Journal of Korean Society of
Coastal and Ocean Engineers
Vol. 12, No. 3, pp. 109~115, September 2000

Dynamic Analysis of Riser with Vortex Excitation by Coupled Wake Oscillator Model
연계 후류진동 모델 적용을 통한 와류방출 가진에 의한 라이저의 동적해석

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Abstract ☐ Numerical model is proposed to estimate dynamic responses of riser with vortex excitation by inline current. Galerkin's finite decomposition method is implemented for the development of a numerical model and vortex excitation is modeled by coupled wake oscillator proposed by Blevins. The numerical results are inspected through the physical interpretation to give the verification and usefulness of the suggested numerical model.

Keywords : vortex excitation, riser, coupled wake oscillator model, dynamic analysis

要 旨 : 면내 조류흐름에 의해 발생하는 와류방출 가진으로 인한 라이저의 동적거동해석을 위한 수치해석 모델을 개발한다. 수치해석 모델개발을 위해 Galerkin의 유한요소 근사화법을 적용하였으며 와류방출 가진은 Blevins에 의해 제안된 연계 후류진동 모델을 사용하였다. 와류방출로 인한 라이저의 일반적인 거동특성을 수치해석 결과와 비교분석함으로써 모델의 유용성을 검증하였다.

핵심용어 : 와류방출 가진, 라이저, 연계 후류진동 모델, 동적해석

1. INTRODUCTION

The vibration induced in elastic structures by vortex shedding is of practical importance because of its potentially destructive effect on pipelines or marine risers. Vortex-induced forces may excite the riser in its normal modes of transverse vibration. When the vortex shedding frequency approaches the natural frequency of a marine riser, the cylinder takes control of the shedding process causing to be shed at a frequency close to its natural frequencies. This phenomenon is called vortex shedding "lock-in" or synchronization. Under "lock-in" conditions, large resonant oscillations occur and lift forces are amplified due to increased vortex strength and spanwise correlation along the cylinder. This is attributed to a substantial increase in in-line drag. Large responses in both directions give rise to oscillatory stresses. If these stresses persist, significant fatigue damage may occur. The investigations of the vortex-induced vibration of structures have been studied continuously and widely up to now since observations and measurements were made on both rigid (Feng, 1968; Sarpkaya, 1977) and flexible (Ramberg and Griffin, 1974; Every *et al.*, 1982) cylinder vibrating transversely in uniform cross-flows. Iwan and Blevins (1974) proposed the two dimensional coupled wake oscillator model which has an advantages as an numerical model. Later, Blevins (1990) modified the model in order to include spanwise resonance band for nonuniform flow.

Several works have been done to obtain analytical and numerical solution. However, the numerical analysis by self excited oscillator has not quite been done yet. Therefore, the objective of this study is to propose a numerical model for the analysis of vortex-induced vibration to give useful tool. Iwan-Blevin's model (Blevins, 1990; Iwan

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and Jones, 1987), which is one of the self-excited oscillator models, is implemented as a vortex shedding model and Galerkin finite element processes are adopted to solve a mathematical model.

2. FORMULATION

The riser system can be modeled as a long tubular beam connecting the drilling platform with the wellhead at the seabed. A tensioning system is installed on the drilling platform and applies tension at the top of the riser. This tension provides part of the support required to keep the riser tight and prevent buckling or collapse. High pressure drilling mud is circulated along the annulus between the riser and the drilling string. It exerts static pressure force, vertical and torsional frictional forces on the riser. And also, the source of external forces exerted on the riser are the ocean currents.

2.1 Equations of Riser Pipe

The nonlinear vector equation of motion for a rod with circular section (Hong, 1995; Nordgren, 1974) is given by

$$m_t\ddot{\boldsymbol{r}}+(EI\boldsymbol{r}'')-[(T_e-EIx^2)\boldsymbol{r}']-H(\boldsymbol{r}'\times\boldsymbol{r}'')=\boldsymbol{q},\ x^2=\boldsymbol{r}''\cdot\boldsymbol{r}'' \quad (1)$$

where prime denotes the differentiation with respect to undeformed arclength s and dot with respect to time t. EI is the flexural rigidity of pipe, H torsional component of external moment and T_e effective tension. $\boldsymbol{q}$ is the forcing vector acting on the riser by external flow and m_t is the total mass including pipe, internal fluid, and added mass.

For nearly vertical risers, introducing a rectangular Cartesian coordinate system x_i (i=1, 2, 3) with the x_3 axis vertically upward and then, restricting attention to the riser whose deformation lies wholly in a single plane and neglecting nonlinear terms, the vector Eq. (1) reduces to

$$m_t\ddot{x}_1+(EIx_1'')''-(T_ex_1')+Hx_2'''=q_1 \quad (2)$$

$$m_t\ddot{x}_2+(EIx_2'')''-(T_ex_2')'-Hx_1'''=q_2 \quad (3)$$

$$m_t\ddot{x}_3-T_3'=q_3 \quad (4)$$

For most riser problems in intermediate water depth, longitudinal vibration is unimportant and then, we may neglect the longitudinal inertia term in Eq. (4). Further, it is convenient to include the hydrostatic effects of internal and external fluid pressures by defining effective weight per unit length w and effective tension T_e as

$$w=w_r+\gamma_iA_i-\gamma_oA_o \quad (5)$$

$$T_e=T+p_iA_i-p_oA_o \quad (6)$$

where w_r=riser weight per unit length γ_i, γ_o=specific weight of internal and external fluid, p_i, p_o=internal and external static pressure of riser, A_i, A_o=internal and external area of riser, and T is actual tension.

Eq. (5) leads to q_3=-w. Eq. (6) can then be integrated to give

$$T_e=TTR-\int_0^l wds\ \ or\ \ T_e=TTB+\int_0^l wds \quad (7)$$

where TTR and TTB are, respectively, the top tension and the bottom tension.

Thus, the effective tension is independent of the horizontal deflections and then, with no torsion, Eqs. (2) and (3) becomes

$$m_t\ddot{x}_1+(EIx_1'')''-wx_1'-T_ex_1''=q_1 \quad (8)$$

$$m_t\ddot{x}_2+(EIx_2'')''-wx_2'-T_ex_2''=q_2 \quad (9)$$

The riser support can be modeled by the linear translational spring providing the restoring boundary forces. The internal boundary conditions associated with Eqs. (8) and (9) can be found by modifying the direction cosines of nonlinear boundary condition at the top of the riser into 0, 0, 1, and are given as followings:

$$F_{1u}=-K_1x_{1u}$$
$$F_{2u}=-K_2x_{2u} \quad (10)$$

the subscript u indicates the upper end of the riser, TTR is the tension applied at top of the riser by the tensioning system and K_1, K_2, are spring constants supplied by the restoring boundary forces.

2.2 Vortex Excitation Model

The mathematical model of vortex excited vibration is presented using the two dimensional coupled wake oscillator model proposed by Blevins (1990) for the Reynolds number of 10^3 to 10^5. There are many vortex excitation models such as harmonic model, wake oscillator model, and etc. Among these models, Iwan-Blevin's model has an advantage as a numerical model. This model has a nature of self excited vortex shedding which means that the fluid behavior may be modeled by a simple, nonlinear, and self-excited oscillator. Attention is confined to plane vibrations (x_2=0) transverse to a steady current in direction x_2. In-line vibration is treated in the usual way and no coupling is considered in this model.

First, a so-called hidden flow variable w is defined as a fluid motion transverse to a steady current and it, in turn, produces transverse force to cylinder. This transverse force, which induces the motion of the cylinder, is a forcing component in oscillator equation. The net force per unit pipe length on cylinder is evaluated using the momentum equation for the control volume containing cylinder and is given by

$$q_1 = a_4\rho_w D_o U_c(\dot{w}-\dot{x}_1) \tag{11}$$

and the fluid oscillator equation is

$$\ddot{w}+\omega_s^2 w=(a_1'-a_4')U_c\dot{w}/D_o - a_2'\dot{w}^3/(U_c D_o) + a_4' U_c \ddot{x}_1/D_o \tag{12}$$

where ω_s is the vortex shedding frequency given by $\omega_s=2SU_c/D_o$ and Stroudal number S is taken as 0.20. The constants a_i (i=4) and a_i' (i=0,1,2) are determined on the basis of the results of various vortex experiments for subcritical Reynolds numbers (Blevins, 1990). Based on the results of various vortex experiments for subcritical Reynolds numbers, Iwan and Blevins(1974) took a_1'=0.916, a_1'=0.916,, a_2'=0.416, a_4'=0.792, a_4=0.38. Later, Blevins (1990) considered the spanwise coupling along the riser because portions of the riser span are not in resonance for nonuniform flow. In order to apply the model to nonuniform flow, the span of the riser is divided into segments that are within the resonance band and the remainder, which in outside the resonance band.

The evaluation of the net force on cylinder under the consideration of the spanwise coupling is obtained from the modification of Eq. (11) and is given by

$$q_1 = a_4\rho_w D_o U_c(\dot{w}-\dot{x}_1)p(x_3) - \frac{1}{2}\rho_w D_o U_c C_D \dot{x}_1(1-p(x_3)) \tag{13}$$

where parameter $p(x_3)$ defines the spanwise region of the resonance band as zero or one.

Since the model derived by Blevin and Iwan is based on the assumption of small amplitude, this model is applied to the approximated equation of motion (8), which gives

$$m_r\ddot{x}_1 + (EIx_1'')'' - wx_1' - T_e x_1'' = a_4\rho_w D_o U_c(\dot{w}-\dot{x}_1)p(x_3) - \frac{1}{2}\rho_w D_o U_c C_D \dot{x}_1(1-p(x_3)) \tag{14}$$

From the above equation, we can recognize that the riser system may respond resonantly to vortex shedding along the part of its span in the lock-in band and the system oscillation will be damped by the external fluid out of that part.

3. NUMERICAL SOLUTION

3.1 Numerical Model

Transverse vibration and in-line dynamic response of current-vortex model are estimated using finite element method in which the Galerkin finite element procedure is applied.

The mathematical model of current induced vibration can be obtained by combining Eqş. (9), (12) and (14):

$$m_r\ddot{x}_1 + (EIx_1'')'' - wx_1' - T_e x_1'' = a_4\rho_w D_o U_c(\dot{w}-\dot{x}_1)p(x_3) - \frac{1}{2}\rho_w D_o U_c C_D \dot{x}_1(1-p(x_3)) \tag{15}$$

$$\ddot{w}+\omega_s^2 w=(a_1'-a_4')U_c\dot{w}/D_o - a_2'\dot{w}^3/(U_c D_o) + a_4' U_c \ddot{x}_1/D_o \tag{16}$$

$$m_r\ddot{x}_2 + (EIx_2'')'' - wx_2' - T_e x_2'' = q_2 \tag{17}$$

These three equations represent the transverse vortex-induced vibration, the fluid oscillator equation about the hidden flow variable w, and the riser vibration in the current direction respectively. The forcing term q_2 in the right side of Eq. (17) can be derived by taking only the drag term due to current and given as,

$$q_2 = -\frac{1}{2}\rho_w C_D D_o|\dot{x}_2 - U_c|(\dot{x}_2 - U_c) \tag{18}$$

The weak form of the governing equations are produced in multiplying the residual by a sufficiently smooth test function, integrating the product by parts, and equating the result to zero. And then, implementation of Hermite polynomials as basis functions (N_i, N_j) yields the dynamic equilibrium equations constructed in terms of the unknown deformations at node i, j of a elememt. For the construction of the matrix equation, the usual processes are applied and the resulting matrix equations are:

$$[\hat{M}_{ij}]\{\ddot{x}_{1j}(t)\}+[\hat{C}_{fij}]\{\dot{x}_{1j}(t)\}+[\hat{T}_{ij}+\hat{K}_{ij}]\{x_{1j}(t)\}=[\hat{F}_{ij}]\{\dot{w}_j(t)\} \tag{19}$$

$$[\overline{M}_{ij}]\{\ddot{w}_j(t)\}+[\overline{C}_{fij}]\{\dot{w}_j(t)\}+[\overline{K}_{ij}]\{w_j(t)\}=[\overline{F}_{ij}]\{\dot{x}_{1j}(t)\}-\{\bar{f}_j(t)\} \tag{20}$$

$$[\tilde{M}_{ij}]\{\ddot{x}_{2j}(t)\}+[\tilde{T}_{ij}+\tilde{K}_{ij}]\{x_{2j}(t)\}=\{\tilde{f}_j(t)\} \tag{21}$$

wher

x_{1j}, x_{2j}=deformation at j-th node, w_j=hidden flow variable at j-th node

$$\hat{M}_{ij}=\tilde{M}_{ij}=\int_{\Omega_e} m_t N_i N_j \, dx_3$$

$$\hat{C}_{fij}=\int_{\Omega_e}\left[a_4\rho_w D_o U_c p(x_3)-\frac{1}{2}\rho_w D_o C_D(1-p(x_3))\right] N_i N_j dx_3$$

$$\hat{T}_{ij}=\tilde{T}_{ij}=\int_{\Omega_e} T_e N_i' N_j' \, dx_3$$

$$\hat{K}_{ij}=\tilde{K}_{ij}=\int_{\Omega_e} EI_e N_i' N_j' \, dx_3,$$

$$\hat{F}_{ij}=\int_{\Omega_e} a_4\rho_w D_o U_c p(x_3) N_i N_j dx_3$$

$$\tilde{f}_j=\int_{\Omega_e}-\frac{1}{2}\rho_w C_D D_o|\dot{x}_{2h}-U_c|(\dot{x}_{2h}-U_c)N_j dx_3$$

$$\overline{M}_{ij}=\int_{\Omega_e} N_i N_j \, dx_3, \quad \overline{C}_{ij}=\int_{\Omega_e}(a_4'-a_1')(U_c/D_o)N_i N_j \, dx_3$$

$$\overline{K}_{ij}=\int_{\Omega_e}\omega_s^2 N_i N_j \, dx_3, \quad \overline{F}_{ij}=\int_{\Omega_e} a_4'(U_c/D_o)N_i N_j \, dx_3$$

$$\bar{f}_j=\int_{\Omega_e}[a_2'/(U_c/D_o)]\dot{w}_h^3 N_j \, dx_3$$

$$x_{2h}(x_3,t)=\sum_{i=1}^{4}x_{2i}(t)N_i(x_3), \quad w_h(x_3,t)=\sum_{i=1}^{4}x_i(t)N_i(x_3)$$

where Ni, N_j=a set of unique local cubic Hermite polynomial shape functions as an interpolation function Having calculated the matrices and equations describing our approximation over each finite element, the next step is to assemble the equations describing the approximation on the entire mesh by adding up the contributions to these equations furnished by each element. This assembling procedure is omitted because it is well known and referenced in several books and papers.

Eqs. (19) and (20) represent the self excited oscillation. These harmonic responses due to vortex shedding are calculated by forcing the riser with an imposed initial force. The vortex driven oscillation usually reaches to a steady state in about 10-15 cycles for top hinged case. Also, the slight modulation in amplitude, which results from a third power term in the hidden flow variable w on the right side of fluid oscillator equation, has been detected.

3.2 Numerical Solution

In the above section, the matrix equation of equilibrium for current-vortex model (Eqs. (19)-(22)) is derived. As discussed in the section 3.1, Eqs. (19) and (20) represent the self-excited oscillator equations, while Eq. (21) presents inline vibration due to current. From the inspection of coefficient matrices and vectors in Eq. (22), it may be recognized that the drag force in inline vibration equation and the fluid force in fluid oscillator equation have some nonlinear terms. In other words, there is no highly nonlinear terms in the equation to be solved. Therefore simple iteration can be applied for the convergence of solution. Thus, the application of Newmark method combined with the simple iteration is enough to solve the system of equations. Time step of Δt=0.05 sec is used for numerical integration.. Using the method mentioned above, a finite element code, which is called CODEV, is developed.

For the verification of CODEV, the numerical results are inspected through the physical interpretation with the

Table 1. Design properties and data for a drilling riser system for use in the northern North Sea.

Outside diameter	D=0.61 m with 16 mm wall thickness
Constant of elasticity	$E=2.1\times10^6$ kg/cm^2
Sectional moment of inertia	I=131018 cm^4
Riser length	L=152 m
Riser mass	m=10 kg/cm (include mass of drilling mud and sea water)
Bottom tension	TTB=1200 kN
Effective weight per unit length	w=3.86 kN/m
Mean Tension	$\overline{T}_e$=1453 kN
Density of drilling mud	ρ_m=1.36 t/m^3
Density of sea water	ρ_w=1.036 t/m^3

generally expected trends. The transverse vibration due to vortex shedding is estimated and the effect on riser vibration is investigated. The example data, which is a typical drilling riser system used in the northern North sea and repeated from Chen (1990), is given in Table 1. In addition, the mass coefficient C_M=2.0, added mass coefficient C_A=C_M-1, and the drag coefficient C_D=0.8 are used.

The first concern is that the deflected shape of the riser should be symmetrical when the riser system with the exclusion of internal tension is subject to a uniform current. The numerical results show that nodal displacements and rotations are exactly symmetrical or skew-symmetrical with respect to middle node, that is, the deflected shapes are exactly symmetrical when there are no internal tension.

Next, the response of riser with the inclusion of internal tension is examined. Fig. 1 presents maximum transverse displacement versus current velocity. Since the transverse vibrations due to current are controlled by the oscillating vortex generated around the riser, they must have resonant bands near the critical current velocity in which the vortex shedding frequency accords with the system frequency. For instance, the first system frequency is already known from Table 2 estimated by Hong (1996) using semi analytical method. The vortex shedding frequency can be also calculated from Eq. (12). Taking Strouhal number S as 0.20, the critical velocity can be obtained from the following equation:

$$\omega_s = 2\pi S u_c / D_o = \omega_1$$
$$\omega_s = 2\pi(0.20)u_c/[(2.0)(0.3048)] = 0.81701$$
$$u_c = 0.40 \text{ m/s} \tag{23}$$

This critical velocity is consistent with the velocity corresponding to the first peak as shown in Fig. 1. In the same manner, the critical velocities for second and third peak in Fig. 1 are compared with the values calculated using Eq. (23) and found to be consistent with each other. While the loading due to vortex shedding in the transverse direction oscillates well to induce harmonic riser motion, the constant load due to current in inline direction causes a steady static response.. This inline displacement simply becomes larger as the current velocity increases as shown in Fig. 2. Fig. 3 plots the time history of the transverse displacement at the middle point of the riser. In this figure, three different current velocities, over, just at and below first critical velocity are chosen to demonstrate the magnification in amplitude of oscillating displacement resulting from the current velocity that generates resonant vorticies. The middle figure corresponds to the case of resonance. As discussed already, all three figures show harmonically oscillating ones due to the vortex shedding. These harmonic responses due to vortex shedding are calculated by forcing the riser with imposed initial force. The vortex driven oscillation usually reaches to steady state in about 10-15 cycles for top-hinged case. Also, the slight modulation in amplitude can be detected from the figure. This modulation results from a third power term in the hidden flow variable w on the right side of the fluid oscillator Eq. (12).

This can be proved from the fact that this modulation would disappear if the third power term is excluded from the fluid oscillator equation. Fig. 4 shows the time history of inline displacement at the middle of the riser. As expected, the displacement in current direction approa-

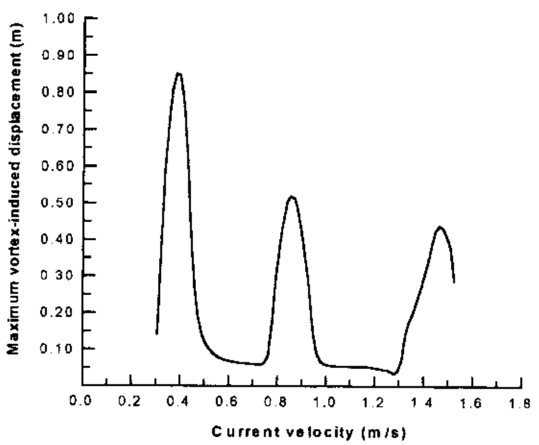

Fig. 1. Maximum vortex-induced displacement in transverse direction to uniform inline current.

Table 2. System frequencies of the 1st to 10th modes for a drilling riser system.

Mode i	1	2	3	4	5	6	7	8	9	10
System Frequency ω_i(*rad/s*)	0.81701	1.80326	3.08550	4.73421	6.78562	9.25782	12.1601	15.4975	19.2727	23.4889

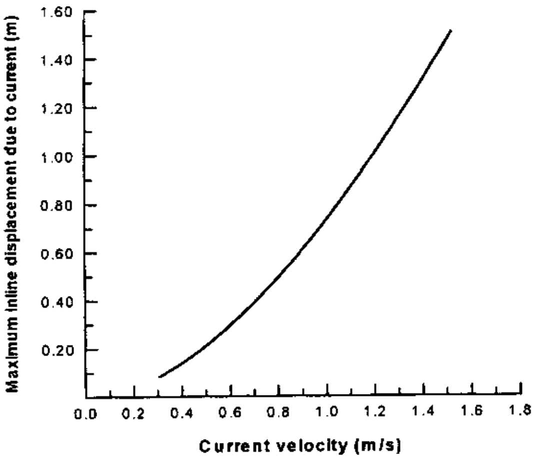

Fig. 2. Maximum inline displacement in current direction to uniform inline current.

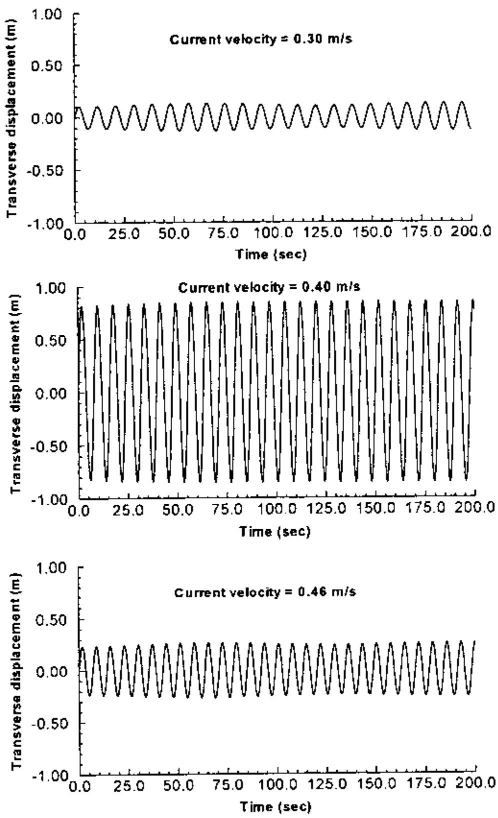

Fig. 3. Transverse displacement history due to inline current at the middle node of riser.

ches to a steady static state after a few cyclic fluctuations. Fig. 5 shows the time-dependent trajectories of displace-

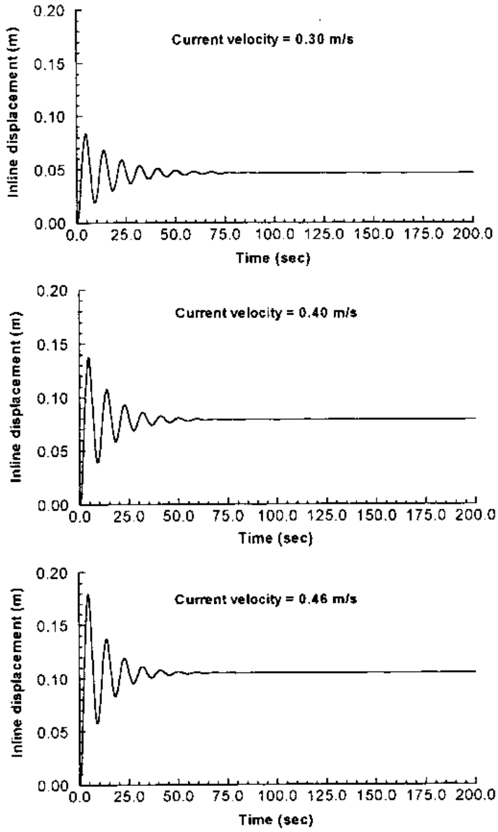

Fig. 4. Inline displacement history due to inline current at the middle node of riser.

ment vector at the middle of the riser for three different current velocity, the current below the first critical velocity, same as the first critical velocity, and above the first critical velocity. As discussed in Figs. 1 and 4, similar differences in the displacement trend between transverse direction and inline direction are also found in Fig. 5. It can also be recognized that the trajectory shape changes from almost closed oval to irregular zigzag. This phenomenon results from the shortening of the response period due to the increase of vortex shedding frequency as shown in Fig. 3. Therefore, the riser system seems to behave as expected from physical point of view.

4. CONCLUSIONS AND FURTHER STUDY

A computational model for vortex induced vibrations

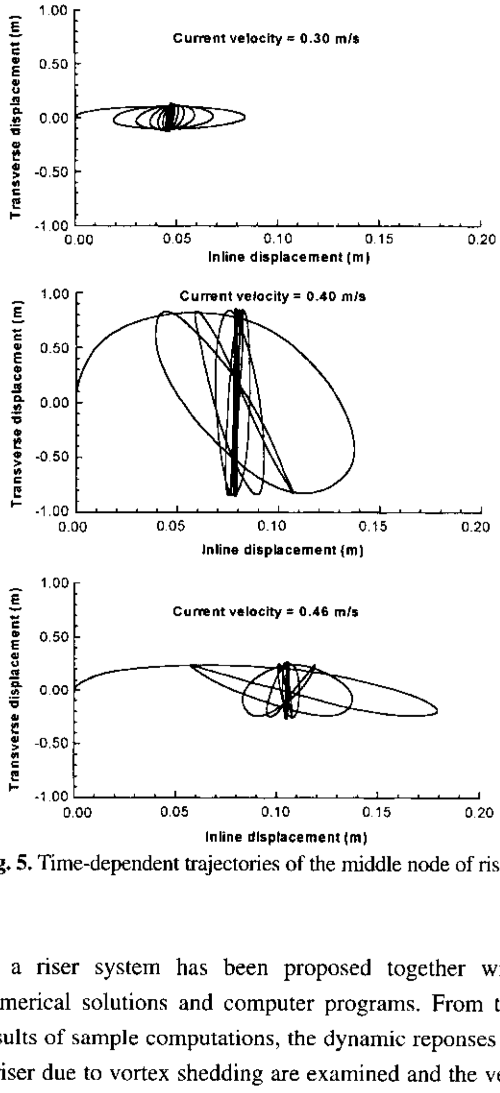

Fig. 5. Time-dependent trajectories of the middle node of riser.

of a riser system has been proposed together with numerical solutions and computer programs. From the results of sample computations, the dynamic reponses of a riser due to vortex shedding are examined and the verification is given to the suggested model in this study.

In this study, Blevin's model, which is one of the self-excited oscillator models, has been implemented as a vortex shedding model. In this model, the nonlinear coupling between inline and transverse vibration, and the extension of the applicable range of Reynolds number may be required for the development of the refined model to investigate the effect of internal flow more precisely.

ACKNOWLEDGEMENTS

This paper was supported by the Dong-A University Research Fund, in 1999. The support of Dong-A University are greatly appreciated.

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Received April 24, 2000
Accepted July 7, 2000