A V-Shaped Lyapunov Function Approach to Model-Based Control of Flexible-Joint Robots

Ho-Hoon Lee*
Department of Mechanical Engineering, University of Suwon
Seung-Gap Choi
Instrumentation & Control Research Group, POSCO Technical Research Laboratories

This paper proposes a V-shaped Lyapunov function approach for the model-based control of flexible-joint robots, in which a new model-based nonlinear control scheme is designed based on a V-shaped Lyapunov function. The proposed control guarantees global asymptotic stability for link trajectory control while keeping all internal signals bounded. Since joint flexibility is used as a control parameter, the proposed control is not restricted by the degree of joint flexibility and can be applied to flexible-joint, partly-flexible-joint, or rigid-joint robots without modification. The effectiveness of the proposed control has been shown by computer simulation.

Key Words : Flexible-Joint Robot, Model-Based Control, Adaptive Control, Lyapunov Stability Theorem, Barbalat's Lemma

1. Introduction

Industrial robots, by nature, include a certain degree of joint flexibility due to the flexibility of the shafts and harmonic drives in the joints, which may induce mechanical vibrations and degrade control performance. In fact, Sweet and Good (1984) experimentally showed that joint flexibility should be taken into account for high performance control. Spong (1989) proposed a dynamic modeling of a flexible-joint robot, which is reduced to that of the corresponding rigid-joint robot as the joint stiffness approaches infinity. Since then, the control of flexible-joint robots has attracted considerable attention.

Extensive research has been performed for the control of flexible-joint robots. Numerous full state feedback controls have been designed based on the integral manifold approach (Spong 1989 and Khorasani 1992) and the integrator backstepping technique (Nicosia and Tomei 1991, Dawson et al. 1991, Yuan and Stepanenko 1993, and Bridges et al. 1995). These full state feedback controls normally require the velocity and position measurements of the links and actuators. In practice, the angular velocity is usually computed by numerical differentiation of the angular position. For slow motions, the angular velocity can be measured with considerable accuracy by measuring the duration between the two consecutive encoder pulses.

In addition, various partial state feedback controls were also proposed for set-point regulation (Tomei 1991, Ailon et al. 1993, and Kelly et al. 1994). Recently, several tracking controllers have been designed based on model-based observers (Chang et al. 1996 and Lim et al. 1997), where only the position measurements of the links and actuators are used in the control. These partial state feedback controls estimate the velocities of the links and actuators using model-based observers; consequently, the resulting control schemes tend to be complicated and usually

* Corresponding Author,
E-mail : hhlee@mail.suwon.ac.kr
TEL : +82-31-220-2608 ; **FAX** : +82-31-220-2494
Department of Mechanical Engineering, University of Suwon, Gyunggi-do 445-743, Korea. (Maunuscript **Received** February 21, 2000 ; **Revised** August 21, 2000)

require exact knowledge of the dynamic model and its parameters. In contrast, full state feedback controls normally lead to simpler control schemes; however, they require velocity sensors or additional circuitry for the measurement of the duration between the two consecutive encoder pulses. Therefore, there should be tradeoffs between the full and partial state feedback controls.

This paper proposes a V-shaped Lyapunov function approach for the model-based control of flexible-joint robots, where a new model-based nonlinear control scheme is designed based on a V-shaped Lyapunov function. The proposed control is a full state feedback control of simple structure, the complexity of which is close to that of the full state feedback control of rigid-joint robots (Slotine and Li 1987). The proposed control guarantees global asymptotic stability for link trajectory control while keeping all internal signals bounded. Since joint flexibility is used as a control parameter, the proposed control is not restricted by the degree of joint flexibility and can be applied to flexible-joint, partly-flexible-joint, or rigid-joint robots without modification.

The remainder of this paper is organized as follows. In Section 2, the dynamic model of a flexible-joint robot is described. In Section 3, a model-based nonlinear control scheme is designed based on a V-shaped Lyapunov function, and the nonlinear control is furthermore extended to an adaptive control. In Section 4, the proposed control is evaluated through computer simulations. Finally in Section 5, the conclusions are drawn for this study.

2. Modeling of a Flexible-Joint Robot

The motion of a flexible-joint robot can be described by the following state-space equations (Spong 1989):

$$M(q)\ddot{q}+C(\dot{q},q)\dot{q}+D\dot{q}+g(q)=K(p-q), \tag{1}$$

$$J\ddot{p}+B\dot{p}+K(p-q)=u, \tag{2}$$

where q and p are the $n\times 1$ state vectors for the links and actuators, respectively; n is the number of joints; $g(q)$ and u are the $n\times 1$ gravitational force vector and input vector, respectively; $M(q)$ is the $n\times n$ positive definite symmetric mass matrix of the links; J is the $n\times n$ positive definite diagonal inertia matrix of the actuators; $C(\dot{q},q)$ is the $n\times n$ Coriolis and centrifugal force matrix satisfying $(\dot{M}-2C)=(2C-\dot{M})^T$; D and B are the $n\times n$ viscous damping matrices of the links and actuators, respectively; K is the $n\times n$ stiffness matrix of the joints. When $K>0$, the link dynamics (1) is controllable.

3. Control Design

In this section, a model-based nonlinear control law will be designed based on the Lyapunov stability theorem.

First, the following error signals are defined:

$$e_p=r_p-p, \tag{3}$$

$$e_q=r_q-q, \tag{4}$$

$$s_q=\dot{e}_q+K_q e_q, \tag{5}$$

$$z_q=\dot{r}_q+K_q e_q, \tag{6}$$

where r_q and r_p are the reference trajectories of q and p, respectively; K_q is the $n\times n$ positive definite symmetric matrix.

Then the following error dynamics can be derived from the dynamic model (1) and (2) and the error signals defined in Eqs. (3) ~ (6) :

$$M(q)\dot{s}_q+C(\dot{q},q)s_q=W_q x, \tag{7}$$

$$J\ddot{e}_p=W_p x-u, \tag{8}$$

with

$$W_q x\equiv M(q)\dot{z}_q+C(\dot{q},q)z_q+D\dot{q}+g(q)-K(p-q), \tag{9}$$

$$W_p x\equiv J\ddot{r}_p+B\dot{p}+K(p-q), \tag{10}$$

where W_q and W_p are the $n\times n$ regressor matrices for the links and actuators, respectively, x is the $n_x\times 1$ constant parameter vector, and n_x is the number of parameters in the dynamic model.

In this study, the reference trajectories r_q and r_p are assumed to be selected such that they satisfy the dynamic model (1) and (2). In addition, the frequency contents of r_q and r_p are assumed to be set much smaller than the natural frequencies of the joints defined in the dynamic model (2). Finally, the actuator torque is assumed to be sufficient enough to track the reference trajectories.

Theorem: Consider a flexible-joint robot whose dynamics is described by Eqs. (1) and (2), and suppose that the dynamic system is controllable, i.e. $K>0$. Then the following model-based control (11) guarantees that s_q, s_{qi}, $\dot{e}_p$, e_p, $k \in L_\infty$, and that $e_q(t) \to 0$, $\dot{e}_q(t) \to 0$, and $\dot{e}_p(t) \to 0$, asymptotically as $t \to \infty$:

$$u = u_p + \sigma_q u_q + \sigma_f \,|\, k \,|\, K^{-1} \dot{e}_p, \qquad (11)$$

with

$$\dot{k} = \sigma_k[u_q^T(\sigma_q \dot{e}_p - s_q)\,\mathrm{sgn}(k) + \sigma_f \dot{e}_p^T K^{-1} \dot{e}_p k], \qquad (12)$$

$$u_q \equiv W_q x + K_{pq} s_q + K_{qi} s_{qi}, \qquad (13)$$

$$u_p \equiv W_p x + \sigma_d K^{-1} \dot{e}_p + \sigma_p K^{-1} e_p, \qquad (14)$$

$$s_{qi} \equiv \int_0^t s_q(\tau)\, d\tau, \qquad (15)$$

$$\mathrm{sgn}(k) \equiv \begin{cases} 1, & k \geq 0, \\ -1, & k < 0, \end{cases} \qquad (16)$$

where σ_q, σ_f, σ_k, σ_d, and σ_q are some positive constants and σ_q is set such that $\sigma_q K$ is sufficiently large, and K_{pq} and K_{qi} are the $n \times n$ positive definite and positive semi-definite symmetric matrices, respectively.

Remark 1: Since $K(p-q)$ represents the joint torque for the link dynamics (1), u_q defined in Eq. (13) is the joint torque error; refer to Slotine and Li (1987). Hence the proposed control (11) is composed of the actuator control u_p, the joint torque error $\sigma_q u_q$, and the flexibility compensation $\sigma_f \,|\, k \,|\, K^{-1} \dot{e}_p$.

Remark 2: As the joint becomes rigid, $K \to \infty$ and $p \to q$. Then, with $\sigma_q = 1$, the model-based control (11) is reduced to that of rigid-joint robots; refer to Slotine and Li (1987). If $K \geq 0$ (uncontrollable), the control input u is not bounded; consequently, the stability proof becomes invalid.

Proof of Theorem: Consider the following V-shaped Lyapunov function candidate:

$$V(t) = \frac{1}{2}(s_q^T M s_q + s_{qi}^T K_{qi} s_{qi} + \dot{e}_p^T J \dot{e}_p + \sigma_p e_p^T K^{-1} e_p) + \frac{1}{\sigma_k} |\, k \,| \geq 0. \qquad (17)$$

Take the time derivative of $V(t)$ along the trajectories of the error dynamics (7) and (8) and the control law (11):

$$\begin{aligned} \dot{V} &= s_q^T M \dot{s}_q + \frac{1}{2} s_q^T \dot{M} s_q + s_q^T K_{qi} s_{qi} \\ &\quad + \dot{e}_p^T J \ddot{e}_p + \sigma_p e_p^T K^{-1} \dot{e}_p + \frac{1}{\sigma_k} \frac{k}{|\, k \,|} \dot{k} \\ &= s_q^T (W_q x + K_{qi} s_{qi}) \\ &\quad + \dot{e}_p^T (W_p x + \sigma_p K^{-1} e_p - u) + \frac{1}{\sigma_k} \frac{k}{|\, k \,|} \dot{k} \\ &= - s_q^T K_{pq} s_q - \sigma_d \dot{e}_p^T K^{-1} \dot{e}_p, \\ &\qquad \text{when } k \neq 0 \text{ or } k = 0 \text{ with } \dot{k} = 0, \end{aligned} \qquad (18)$$

where the properties $M = M^T$ and $(\dot{M} - 2C) = (2C - \dot{M})^T$ have been used in the derivation.

When $k \neq 0$, $\dot{V}$ in Eq. (18) is clearly valid. When $k=0$ and $\dot{k}=0$, $d\,|\, k \,|\,/dt = \dot{k}k/\,|\, k \,| = 0$ and $u_q^T(\sigma_p \dot{e}_p - s_q) = 0$ according to Eq. (12). Therefore $\dot{V}$ is definitely valid whenever $k \neq 0$ or $k=0$ with $\dot{k}=0$.

Now, suppose that $\dot{V}$ in Eq. (18) is valid at t_i^-, and that k changes instantaneously such that $k=0$ and $\dot{k} \neq 0$ ($u_q^T(\sigma_p \dot{e}_p - s_q) \neq 0$ for this case) at t_i. Then V is not differentiable at t_i. Since u_q is well defined and continuous for all time $t \geq 0$, k, the solution of Eq. (12), is well defined and is continuous for all time $t \geq 0$; hence k is continuous at t_i, i.e. $k(t_i^-) = k(t_i^+)$. Since V is a continuous function of k, V remains continuous at t_i, i.e. $V(t_i^-) = V(t_i^+)$. When $k=0$ and $\dot{k} \neq 0$ at t_i, Eq. (12) guarantees $k \neq 0$ at t_i^+ and hence $\dot{V}$ is valid at t_i^+. Accordingly, $\dot{V}$ in Eq. (18) is valid for all time $t \geq 0$.

The Lyapunov function V in Eq. (17) and its time-derivative $\dot{V}$ in Eq. (18) imply that s_q, s_{qi}, $\dot{e}_p$, e_p, $k \in L_\infty$. Then, $\dot{s}_q$, $\ddot{e}_p \in L_\infty$ follows from the error dynamics (7) and (8) and the control law (11). In addition, integration of $\dot{V}$ in Eq. (18) yields $\int_0^\infty \|s_q\|^2 d\tau < \infty$ and $\int_0^\infty \|\dot{e}_p\|^2 d\tau < \infty$ so that s_q, $\dot{e}_p \in L_2$, where $\|\cdot\|$ denotes the Euclidean norm of the argument vector. Then, as a consequence of Barbalat's Lemma, $s_q(t) \to 0$ and $\dot{e}_p(t) \to 0$, and hence $e_q(t) \to 0$ and $\dot{e}_q(t) \to 0$, asymptotically as $t \to \infty$. Q. E. D.

Remark 3: For possible improvement in control performance, the proposed nonlinear control can be readily extended to an adaptive control by replacing the real parameter x in the theorem with the computed parameter $\tilde{x}$ defined as $\dot{\tilde{x}} = K_x(W_q^T s_q + W_p^T \dot{e}_p)$, where $K_x = K_x^T > 0$. Then the global asymptotic stability of the resulting adaptive control can be easily proven by adding $(\tilde{x}_e^T K_x^{-1} \tilde{x}_e)/2$ to the Lyapunov function $V(t)$ defined in Eq. (17), where $\tilde{x}_e \equiv \tilde{x} - x$.

Remark 4: When there exist parametric uncertainties, the reference trajectories satisfying the dynamic model (1) and (2) cannot be generated, which implies that both e_q and e_p cannot be driven to zero as $t \rightarrow \infty$. For this reason, in the theorem, the actuator tracking error e_p is not driven to zero as $t \rightarrow \infty$, which allows a certain level of actuator tracking errors to prevent the additional link tracking error e_q due to parametric uncertainties.

Remark 5: The proposed V-shaped Lyapunov function approach has been successfully applied to the control of flexible-link robots (Lee 1999). The motions of industrial overhead and rotary cranes can be described by the dynamic model of a flexible-link robot (Lee 1998 and 1998a and Lee and Cho 1997 and 1999); consequently, the proposed V-shaped Lyapunov function approach can also be applied to the control of industrial overhead and rotary cranes.

4. Simulation Results

As an example, the proposed control has been applied to the control of a two-link flexible-joint robot shown in Fig. 1. The following matrices are obtained for the dynamic model of this robot:

$$M = \begin{bmatrix} x_1 + 2x_2\cos(q_2) + x_3 & x_1 + x_2\cos(q_2) \\ x_1 + x_2\cos(q_2) & x_1 \end{bmatrix} \tag{19}$$

$$C = \begin{bmatrix} -x_2\sin(q_2)\dot{q}_2 & x_2\sin(q_2)(\dot{q}_1 - \dot{q}_2) \\ x_2\sin(q_2)\dot{q}_1 & 0 \end{bmatrix}, \tag{20}$$

$$g = \begin{pmatrix} x_6\cos(q_1+q_2) + x_7\cos(q_2) \\ x_6\cos(q_1+q_2) \end{pmatrix}, \tag{21}$$

$$D = \begin{bmatrix} x_4 & 0 \\ 0 & x_5 \end{bmatrix}, \tag{22}$$

$$K = \begin{bmatrix} x_8 & 0 \\ 0 & x_9 \end{bmatrix}, \tag{23}$$

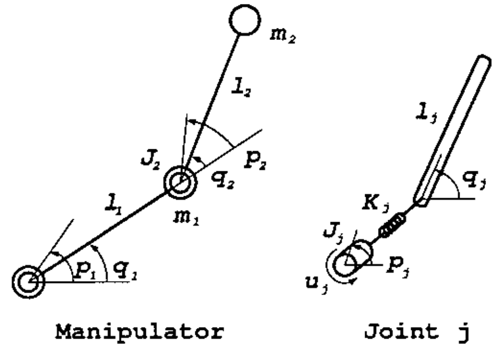

Fig. 1 Model of a two-link flexible-joint robot

$$J = \begin{bmatrix} x_{10} & 0 \\ 0 & x_{11} \end{bmatrix}, \tag{24}$$

$$B = \begin{bmatrix} x_{12} & 0 \\ 0 & x_{13} \end{bmatrix}, \tag{25}$$

where $x_1 = l_2^2 m_2$, $x_2 = l_1 l_2 m_2$, $x_3 = l_1^2(m_1 + m_2)$, $x_6 = l_2 m_2 g$, and $x_7 = l_1(m_1 + m_2)g$. In this paper, the integer subscripts i and ij denote the ith components of the vectors and the ijth components of the matrices, respectively.

Then the regressor matrices W_q and W_p and the parameter vector x defined in Eqs. (9) and (10) are obtained as

$$W_q = \begin{bmatrix} w_{11} & w_{12} & \dot{z}_{q1} & \dot{q}_1 & 0 & w_{16} & w_{17} & w_{18} & 0 & 0 & 0 & 0 & 0 \\ w_{21} & w_{22} & 0 & 0 & \dot{q}_2 & w_{26} & 0 & 0 & w_{29} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{26}$$

$$W_q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_{38} & 0 & \ddot{r}_{p1} & 0 & p_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_{49} & 0 & \ddot{r}_{p2} & 0 & p_2 \end{bmatrix}, \tag{27}$$

$$x = (x_1 \ x_2 \ \cdots \ x_{12} \ x_{13})^T, \tag{28}$$

where

$$\begin{aligned} w_{11} &= w_{21} = \dot{z}_{q1} + \dot{z}_{q2}, \\ w_{12} &= \cos(q_2)(2\dot{z}_{q1} + \dot{z}_{q2}), \\ &\quad -\sin(q_2)(\dot{q}_2 z_{q1} + \dot{q}_1 z_{q2} + \dot{q}_2 z_{q2}), \\ w_{16} &= w_{26} = \cos(q_1 + q_2), \\ w_{17} &= \cos(q_1), \\ w_{18} &= -w_{38} = q_1 - p_1, \\ w_{22} &= \cos(q_2)\dot{z}_{q1} + \sin(q_2)\dot{q}_1 z_{q1}, \\ w_{29} &= -w_{49} = q_2 - p_2 \end{aligned} \tag{29}$$

The proposed control has been applied to the robot shown in Fig. 1. Parts of numerical values of the robot parameters are taken from a Unimation PUMA 560 arm: $m_1 = 15.91$ kg, $m_2 = 11.36$ Kg, and $l_1 = l_2 = 0.432$ m. For the rest of the

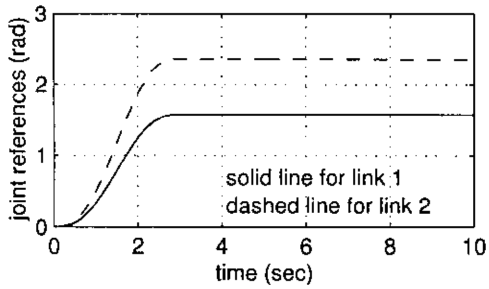

Fig. 2 Link reference trajectories

dynamic parameters, the following numerical values are used: $x_4=10$ Nm/s, $x_5=3$ Nm/s, $x_8=200$ Nm/rad, $x_9=200$ Nm/rad, $x_{10}=5$ Nm/s^2, $x_{11}=5$ Nm/s^2, $x_{12}=2$ Nm/s, and $x_{13}=2$ Nm/s. Note that the joint stiffness ($x_8=x_9=200$ Nm/rad) is extremely low considering the mass and link length of the robot.

The link reference trajectory r_q has been chosen as

$$r_{q1}=\frac{\pi}{2}\left[15\left(\frac{t}{t_{q1}}\right)^7-70\left(\frac{t}{t_{q1}}\right)^6+126\left(\frac{t}{t_{q1}}\right)^5-105\left(\frac{t}{t_{q1}}\right)^4+35\left(\frac{t}{t_{q1}}\right)^3\right], \quad (30)$$

$$r_{q2}=\frac{3\pi}{4}\left[15\left(\frac{t}{t_{q2}}\right)^7-70\left(\frac{t}{t_{q2}}\right)^6+126\left(\frac{t}{t_{q2}}\right)^5-105\left(\frac{t}{t_{q2}}\right)^4+35\left(\frac{t}{t_{q2}}\right)^3\right], \quad (31)$$

where $t_{q1}(=3\text{s})$ and $t_{q2}(=3\text{s})$ are the expected duration of motion for links 1 and 2, respectively. The link reference trajectory r_q is shown in Fig. 2.

Since r_q is available, the actuator reference trajectory r_p can be computed using the dynamic model (1) with q and p replaced by r_q and r_p, respectively:

$$r_p=f(r_q,\ \dot{r}_q,\ \ddot{r}_q) \equiv r_q+K^{-1}[M(r_q)\ddot{r}_q+C(\dot{r}_q,\ r_q)\dot{r}_q+D\dot{r}_q+g(r_q)], \quad (32)$$

$$\dot{r}_p=\frac{d}{dt}f(r_q,\ \dot{r}_q,\ \ddot{r}_q), \quad (33)$$

$$\ddot{r}_p=\frac{d^2}{dt^2}f(r_q,\ \dot{r}_q,\ \ddot{r}_q). \quad (34)$$

The following control gains were used in this simulation: $\sigma_k=0.001$; $\sigma_q=5$; $k(0)=10$; $K_q=\text{diag}(10, 10)$; $K_{pq}=\text{diag}(20, 20)$; $K_{qi}=\text{diag}(3, 3)$; $\sigma_d K^{-1}=\text{diag}(20,\ 20)$; $\sigma_p K^{-1}=\text{diag}(30,\ 30)$; $\sigma_f K^{-1}=\text{diag}(10,\ 10)$. The sampling rate was chosen to be 500 Hz. One sampling delay was included in the control inputs as in practical applications. The trapezoidal rule was used for the integration in the proposed control. The dynamics of the robot was solved using the fourth order Runge-Kuta formula with adaptive stepsize (Press et al. 1986).

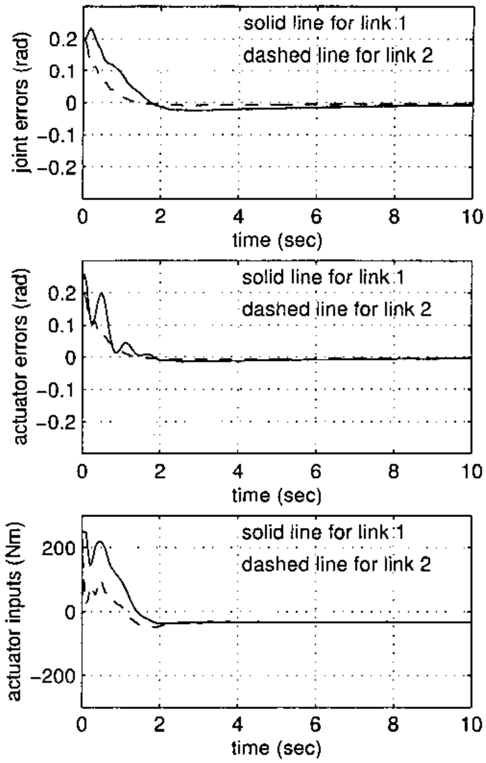

Fig. 3 Simulation results without parametric uncertainties

Figures 3 and 4 show the simulation results of the proposed control without and with parametric uncertainties, respectively. The solid and dashed lines in the figures represent the experimental results for links 1 and 2, respectively. For the simulation of parametric uncertainties, the nominal value of the parameter x was set to seventenths of the true value of the parameter x, and was used for the control computation and for the generation of actuator reference trajectories. The initial errors in the link trajectories are 0.2 rad, which is about 11 degrees, for both links.

As expected, the control performance without

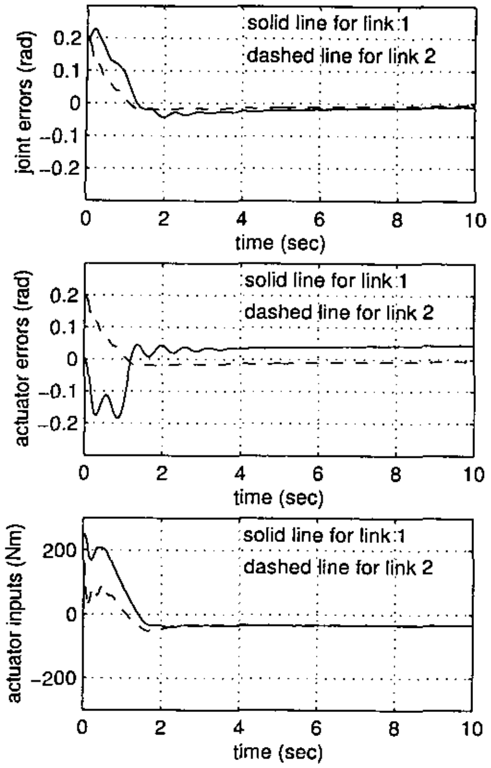

Fig. 4 Simulation results with parametric uncertainties

parametric uncertainties is better than that with parametric uncertainties. Without parametric uncertainties, all the tracking errors are reduced asymptotically to zero; see Fig. 3. With parametric uncertainties, the link tracking errors are reduced asymptotically to zero, but the actuator tracking errors approach some nonzero constants; see Fig. 4. The parametric uncertainties normally cause a certain level of error in the actuator reference trajectories (32) ~ (34), which contribute to actuator tracking errors. The control objective is not, however, to drive the actuator tracking errors to zero. Note that the tolerance of thirty-percent error in the model parameters is quite realistic in industrial applications.

The overall control performance does not require the adaptation of the model parameters. The actuator inputs are also in very good shape for both cases. The tracking errors can be considerably reduced with high control gains. The link tracking errors can be rapidly driven to zero when the integral gain K_{qi} is high; however, in this case the overshoot also becomes larger.

In conclusion, the computer simulations are in good agreement with the theoretical results. That is, the link tracking errors are asymptotically stable with all the internal signals bounded even though extremely low values of joint stiffness are used in the simulation.

5. Conclusion

In this study, the V-shaped Lyapunov function approach has been successfully applied to the model-based control of flexible-joint robots. The simulation results have shown the effectiveness of the proposed design approach and the resulting model-based control for flexible-joint robots. The proposed control is simple with fewer control parameters to determine; the complexity of the proposed control is close to that of the full state feedback control of rigid-joint robots.

The link tracking errors are asymptotically stable with all the internal signals bounded, and the actuator inputs are in very good shape for all cases. The control performance is little affected by errors in the actuator reference trajectories due to parametric uncertainties. Since joint flexibility is used as a control parameter, the proposed control is not restricted by the degree of joint flexibility and can be applied to flexible-joint, partly-flexible-joint, or rigid-joint robots without modification. Consequently, it can be concluded that the proposed control has a great potential for practical application.

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