wang (1990), Sohn and Reshotko (1991) and Kim et al. (1994)) indicated that the non-turbulent and turbulent parts in transitional flows were not Blasius and fully turbulent flows, respectively. Using the data of Sohn and Reshotko (1991), Jeon and Kang (1995) reported that the near-wall velocity profiles with similar values of maximum intermittency coincided well with each other.

In the present study, we assume that a similar velocity profile in transition zone exists as a function of representative intermittency as well as wall unit y^+. As shown in Figs. 3, 5, 6 and 11, the extended wall function of Eq. (1) with free parameter K_1, which is determined from the CPM process, reasonably predicted the near-wall velocity profiles of transitional boundary layer within the applying range i. e. $y^+<30$. The value of K_1 in Eq. (1) can be characterized from the state of transition and therefore, closely related to near-wall intermittency. Consequently, K_1 is supposed to correlate strongly with representative intermittency of each boundary layer.

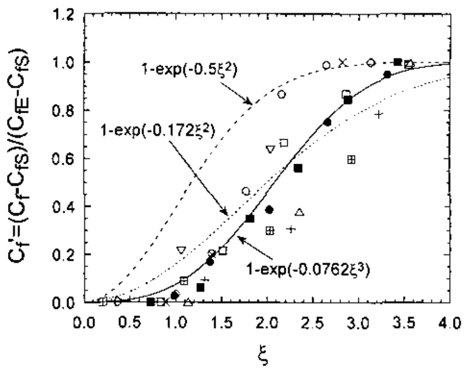


Fig. 14 Normalized wall skin friction versus non-dimensional distance in the transition region (each symbol is the same one in the legend of Fig. 13)

To estimate the empirical correlation between K_1 and Γ, we compiled the available databases that include mean velocity profiles, skin friction coefficients and representative intermittencies. In addition, to find a physically reasonable correlation, the universal intermittency distribution (Eq. (1)) was considered in the present study. On the other hand, according to the transition zone model of Dhawan and Narasimha (1958), the normalized wall skin friction distribution can be expressed as follows;

$$C_f'=\frac{(C_f-C_{fs})}{(C_{fE}-C_{fs})}=1-\exp(-0.5\xi^2) \quad (3a)$$

where ξ is determined from the Narasimha's technique introduced in Section 3.3 and C_{fs}, C_{fE}, denote the wall skin friction coefficient at the start and end points of transition, respectively. The C_{fs}, C_{fE}, can not be exactly selected in the discrete experimental data and hence, are set to be the minimum value in a laminar region and the maximum value in a turbulent region, respectively. As shown in Fig. 14, experimental data do not agree with Eq. (3a) though there are large scatters in data set. This fact shows the limitation of the Narasimha's transition model for prediction of mean quantities. We obtained more reasonable curves for wall shear stress distribution using the experimental data as presented in Fig. 14. The

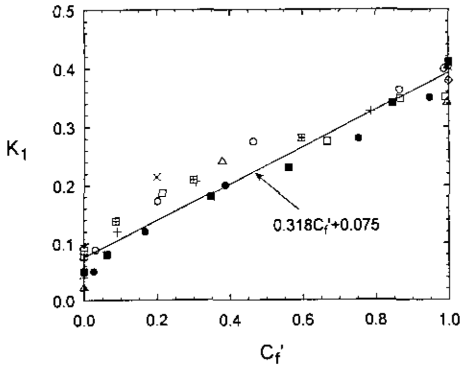


Fig. 15 Free parameter K_1 versus normalized wall skin friction in the transition region (each symbol is the same one in the legend of Fig. 16)

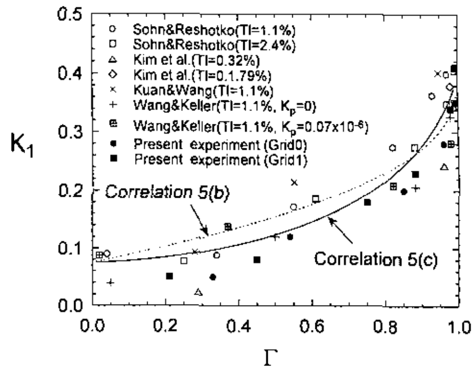


Fig. 16 Free parameter K_1 versus representative intermittency in the transition region

fitting curves are

$$C_f' = 1 - \exp(-0.172\xi^2) \quad (3b)$$
$$C_f' = 1 - \exp(-0.0762\xi^3) \quad (3c)$$

The relation Eq. (3b) has the same form with Eq. (3a) but the coefficient of exponent is changed. Eq. (3c) is in good agreement with the present experimental data even though this relation is acquired from the whole experimental data. As mentioned above, we compiled limited databases to acquire the relation between K_1 and Γ. Therefore, more databases are required to evaluate the $C_f' \sim \xi$ relation. If nondimensional distance ξ is eliminated in Eqs. (2), (3a), (3b) and (3c), the correlation between $C'f$ and Γ are obtained. To connect representative intermittency and free parameter, we simply assumed that K_1 varied linearly with the normalized wall skin friction C_f';

$$K_1 = mC_f' + K_{1s} \quad (4)$$

K_{1S} is the initial value of free parameter when skin friction has its minimum value in laminar boundary layers. From compiling all experimental data, linearly fitting curve is gained as exhibited in Fig. 15 where m, K_{1S} are 0.318 and 0.075 respectively. If C_{fS} is the exact value of transition onset, K_{1S} is ideally zero. However, because of uncertainty in the measured near-wall velocities and skin friction coefficients, K_{1S} has different value in each measurement (Fig. 15). More experimental data should be analyzed to verify Eq. (4). Based on the $C_f' \sim \Gamma$ relation and Eq. (4), three approximate functions of free parameter K_1 corresponding to Eqs. (3a), (3b) and (3c) are expressed as

$$K_1 = m\{1 - (1-\Gamma)^{1.214}\} + K_{1S} \quad (5a)$$
$$K_1 = m\{1 - (1-\Gamma)^{0.418}\} + K_{1S} \quad (5b)$$
$$K_1 = m[1 - \exp\{-0.288(-\ln(1-\Gamma))^{1.5}\}] + K_{1S} \quad (5c)$$

Figure 16 presents these curves with the present and other experimental data. The relation (5c) is the most reasonable among these curves although the experimental data shows large scattering.

5. Conclusions

Using the principle of Computational Preston Tube Method, the wall skin friction was estimated from the near-wall velocity profiles in transitional boundary layers. Compared with the DNS and experimental data, it was confirmed that the present method could predict wall skin friction in the transition region with reasonable accuracy regardless of freestream conditions (e. g. turbulence intensity, streamwise pressure gradient). The analysis using DNS data provided the applicable range of y^+ for the present method. The results of near-wall hot-wire measurement also supported the validity of the CPM. Intermittency profiles, which were estimated from the conditional sampling technique, showed typical characteristics of transitional boundary layer. The relation between the representative intermittency and free parameter K_1 of the extended wall function of CPM was empirically proposed. This correlation is expected to be helpful for numerical simulation of transitional boundary layers.

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References

Cantwell, B. J., Coles, D. and Dimotakis, P. E., 1978, "Structure and Entrainment in the Plane of Symmetry of a Turbulent Spot," *J. of Fluid Mechanics*, Vol. 87, pp. 641~672.

Cebeci, T. and Smith, A. M. O., 1974, *Analysis of Turbulent Boundary layer*, Academic Press, New York.

Dhawan, S. and Narasimha, R., 1958, "Some Properties of Boundary Layer Flow During the Transition from Laminar to Turbulent Motion," *J. of Fluid Mechanics*, Vol. 3, pp. 418~436.

Escudier, M. P. and Acharya M., 1987, "Critique of the Computational Preston Tube Method," *Experiments in Fluids*, Vol. 5, pp. 59~62.

Gostelow, J. P., Blunden, A. R. and Walker, G. J., 1994, "Effects of Free-Stream Turbulence and Adverse Pressure Gradients on Boundary Layer Transition," *ASME J. of Turbomachinery*, Vol. 116, pp. 392~404.

Hinze, J. O., 1975, Turbulence, McGraw Hill, New York.

Hedley, T. B. and Keffer, J. F., 1974, "Turbulent/Non-Turbulent Decisions in an Intermittent Flow," *J. of Fluid Mechanics*, Vol. 64, pp. 625~644.

Keller, F. J. and Wang, T., 1995, "Effect of Criterion Functions on Intermittency in Heated Transitional Boundary Layers with and without Streamwise Acceleration," *ASME J. of Turbomachinery*, Vol. 117, pp. 154~165.

Kim, J., Simon, T. W. and Kestoras, M., 1994, "Fluid Mechanics and Heat Transfer Measurements in Transitional Boundary Layers Conditionally Sampled on Intermittency," *ASME J. of Turbomachinery*, Vol. 116, pp. 405~416.

Klebanoff, P. S., 1955, "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," *NACA Rep.* No. 1247.

Kreplin, H. P. and Höhler, G., 1992, "Application of the Surface Hot Film Technique to Laminar Flow Technology," *DGLR-Rep.* 92-06, pp. 123~131.

Kuan, C. L. and Wang, T., 1990, "Investigation of the Intermittent Behavior of Transitional Boundary Layer Using a Conditional Averaged Technique," *Experimental Thermal and Fluid Science*, Vol. 3, pp. 157~173.

Jeon, W. P. and Kang, S. H., 1995, "Measurements of Transitional Boundary Layer on a Flat Plate Using a Computational Preston Tube Method," *Experiments in Fluids*, Vol. 20, pp. 29~37.

Mayle, R. E., 1991, "The Role of Laminar-Turbulent Transition in Gas Turbine Engines," *ASME J. of Turbomachinery*, Vol. 113, pp. 509~537.

Narasimha, R., Devasia, K. J., Gururani, G. and Badri Narayanan, M. A., 1984, "Transitional Intermittency in Boundary Layers Subjected to Pressure Gradient," *Experiments in Fluids*, Vol. 2, pp. 171~176.

Nitsche, W., Thünker, R. and Haberland, C., 1983, "A Computational Preston Tube Method," *Turbulent Shear Flows 4*, pp. 261~276.

Nitsche, W., 1987, "Rebuttal to the 'Critique of the Computational Preston Tube Method'," *Experiments in Fluids*, Vol. 5, pp. 63~65.

Preston, J. H., 1954, "The Determination of Turbulent Skin Friction by Means of Pitot Tubes," *J. of the Royal Aeronautical Society*, Vol. 58, pp. 109~121.

Rai, M. M. and Moin, P., 1993, "Direct Numerical Simulation of Transition and Turbulence in a Spatially Evolving Boundary Layer," *J. of Computational Physics*, Vol. 109, pp. 169~192.

Schlichting, H., 1979, *Boundary layer theory*, McGraw Hill, New York.

Savill, A. M., 1993, "Further Progress in the Turbulence Modelling of By-Pass Transition," *Proceeding of Second International Symposium on Engineering Turbulence Modeling and Measurements*, Florence, Italy, pp. 583~592.

Sohn, K. H. and Reshotko, E., 1991, "Experimental Study of Boundary Layer Transition with Elevated Freestream Turbulence on a Heated Flat Plate," *NACA CR* 187068.

Szablewski, W., 1969, "Turbulente Grenzschichten in Ablösenahe," *Z. Angew, Math.* , Vol. 49, p. 215.

Wang, T. and Keller, F. J., 1999, "Intermittent Flow and Thermal Structures of Accelerating Transitional Boundary Layers: Part 1-Mean Quantities," *ASME J. of Turbomachinery*, Vol. 121, pp. 98~105.

Weiser, N., Mirow, P.· and Nitsche, W., 1989, "Statische und Dynamische Meβverfahre zur Transitionsbestimmung an Laminarprofilen," *DGLR-jahrbuch 1*, pp. 427~434.