Robust Backstepping Control for Nonvanishing Parametrization[1)]

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Abstract - In this paper, a design method of a controller is presented for a class of nonlinear systems which have time-varying parametric uncertainty. Some features of this controller are that it can tackle 1) nonlinear parametrization (i.e. uncertain parameters enter the system in the nonlinear form) and 2) nonvanishing perturbation (i.e. uncertainty need not vanish at the origin). The class of systems considered in this paper has the triangular structure for which the well-known backstepping design can be applied. The uncertain parameter is assumed to be contained in the bounded set whose size can be arbitrarily large. Also, the uncertain term is assumed to vanish when the parameter has the nominal value. Using the proposed control, all the states of the uncertain system are globally uniformly bounded and converge to a compact set whose size is designable. In particular, the first state of the system can be made arbitrarily small, which can be seen by the presented simulation result.
keyword - Robust control, Nonvanishing perturbation, Backstepping

1. Introduction

Consider a single input, uncertain nonlinear system

$$\dot{x} = f(x) + q(x, \theta) + g(x)u \tag{1}$$

in which $x \in R^n$ is the state, $u \in R$ is the control input, f and g are assumed to be smooth vector fields with the property $f(0) = 0$, and q is smooth in x and θ where $\theta \in R^p$ is unknown parameter vector.

Robust control of the class of system (1) has been an important issue for the past several decades. Early results, e.g. [1], were based on the matching condition (i.e., there exists a smooth function $\lambda(x, \theta)$ such that $q(x, \theta) = \lambda(x, \theta)g(x)$). More recent results were obtained under the parametric-strict-feedback or parametric-pure-feedback condition and by using the backstepping technique (see [2]). They used overparametrized adaptive control scheme, which is improved to a non-overparameterized scheme by [3]. These results are comprehensively described in the book of [4]. Most of adaptive results were based on the conditions that 1) uncertain parameter θ is constant, and 2) uncertain parameter enters linearly, i.e, $q(x, \theta) = q'(x)\theta$. On the other hands, [5] developed a nonadaptive robust control scheme, which allows time-varying uncertainty $\theta(t)$ and nonlinear parametrization $q(x, \theta)$. The regulation result of [5] is well extended under the existence of zero dynamics in [6], [7].

However, most of the aforementioned results are based on somewhat restrictive 'vanishing' assumption, that is, $q(0, \theta) = 0$ for all θ. It is because that, with the vanishing assumption, the equilibrium $x = 0$ is preserved though the parametric perturbation exists, and hence the analysis or synthesis of stability of the origin is possible. Indeed, without the vanishing assumption, the origin may not be the equilibrium point except the matching case because the equality $q(0, \theta) + g(0)u = 0$ is not always satisfied for any u. Therefore, without the vanishing assumption, the equilibrium may be changed by the parameter perturbation, thus the regulation to the origin is generally impossible. Instead, the best thing to be pursued is that uniform asymptotic convergence to a small set containing the origin.

Vanishing assumption is, in general, disappointing one since there are many cases that it doesn't meet for real physical plants. Nevertheless, there have been relatively few results for the nonvanishing case. Earlier results can be found in [8], but they restricted the growth of nonlinearity of the vector field $q(x, \theta)$. The adaptive controllers in the book of [4] can be applied to the nonvanishing case but unfortunately they require constant uncertainty and linear parametrization. In [9] they concerned the case that the equilibrium was fixed but affected by the unknown parameter θ. They treated the unknown equilibrium as an uncertain parameter and introduced an adaptive controller which estimates the equilibrium. However, their results lead to a local stabilizing controller which guarantees the convergence to the unknown equilibrium point from the small neighborhood of the origin.

In this paper, a non-adaptive robust controller is designed for nonlinearly parametrized system. It does not require that θ is constant nor $q(x, \theta)$ vanishes at the origin $x = 0$.

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Moreover, the result has a global property and does not restrict the nonlinear growth.

The paper is organized as follows. In Section 2, the system (1) is considered assuming the matching condition. In Section 3, in order to remove the matching condition, the following strict-feedback system is treated.

$$\begin{aligned} \dot{z}_1 &= z_2 + \phi_1(z_1, \theta) \\ \dot{z}_2 &= z_3 + \phi_2(z_1, z_2, \theta) \\ &\vdots \\ \dot{z}_n &= v + \phi_n(z_1, \cdots, z_n, \theta) \end{aligned} \tag{2}$$

It is well-known that, under a geometric condition [10, p.96], the system (1) is globally feedback transformed to (2) by a global state transformation and a regular feedback. It should be noted that the condition $\phi_i(0, \theta) = 0$ is not required for the transformation. Finally, example and conclusions are presented in Section 4 and 5, respectively.

Notations: A function is said to be C^k when it is continuously differentiable k times, and a *smooth* function implies C^∞ function. $V: R^n \to R$ is said to be *positive definite* if $V(0) = 0$ and $V(x) > 0$ for all $x \neq 0$, and *proper* if, for any $a \in R$, the set $V^{-1}([0, a]) = \{x \in R^n : 0 \le V(x) \le a\}$ is compact. Lie derivative of a vector field f with respect to V is defined to be $L_f V(x) = dV(x) \cdot f(x)$ where $dV(x)$ is the gradient of the function V.

2. Uniform Boundedness and Convergence

In this section, we begin with two mild assumptions for system (1). These assumptions are practically reasonable since the size of B_ρ is not restricted in Assumption 1. For Assumption 2, the following remark explains the reason.

Assumption 1. The unknown parameter $\theta(t)$ is globally uniformly bounded with a known bound. More specifically,

$$\theta(t) \in B_\rho$$

where $B_\rho = \{\theta \in R^p : |\theta| \le \rho\}$ is a closed ball and ρ is a known value.

Assumption 2. For the system (1), $q(x, 0) = 0$ for all x.

Remark 1. It should be noted that Assumption 2 differs from the usual assumption that $q(0, \theta) = 0$ for all θ. Indeed, for the system $\dot{x} = f(x, p)$ with a parameter vector p, θ can be defined as the deviation of p from the nominal value p^*, i.e., $\theta = p - p^*$. If we define $q(x, \theta) = f(x, \theta + p^*) - f(x, p^*)$, the system equation becomes $\dot{x} = f(x, p^*) + q(x, \theta)$. This satisfies Assumption 2.

When the matching condition is satisfied we have obtained the following result under the above assumptions. For notational simplicity, some arguments of a function will be frequently omitted in the proofs.

Theorem 1. Assume that the nominal system $\dot{x} = f(x) + g(x)u$ is globally asymptotically stabilizable. In other words, assume that there are a smooth feedback control $u = \alpha(x)$ and two smooth, positive definite and proper functions $V: R^n \to R$ and $W: R^n \to R$ such that

$$\frac{\partial V}{\partial x}(x)[f(x) + g(x)\alpha(x)] \le -W(x) \le 0,$$

for all $x \in R^n$. Also, assume that $q(x, \theta)$ satisfies the matching condition. Then, under Assumptions 1 and 2, there exist a smooth function $s(x)$ such that, with a control

$$u = \alpha(x) - \varkappa L_g V(x) s(x) \tag{3}$$

for any $\varkappa > 0$, the derivative of $V(x)$ along the trajectory of (1) satisfies that

$$\dot{V}(x) \le -W(x) + \frac{|\theta(t)|^2}{4\varkappa}. \tag{4}$$

Proof. First of all, by the matching condition, there exists the smooth function $\lambda(x, \theta)$ such that

$$q(x, \theta) = g(x)\lambda(x, \theta).$$

In addition, by Assumption 2, there exists the smooth $(1 \times p)$ vector function $\bar{\lambda}(x, \theta)$ such that [11, p.39]

$$\lambda(x, \theta) = \bar{\lambda}(x, \theta)\theta.$$

Then, it is not difficult to find a smooth function $s(x)$ such that

$$s(x) \ge |\bar{\lambda}(x, \theta)|^2, \quad \forall \theta \in B_\rho \tag{5}$$

since B_ρ is a known compact set.

Now, the derivative of the Lyapunov function candidate $V(x)$ along the trajectory of (1) with $u = \alpha(x) - \varkappa L_g V s(x)$ is given as

$$\begin{aligned} \dot{V} &= L_f V + L_q V + L_g V\alpha - \varkappa(L_g V)^2 s(x) \\ &= L_f V + L_g V\alpha + L_g V\bar{\lambda}(x, \theta)\theta - \varkappa(L_g V)^2 s(x), \end{aligned}$$

by Young's inequality $xy \le \varkappa x^2 + \frac{y^2}{4\varkappa}$,

$$\le -W(x) + \frac{|\theta|^2}{4\varkappa} + \varkappa(L_g V)^2|\bar{\lambda}(x, \theta)|^2 - \varkappa(L_g V)^2 s(x)$$

$$\leq -W(x) + \frac{|\theta(t)|^2}{4\varkappa}$$

This completes the proof.

Remark 2. Note that since $V(x)$ is smooth and positive definite (i.e., $V(x)>0$ for $x \neq 0$ and $V(0)=0$), the function $V(x)$ has its minimum value at the origin. Therefore, $L_gV(0)=0$. Then, when α vanishes at the origin, u also vanishes i.e. $u(0)=0$. This property is further utilized in the next section with the smooth property of u.

In Theorem 1, since $V(x)$ and $W(x)$ are positive definite and proper there exist class-K_∞ functions γ_1 and γ_2 such that [8, p.138]

$$V(x) \leq \gamma_1(|x|), \quad \gamma_2(|x|) \leq W(x). \tag{6}$$

This fact, together with (4), implies that the state $x(t)$ is globally uniformly bounded and asymptotically converges to a $\varkappa$-parametrized compact set

$$S(\varkappa) = \left\{x : V(x) \leq \gamma_1 \circ \gamma_2^{-1}\left(\frac{\rho^2}{4\varkappa}\right)\right\},$$

which is positively invariant set. Indeed, from (4) and (6) it follows that

$$|x(t)| > \gamma_2^{-1}\left(\frac{\rho^2}{4\varkappa}\right) \Rightarrow \dot{V} < 0.$$

Hence, for $x \notin S(\varkappa)$, $\dot{V}(x) < 0$. For more rigorous arguments, refer to [8] or [12, Proof of Lemma 2.2].

Remark 3. The value of $\varkappa$ is a designer's choice. Although increasing $\varkappa$ results in a high gain control, it reduces the size of set $S(\varkappa)$. In this way, the converging set can be made arbitrarily small.

Remark 4. From (6), the closed-loop system (1)-(3) satisfies

$$\dot{V}(x) \leq -\gamma_2(|x|) + \frac{1}{4\varkappa}|\theta|^2.$$

Thus, the function $V(x)$ may be viewed as an ISS-Lyapunov function[2)] when $\theta(t)$ is regarded as an input. However, this fact does not imply that the system (1) is made input to state stable (ISS) with respect to θ by the control (3). This is due to the fact that $s(x)$ is constructed under Assumption 1, that is, using the knowledge of ρ. (The definition of ISS-property does not restrict the size of input.) Nevertheless, when $|\theta(t)| \leq \rho$, the ISS-property holds. This can be interpreted as *weak ISS-property*.

The result of Theorem 1 is somewhat disappointing since it requires matching condition. However, it can be conjectured that the matching condition of Theorem 1 can be removed by the backstepping technique. Next section shows that the conjecture is true. Before that, a corollary is provided which helps the exposition of next section.

Corollary 1. Consider a scalar system

$$\dot{z} = q(z, \theta(t)) + u \tag{7}$$

where $\theta(t) \in R^p$ is unknown. Suppose the above system satisfies Assumptions 1 and 2. Then, for any constants $c>0$ and $\varkappa>0$, there exists a smooth function $s(z)$ such that a function $V(z) = \frac{1}{2}z^2$ satisfies the following inequality

$$\dot{V}(z) \leq -cz^2 + \frac{|\theta(t)|^2}{4\varkappa}$$

with a control

$$u = -cz - \varkappa z s(z). \tag{8}$$

Proof. With $V(z)=\frac{1}{2}z^2$, $W(z) = cz^2$ and $\alpha(z) = -cz$, all the assumptions of Theorem 1 are satisfied. This is because that a scalar system always satisfies the matching condition.

3. Recursive Strategy using Backstepping

Suppose that the system (1) is transformed to (2) by a global state transformation $z = \Phi(x)$ ($0 = \Phi(0)$) and a regular feedback $u = a(x) + b(x)v$, $b(x) \neq 0$. (See [10] for further conditions and other discussions.) Assumption 2 can be interpreted appropriately in z-coordinates as follows.

Assumption 3. For the system (2),

$$\phi_i(z_1, \cdots, z_i, 0) = 0, \quad 1 \leq i \leq n, \ \forall z.$$

Since $\phi_i(z_1, \cdots, z_i, \theta)$ is i-th element of the vector $\left[\frac{\partial \Phi}{\partial x} q(x, \theta)\right]_{x=\Phi^{-1}(z)}$, Assumption 3 derives from Assumption 2. Notice that it is not required that $\phi_i(0, \cdots, 0, \theta) = 0$.

Throughout the following lemma, define $z_0 := 0$, $v_0 := 0$, Z_i as $(z_1, \cdots, z_i)$, and D_xV as $\frac{\partial V}{\partial x}$ for nota-

2) For the definition of ISS-Lyapunov function, refer to [13, p.352].

tional convenience.

Lemma 1. Consider the i-th order system

$$\begin{aligned} \dot{z}_j &= z_{j+1} + \phi_j(Z_j, \theta), \quad 1 \le j \le i-1 \\ \dot{z}_i &= v_i + \phi_i(Z_i, \theta) \end{aligned}$$

for which Assumptions 1 and 3 hold. Suppose that there exist a positive constant κ, c_j and a smooth function $v_j(Z_j)$, $1 \le j \le i$, such that

1) $v_j(0) = 0, \ 1 \le j \le i$.

2) the derivative of $V_i(Z_i) = \frac{1}{2}\sum_{j=1}^{i}(z_j - v_{j-1}(Z_{j-1}))^2$ satisfies

$$\dot{V}_i \le -\sum_{j=1}^{i} c_j(z_j - v_{j-1}(Z_{j-1}))^2 + \frac{i}{4\kappa}|\theta|^2. \tag{9}$$

Under these conditions, when the $(i+1)$-th order system

$$\begin{aligned} \dot{z}_j &= z_{j+1} + \phi_j(Z_j, \theta), \quad 1 \le j \le i \\ \dot{z}_{i+1} &= v_{i+1} + \phi_{i+1}(Z_{i+1}, \theta) \end{aligned}$$

satisfies Assumptions 1 and 3, there exists a smooth function $s_{i+1}(Z_{i+1})$ such that $V_{i+1} = \frac{1}{2}\sum_{j=1}^{i+1}(z_j - v_{j-1})^2$ satisfies the following inequality

$$\dot{V}_{i+1} \le -\sum_{j=1}^{i+1} c_j(z_j - v_{j-1})^2 + \frac{i+1}{4\kappa}|\theta|^2 \tag{10}$$

with the control ($v_{i+1}(0) = 0$)

$$\begin{aligned} v_{i+1} = &-c_{i+1}(z_{i+1} - v_i) - (z_i - v_{i-1}) \\ &+ \sum_{j=1}^{i}(D_{z_j}v_i)z_{j+1} - \kappa(z_{i+1} - v_i)s_{i+1}(Z_{i+1}) \end{aligned} \tag{11}$$

where $c_{i+1} > 0$ is a designer's choice.

Proof. By taking $\tilde{z}_{i+1} = z_{i+1} - v_i$,

$$\begin{aligned} \dot{\tilde{z}}_{i+1} &= v_{i+1} + \phi_{i+1}(Z_{i+1}, \theta) - \sum_{j=1}^{i} D_{z_j}v_i(z_{j+1} + \phi_j(Z_j, \theta)) \\ &= v_{i+1} - \sum_{j=1}^{i} D_{z_j}v_i z_{j+1} + w_{i+1}(Z_{i+1}, \theta) \end{aligned} \tag{12}$$

where

$$w_{i+1}(Z_{i+1}, \theta) = \phi_{i+1}(Z_{i+1}, \theta) - \sum_{j=1}^{i} D_{z_j}v_i\,\phi_j(Z_j, \theta). \tag{13}$$

It is clear that $w_{i+1}(Z_{i+1}, 0) = 0$ from Assumption 3, which guarantees the existence of ($1 \times p$) vector function $\overline{w}_{i+1}$ such that

$$w_{i+1}(Z_{i+1}, \theta) = \overline{w}_{i+1}(Z_{i+1}, \theta)\,\theta. \tag{14}$$

Hence we can find a smooth function $s_{i+1}(Z_{i+1})$ which satisfies

$$s_{i+1}(Z_{i+1}) \ge |\overline{w}_{i+1}(Z_{i+1}, \theta)|^2 \tag{15}$$

for all $\theta \in B_\rho$.

It remains to show (10) with $V_{i+1} = \frac{1}{2}\sum_{j=1}^{i+1}\tilde{z}_j^2 = V_i + \frac{1}{2}\tilde{z}_{i+1}^2$. With the control (11),

$$\begin{aligned} \dot{V}_{i+1} \le &-\sum_{j=1}^{i} c_j\tilde{z}_j^2 + \frac{i}{4\kappa}|\theta|^2 + \tilde{z}_i\tilde{z}_{i+1} \\ &+ \tilde{z}_{i+1}\Big(v_{i-1} - \sum_{j=1}^{i} D_{z_j}v_i z_{j+1} + \overline{w}_{i+1}(Z_{i+1}, \theta)\theta\Big) \\ = &-\sum_{j=1}^{i+1} c_j\tilde{z}_j^2 + \frac{i}{4\kappa}|\theta|^2 + \tilde{z}_{i+1}\overline{w}_{i+1}\theta - \kappa\tilde{z}_{i+1}^2 s_{i+1} \\ \le &-\sum_{j=1}^{i+1} c_j\tilde{z}_j^2 + \frac{i}{4\kappa}|\theta|^2 + \frac{|\theta|^2}{4\kappa} + \kappa\tilde{z}_{i+1}^2|\overline{w}_{i-1}|^2 - \kappa\tilde{z}_{i+1}^2 s_{i+1} \\ \le &-\sum_{j=1}^{i+1} c_j\tilde{z}_j^2 + \frac{i+1}{4\kappa}|\theta|^2 \end{aligned}$$

by (9), (11), (12), (14), (15) and by the inequality $xy \le \kappa x^2 + \frac{y^2}{4\kappa}$. This completes the proof.

Combining Lemma 1 with the Corollary 1 leads to a design step of the control v_n for a class of the system (2) under Assumptions 1 and 3. That is, construct v_1 using Lemma 1 for the first order equation of (2) with z_2 as a control. Then, use (13), (14), (15) and (11) in order to make subsequent virtual controls v_i iteratively.

From now on, let's investigate the performance of the control v_n. Suppose that $\tilde{z}_i = z_i - v_{i-1}(Z_{i-1})$ for $1 \le i \le n$ is a global state transformation for (2). Then, $V_n(z) = \frac{1}{2}|\tilde{z}|^2$. From (10), with $c = \min_{1 \le i \le n} c_i$,

$$\dot{V}_n \le -c\sum_{j=1}^{n}\tilde{z}_j^2 + \frac{n\rho^2}{4\kappa} = -2cV_n + \frac{n\rho^2}{4\kappa}.$$

Thus, by the comparison lemma [8, p.85],

$$\begin{aligned} V_n(z(t)) &\le e^{-2ct}V_n(z(0)) + \int_0^t e^{-2c(t-s)}\frac{n\rho^2}{4\kappa}\,ds \\ &= e^{-2ct}V_n(z(0)) + \frac{1}{2c}(1 - e^{-2ct})\frac{n\rho^2}{4\kappa} \end{aligned}$$

leads to the following inequality,

$$|\tilde{z}(t)|^2 \le e^{-2ct}\Big(|\tilde{z}(0)|^2 - \frac{n\rho^2}{4c\kappa}\Big) + \frac{n\rho^2}{4c\kappa}.$$

By above inequality, the state $\tilde{z}(t)$ is globally uniformly bounded, and converges exponentially to the set

$$\boldsymbol{R}_{\tilde{z}} = \Big\{\tilde{z} : |\tilde{z}| \le \sqrt{\frac{n\rho^2}{4c\kappa}}\Big\}$$

which contains the origin $\tilde{z} = 0$. Hence its global transformation is also globally uniformly bounded, and converges asymptotically to the set

$$R_z = \left\{ z : \sum_{i=1}^{n} | z_i - v_{i-1}(Z_{i-1})|^2 \le \frac{n\rho^2}{4cx} \right\}. \qquad (16)$$

Notice that the origin $z = 0$ is also contained in R_z. These discussion are summarized in the following theorem.

Theorem 2. For a class of the system (2), if Assumptions 1 and 3 hold, there exists a control which guarantees global uniform boundedness of all the states and convergence to the set R_z. One of such controller is constructed by Corollary 1 and Lemma 1.

Unlike the case of Remark 3, it is not possible to make the size of the compact set R_z arbitrarily small in general. Indeed, although it seems that increasing c_i's or x reduces the size of R_z by (16), it is not so simple because v_i's are also dependent on c_i's or x. However, when the output of the system (2) is z_1 ($y = z_1$), the output can be made to converge to an arbitrary small value by increasing c_i's or x. Indeed, by (16),

$$|y|^2 = |z_1|^2 \le \sum_{i=1}^{n} | z_i - v_{i-1}(Z_{i-1})|^2 \le \frac{n\rho^2}{4cx}.$$

The simulation result of next section shows these issues.

4. Example and Simulation

Consider the following uncertain system

$$\begin{aligned} \dot{z}_1 &= z_2 + \theta e^{\theta z_1} \\ \dot{z}_2 &= u \end{aligned} \qquad (17)$$

where θ is unknown parameter satisfying that $-\frac{1}{2} \le \theta \le \frac{1}{2}$. Assumptions 1 and 3 are satisfied. According to (8) and (5), a virtual control $v_1(z_1)$ is constructed as

$$v_1(z_1) = -cz_1 - xz_1 s_1(z_1)$$

where c and x are the design parameters which are positive constants, and $s_1(z_1)$ is chosen as

$$s_1(z_1) = e^{z_1} + e^{-z_1}$$

since $|e^{\theta z_1}|^2 \le e^{z_1} + e^{-z_1}$ for any $\theta \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

Now, define $\tilde{z}_2 = z_2 - v_1(z_1) = z_2 + cz_1 + xz_1 (e^{z_1} + e^{-z_1})$. Then,

Table 1 Results of ten sample runs. ($\theta = 0.2$)

$c*x$	$z_1(0)$	$z_2(0)$	$z_1(\infty)$	$z_2(\infty)$	$\frac{\vert\theta\vert}{\sqrt{2cx}}$
0.5	-0.0483	0.5420	0.0555	-0.2022	0.202
5.8	-0.0172	0.4398	0.0274	-0.2011	0.059
6.7	0.0970	-0.1739	0.0198	-0.2008	0.055
7.4	-0.0934	-0.0240	0.0269	-0.2011	0.052
8.1	-0.0784	-0.9943	0.0219	-0.2009	0.049
8.6	-0.0745	-0.1739	0.0235	-0.2009	0.048
10.0	0.0050	0.3400	0.0220	-0.2009	0.045
20.5	-0.0898	0.8439	0.0138	-0.2006	0.031
42.0	-0.0835	0.3142	0.0107	-0.2004	0.022
56.1	0.0361	-0.2030	0.0091	-0.2004	0.019

$$\dot{\tilde{z}}_2 = u + \lambda(z_1)z_2 + \lambda(z_1)e^{\theta z_1}\theta$$

where $\lambda(z_1) = c + xs_1(z_1) + xz_1(e^{z_1} - e^{-z_1})$. Hence the suggested control is, by (11),

$$u = -c\tilde{z}_2 - z_1 - \lambda(z_1)z_2 - x\tilde{z}_2 s_2(z_1, z_2)$$

where $s_2(z_1, z_2) = \lambda^2(z_1)s_1(z_1)$ from the fact that

$$|\lambda(z_1)e^{\theta z_1}|^2 \le \lambda^2(z_1)s_1(z_1) = s_2(z_1, z_2)$$

for any $\theta \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. (See (5).)

Simulations are performed by MATLAB. The results of several sample runs are summarized at Table 1. It can be noticed that increasing $c*x$ (first column) does not decrease the steady state value of z_2 sufficiently. However, it does decrease $z_1(\infty)$, and which fits *a priori* upper bound (fourth column).

5. Conclusion

In this paper, a robust control scheme is presented which guarantees global uniform boundedness of all the states, and convergence to a designable compact set, under the nonvanishing nonlinear parameterization.

When the matching conditions is satisfied, Theorem 1 shows the motivations of the approach and it is extended to the non-matching case. Especially this design is suitable to a triangular system to which standard backstepping technique can be applied.

References

[1] D.G. Taylor, P.V. Kokotovic, R. Marino and I. Kanellakopoulos, "Adaptive Regulation of Nonlinear

Systems with Unmodeled Dynamics", *IEEE Trans. on Automat. Contr.*, vol. 34, no.4, 405~412, 1989.

[2] I. Kanellakopoulos, P.V. Kokotovic and A.S. Morse, "Systematic Design of Adaptive Controllers for Feedback Linearizable Systems", *IEEE Trans. Automat. Contr.*, vol. 36, 1241~1253, 1991.

[3] M. Krstic, I. Kanellakopoulos and P.V. Kokotovic, "Adaptive Nonlinear Control without Overparameterization", *Systems & Control Lett.*, 19, 177~185, 1992.

[4] M. Krstic, I. Kanellakopoulos and P.V. Kokotovic, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, Inc., 1995.

[5] R. Marino and P. Tomei, "Robust Stabilization of Feedback Linearizable Time-varying Uncertain Nonlinear Systems", *Automatica*, 29, 181~189, 1993.

[6] W. Lin, "Global Robust Stabilization of Minimum-phase Nonlinear Systems with Uncertainty", *Automatica*, vol. 33, 453~462, 1997.

[7] J. Jin, S. Joo, N.H. Jo and J.H. Seo, "Adaptive Regulator of Partially Feedback Linearizable Uncertain Nonlinear Systems", *American Control Conference*, 1997.

[8] H.K. Khalil, *Nonlinear Systems*, 2nd ed.. Prentice-Hall., 1996.

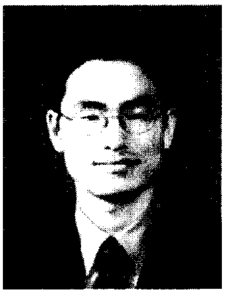
[9] A.S. Bazanella, P.V. Kokotovic and A.S. e Silva, "On the Control of Dynamic Systems with Unknown Operating Point", *European Control Conference*, 1997.

[10] R. Marino and P. Tomei, *Nonlinear Control Design: Geometric, Adaptive and Robust*, Prentice-Hall., 1995.

[11] H. Nijmeijer and A.J. van der Schaft, *Nonlinear Dynamical Control Systems*, Springer-Verlag. 1990.

[12] I. Kanellakopoulos, "Robust Nonlinear Control Design with Input and Measurement Disturbances", *American Control Conference*, 1997.

[13] E.D. Sontag and Y. Wang, "On Characterizations of the Input-to-State Stability Property" *Systems & Control Letters*, vol. 24, 351~359, 1995.



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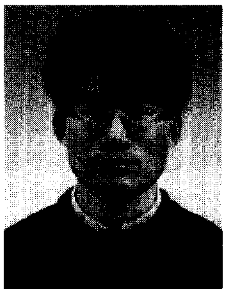

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