

# A Novel Spectral Analysis of Ultrashort Pulse Transmission Using the Pulse-Ordering Concept

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## ABSTRACT

In this paper, we analyze transmission characteristics of ultrashort laser pulses using the properties of high-order pulses which are systematically obtained following their orders. The high-order pulses are easily derived from a modified PRS system model. But we make clear they are very useful to cover wider area and to show more accurate transmission characteristics of ultrashort pulses than Gaussian or Sech pulse approximations used conventionally. These are based on the fact that the spectra and bandwidths of the high-order pulses are beautifully related to their orders. First modifying the generalized PRS system model, we propose a new model for deriving any type of high-order pulse. And we offer a novel analysis method of ultrashort pulse transmission which has any shape and FWHM, using the proposed model. In addition, by fixing the pulse range  $\tau=1$ (ps) and varying the order of the pulse from  $n=1$  to  $n=100$ , we obtain spectra of ultrashort pulses with 1(ps)~150(fs) FWHM's, which are widely used in optical pulse communications. One-step further, we derive PSD's of their pulse-trains when they are applied to Unipolar signaling scheme. These PSD's are decided in the range of possible pulse intervals. All of these results are not only coincided with some conventional experimental works but will be applied to any pioneering ultrashort pulse in the future.

## 1. Introduction

With the trends of high-speed transmission demands, recent systems have been requiring more and more bit-rate capacity and bandwidth characteristics. Moreover, approaching the era of IMT-2000, we can easily expect hundreds of Gbps or a number of Tbps rate on the backbone lines<sup>[1]</sup>. Ultrashort laser pulse transmission systems have been developed for the purpose of achievement these high-speed networks and their practical uses have gradually become reality.

In order to transmit such high-speed digital information along the network, we might use ultrashort pulse with subpico or femto second duration in time-domain. This means the spectrum of the pulse will occupy much wider bandwidth than ever. In addition, the shorter the pulse

duration, the more difficult it becomes to assert its detailed spectral characteristics. In the femto second domain, even the simple concept of pulse duration seems to fade away in a cloud of mushroom. On the other hand, we need an exact measurement of the spectrum when the pulses are applied to the transmission systems as digital elements. Precisely because of the difficulty of asserting the exact pulse shape, it is usually approximated by standard waveforms such as Gaussian, Sech, or Lorentz profile<sup>[2]</sup>.

Although the standard profiles are useful to expect the spectrum of single pulse, they should be applied in a pulse-by-pulse manner and the pulses must be symmetrically built about the peak-point. Problems are occurring in the status of multiple pulse transmission and asymmetric pulse cases<sup>[3]</sup>. We need a systematic measurement technique for the spectra of ultrashort pulses used

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in the transmission systems when their FWHM (full-width-half-maximum) and the corresponding bandwidths are occasionally varying. Moreover, because we may expect various asymmetric types of pulses which have better transmission characteristics, the spectral analysis method need to be able to expanded for them in the near future.

In this paper, we propose a novel method of spectral analysis for the transmission of ultrashort pulses using the pulse-ordering concept. First we modify the generalized PRS<sup>[4]</sup> system model in a recursive manner and so build a systematic formula for deriving the high-order pulses which have narrower widths according to their orders. With this new model, we can generate various types of pulses by setting the initial seed-pulse or changing configuration of the model and systematically derive the spectra of them. These pulses are shown to be good approximations for the ultrashort laser pulses of any type and width. Furthermore, the FWHM widths and the important parameters of pulses are directly related to their orders<sup>[5]</sup>. Although the paper only focuses the cosine pulse as a seed which generates the similar shapes to the conventional profiles, we can obtain the spectra of any other pulses by changing the seed-pulse and applying the same procedure presented here.

It must be noted that the measured width of pulse tends to constrict as the order of the pulse increase. We expect this tendency to play a significant role in the spectral analysis of ultrashort pulse transmission. After fixing all the pulse range  $\tau=1$ (ps) and varying the order of the pulse from  $n=1$  to  $n=100$ , we obtain spectra of ultrashort pulses with 1(ps)~150(fs) FWHM. In addition, we derive PSD's (power spectral density) of their transmission when they are applied in an Unipolar signaling scheme.

The remainder of this paper is organized as follows: In Section II, we describe the modified PRS system model and the pulse-ordering concept for approximation of ultrashort pulse. Section III analyzes the bandwidth of individual pulse. In

Section IV, we develop the main focal points concerning the analysis of the transmission of ultrashort pulses. In Section V, we present the simulation process and consider the results. Finally we draw concluding remarks in Section VI.

## II. Modified PRS Model and the Pulse-Ordering Concept

In order to obtain the spectra of ultrashort pulses systematically, we define the general pulse signal as a product of the  $n$ -th power of a periodic function and a single rectangular pulse centered at the origin:

$$p_n(t) = A\{u(t)\}^n \Pi\left(\frac{t}{2\tau}\right),$$

for  $n = 1, 2, 3, \dots$  (1)

where the function  $u(t)$  is a periodic function with fundamental period  $T_0$ .

It is clear that this definition can be used to represent all types of pulses by changing the function  $u(t)$  without lose of generality.

Because  $u(t)$  is periodic, it can be described by Fourier series:

$$u(t) = \sum_{r=-\infty}^{\infty} C_r \exp(-jr\omega_0 t) \quad (2)$$

and if  $u(t)$  satisfies

$$T_0 u(t) = \int_{-T_0/2}^{T_0/2} u(z) S_N(z-t) dz \quad (3)$$

where

$$S_N(z-t) = \frac{\sin\left\{\left(N+\frac{1}{2}\right)\omega_0(z-t)\right\}}{\sin\left\{\frac{\omega_0}{2}(z-t)\right\}} \quad (4)$$

then it can be a truncated Fourier series<sup>[6]</sup>:

$$u_N(t) = \sum_{r=-N}^N C_r \exp(-jr\omega_0 t) \quad (5)$$

where  $N$  is a finite integer and  $\omega_0 = 2\pi/T_0$ .

When the seed function  $u(t)$  satisfies (3) and

(4), we can obtain the  $n$ -th order pulses given in (1) and their spectra in a recursive manner using the model shown in Fig.1. The model consists of two parts as in the case of the original PRS system model<sup>[4]</sup>: The part denoted as  $K(f)$  is a tapped delay line with  $2N$  delay factors and  $2N+1$  multipliers which come together in a summer. The other denoted as  $G_n(f)$  is an analog filter whose frequency response is equivalent to the overall transfer function of the model for the  $(n-1)$ st order. These two filters are connected in cascade with each other and they are named as 'transversal filter' and 'bandlimiting filter', respectively.

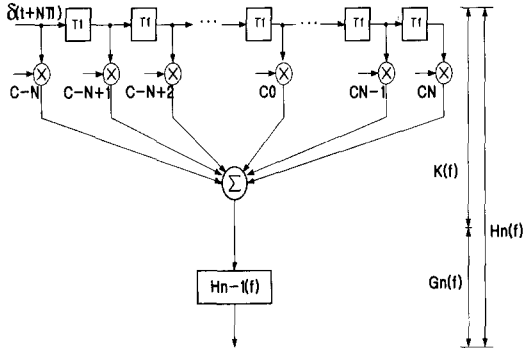


Fig. 1 The Model for Deriving Pulses and Their Spectra (Modified PRS)

From the impulse response of the system shown in Fig.1, we can obtain the  $n$ -th order pulse:

$$p_n(t) = A \left\{ \sum_{r=-N}^N C_r \exp(-jr\omega_0 t) \right\}^n \Pi\left(\frac{t}{2\tau}\right), \quad (6)$$

for  $n=1, 2, 3, \dots$

and its spectrum:

$$P_n(f) = 2A\tau \left( \sum_{r=-N}^N C_r \right)^n \text{sinc}\left(2\tau f + \frac{2\tau}{T_0} \sum_{s=1}^n r_s\right) \quad (7)$$

for  $n=1, 2, 3, \dots$

after some tedious iterations using the recursive property of the model in Fig.1.

In the equation (7) we have used a new notation which defines the depth of summation as

follows:

$$\left( \sum_{r=-N}^N C_r \right)^n \{f(t, \mathbf{r})\} \quad (8)$$

$$\equiv \sum_{r_n=-N}^N C_{r_n} \cdots \sum_{r_1=-N}^N C_{r_1} f(t, r_1, r_2, \dots, r_n)$$

with

$$\left( \sum_{r=-N}^N C_r \right)^0 f(t, \mathbf{r}) \equiv 1, \quad \sum_{s=1}^0 r_s \equiv 0 \quad (9)$$

But the formula (7) requires painstaking calculations as the order of the pulse  $n$  increases although the value of  $N$  is relatively small.

For the sake of overcoming this difficulty, we can expand the equation (7) by means of mathematical induction. When  $N$  is finite or the seed function  $u(t)$  satisfies the conditions (3) and (4), the formula (7) can be rewritten in the following form which contains only one summation with some parameters arranged by a look-up table<sup>[7]</sup>:

$$P_n(f) = 2A\tau \sum_{i=0}^{t_n} b_{n,i} \text{sinc}(2\tau f + d_{n,i}) \quad (10)$$

where

$$t_n = 2nN \quad (11)$$

$$b_{n,i} = \sum_{x=0}^{2N} C_{N-x} b_{n-1, i-(2N-x)} \quad (12)$$

with assumptions

$$\begin{cases} b_{0,0} \equiv 1 \\ b_{p,q} \equiv 0, \text{ when } q < 0 \text{ or } q > 2pN \end{cases} \quad (13)$$

$$d_{n,i} = \frac{2\tau}{T_0} (i - nN), \text{ for } i=0, 1, 2, \dots, t_n \quad (14)$$

In many cases, the FWHM of  $p_n(t)$  should be remarkably decreased in (6) as the pulse order  $n$  increases. Therefore we can freely control the pulse width by assigning the corresponding order. After the adequate order approximating the ultrashort pulse is selected, we can obtain its spectrum using (10) with the helps of (11)~(14).

The pulse-ordering concept described above offers a much easier method for spectral analysis of ultrashort pulses than any other conventional methods based on Gaussian, Sech, or Lorentz profiles. We will show a typical process using Cosine seed-pulse in the next section. This will give us a similar pulse shape as Sech profile but the fundamental differences are their approximation procedures. In addition, it must be noted that the procedure explained here could be applied to any seed-pulse or any structure of the model in Fig.1.

### III. Spectral Analysis of Ultrashort Pulse Using High-Order Cosine Pulse

We analyze high-order cosine pulse and its spectrum to approximate the property of ultrashort pulse using the pulse-ordering concept presented in the previous section. The seed function is given by

$$u(t) = \cos \frac{\pi t}{2\tau} \tag{15}$$

and it satisfies (3) and (4), so it can be written in an exponential form

$$u(t) = \frac{1}{2} \{ \exp(j\omega_0 t) + \exp(-j\omega_0 t) \} \tag{16}$$

where the fundamental period  $T_0 = 4\tau$  and the angular frequency  $\omega_0 = 2\pi/T_0$ .

This is the simplest case of the model in Fig.1 and it can be represented by the simplified model

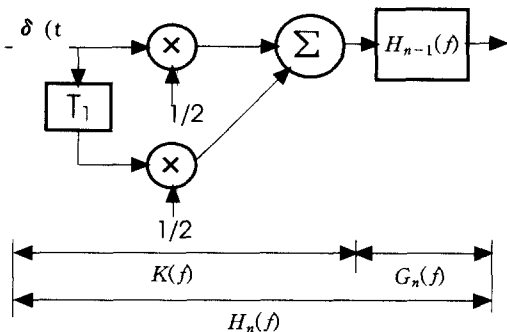


Fig. 2 Model for High-Order Cosine Pulse

shown in Fig.2. We see that the model is a modification of the Class-1 PRS system<sup>[4]</sup>.

In this case,  $N=1, C_{-1}=C_1=1/2$  and other Fourier coefficients are all zero. Inserting (16) into (6) and using (10)~(14), we can obtain the relevant parameters as follows:

$$t_n = 2n \tag{17}$$

$$b_{n,i} = \frac{1}{2} (b_{n-1,i-2} + b_{n-1,i}) \tag{18}$$

with assumptions

$$\begin{cases} b_{0,0} \equiv 1 \\ b_{p,q} \equiv 0, \text{ when } q < 0 \text{ or } q > 2p \end{cases} \tag{19}$$

$$d_{n,i} = \frac{1}{2} (i - n), \text{ for } i=0, 1, 2, \dots, 2n. \tag{20}$$

Using these results we have a useful pair for deriving the shapes of pulses and their spectra with respect to the ordering of them.

$$\begin{aligned} p_n(t) &= A \left\{ \cos \left( \frac{\pi t}{2\tau} \right) \right\}^n \Pi \left( \frac{t}{2\tau} \right) \Leftrightarrow \\ P_n(f) &= \frac{A\tau}{2^{n-1}} \sum_{i=0}^{2n} \binom{2n}{i} \text{sinc} \left( 2\tau f + \frac{d_i}{2} \right) \end{aligned} \tag{21}$$

In time-domain the pulse widths remarkably decrease with increasing orders as shown in Fig.3. On the other hand, the corresponding spectra will be easily derived using (21) and its parameters.

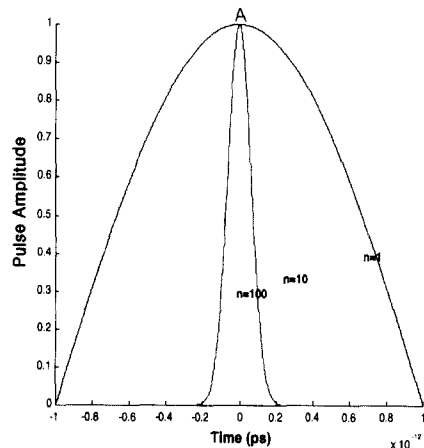


Fig. 3 Width Variation of High-Order Cosine Pulses

As stated above, these parameters can be offered by a look-up table, so we submit Table 1 for this case.

The table includes amplitude varying with the order, coefficients and delay factors of 'sinc(·)' function. It is interesting to note the coefficients column builds a Pascal's triangle. Although TABLE 1 depicts parameters for some lower orders, it can be easily extended to any order by simple arithmetics.

Table 1 Parameters for the Spectra of High-Order Cosine Pulses

n	Constants	Coefficients	Delay-Factors
0	$2A\tau$	1	0
1	$A\tau$	1 1	1-1
2	$A\tau/2$	1 2 1	2 0 2
3	$A\tau/2^2$	1 3 3 1	3 1-1 3
4	$A\tau/2^3$	1 4 6 4 1	4 2 0-2 4
5	$A\tau/2^4$	1 5 10 10 5 1	5 3 1-1 3-5
6	$A\tau/2^5$	1 6 15 20 15 6 1	6 4 2 0-2 4-6

We can easily expect that the cosine pulse with high order approximate ultrashort pulse as for the same reason of Gaussian or Sech profile. In addition, we can analyze multiple pulses with various FWHM's at the same time using the pulse ordering concept presented here, which is not the case of conventional approximations. Moreover, the model in Fig.1 is opened to any other pulse shape satisfying (3) and (4) just by changing the seed function.

After fixing the pulse range  $\tau=1(\text{ps})$  as shown in Fig.3 and varying the order of pulse from  $n=1$  to 100 with an adequate increment, we can obtain the spectra of short pulses with various FWHM's without changing the basic pulse shape. Fig.4 shows the variation of spectra when the FWHM's of pulses are set to 1(ps), 0.4(ps), and 150(fs).

As the orders of pulses increase, we can see their FWHM's will be gradually constricted to any short level and the bandwidths should be wider and wider. Throughout this method, we can

estimate the spectral bands of ultrashort pulses of any type and any width.

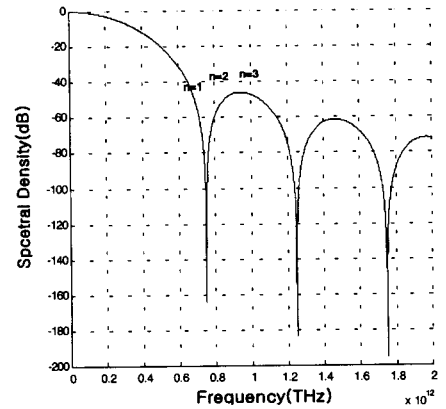


Fig. 4 Spectra of Pulses Based on the Pulse-Ordering Concept

#### IV. Transmission Characteristics of Ultrashort Pulses

In this section we derive PSD(power spectral density) of the pulse-trains when they are applied to unipolar signaling scheme. We selected unipolar scheme because most laser pulse transmission systems are fabricated in this signaling format.

In general PSD of baseband digital signal is influenced by both the spectrum of data and the spectrum of pulse shape used for the line code. But the influence of the correlation of random data disappear from the PSD of the schemes of polar or unipolar<sup>[8]</sup>. It is also noted that a discrete term with respect to the dc component will be shown in the PSD of unipolar signaling scheme.

In the case of ultrashort pulse transmission, we should also consider the relationship between the pulse interval  $T_b$  and the individual width. Fig.5 shows the case of the minimum possible interval for  $n=100$  cosine pulses. Of course the pulse interval has some influences on the PDS of transmission. We can extract the optimum interval for a given frequency environment.

The PSD of unipolar signaling with the basis of power of unity can be given by

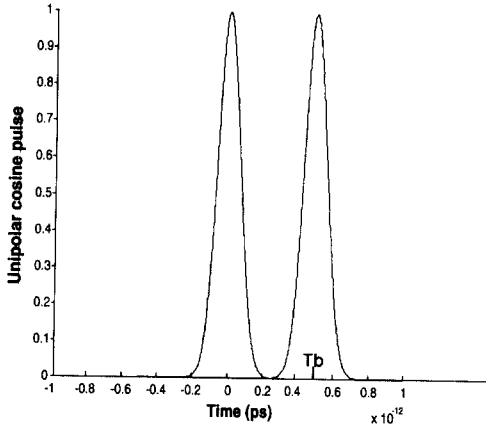


Fig. 5 Minimum Possible Pulse Interval When  $n=100$

$$P_s(f) = \frac{1}{2T_b} |P_n(f)|^2 + \frac{1}{2} \delta(f) \quad (22)$$

where the first term is directly given by the spectrum of the individual  $n$ -th order pulse such as (10) or (21) derived in the previous sections and the second term is the discrete impulse with respect to dc component of the unipolar signaling format.

Fig.6 shows the variation of PSD of random pulse transmission for several pulse intervals when the order is 10. Fig.7 and Fig.8 are about the cases of order 50 and 100, respectively. In these figures, the discrete components are omitted because it doesn't have occupations of frequency bands any more.

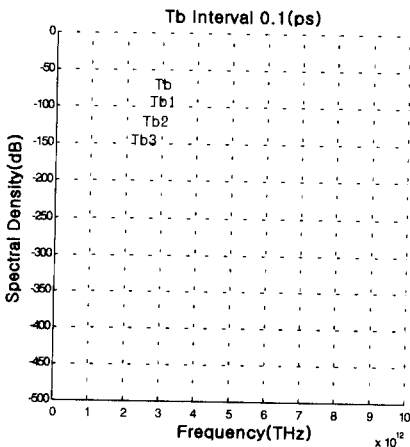


Fig. 6 Variation of PSD According to the Pulse Interval When  $n=10$

As we can see, the shorter the pulse widths or the higher the order of pulses, the more serious influence on the spectral band according to the pulse intervals. The graph indicated by  $T_b$  in each figure expresses the spectrum for the case of the minimum possible pulse interval.

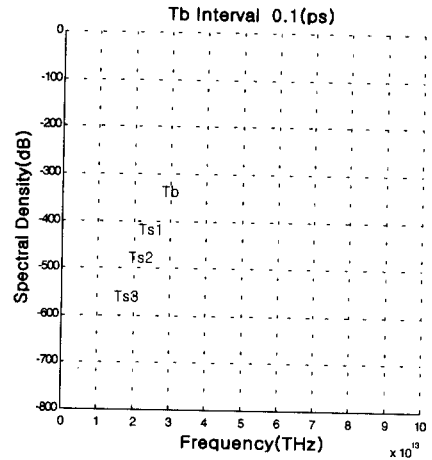


Fig. 7 Variation of PSD According to the Pulse Interval When  $n=50$

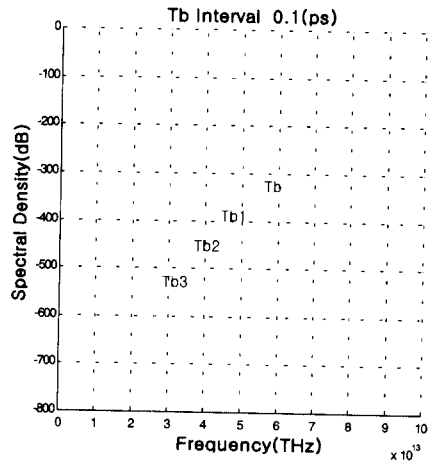


Fig. 8 Variation of PSD According to the Pulse Interval When  $n=100$

## V. Simulations and Results

We now perform computer simulations to prove the conveniences and compactness of the proposed method using MATLAB program. First we match the  $n$ -th order cosine pulses with the ultrashort

pulses according to their FWHM's, and get the transmission bandwidths of them when the minimum possible pulse-interval conditions are applied.

Table 2 shows simulated FWHM's and first-null transmission bandwidths of the pulses when the orders of them are increasing from  $n=1$  to  $n=100$ . It can be noted that the pulse widths are varying from 1(ps) to 150(fs) and the transmission bandwidths from 0.7(THz) to 75(THz). A part of this result can be conformed by several conventional methods. However the method presented here gives us the unique process of analysis in the multiple-width and multiple-type environment.

Table 2. Variation of FWHM and Transmission Bandwidth According to the order

n	FWHM( $10^{-12}$ )	BW <sub>null</sub> ( $10^{12}$ )
1	1.3333332	0.75
10	0.4686060	4.10
20	0.3332820	9.91
30	0.2726500	17.27
40	0.2364032	25.77
50	0.2115188	35.05
60	0.1931836	44.11
70	0.1788842	52.07
80	0.1673846	59.74
90	0.1577818	67.83
100	0.1497619	75.58

Of course the table can be extended to any order of pulse for the corresponding pulse-width by some trivial repetitions and can be modified for any type of seed function which satisfies (3) and (4). Although these are the results of unipolar signaling, any kind of optical pulse transmission method expected in the future can be beautifully applied to the procedure presented here.

As stated earlier, the intervals between pulses put a great influence on the transmission bandwidth. Table 3 shows the variations of first null bandwidth for the pulses of  $n=10, 50$ , and  $100$ , respectively, as the pulse intervals are changed from 100(fs) to 1.5(ps). We also offer Fig.9 to show the tendency of variations of spectral widths according to their orders. As the

order of the pulse increase, the amount of bandwidth variation should be also increase. This tendency coincides with the results obtained from the graphs in Fig.6~8.

Table. 3 Variation of Transmission Bandwidths According to the Pulse Intervals

Pulse Interval(ps)	B <sub>NULL</sub> ( $10^{12}$ )		
	n=100	n=50	n=10
0.1	54.93	28.31	3.62
0.2	43.28	22.70	3.23
0.3	35.82	19.23	2.91
0.4	30.10	17.32	2.66
0.5	26.15	15.38	2.44
0.6	23.45	13.36	2.25
0.7	20.87	12.11	2.10
0.8	18.75	11.39	1.96
0.9	17.35	10.24	1.84
1.0	16.47	9.41	1.73
1.1	14.94	8.81	1.64
1.2	13.82	8.27	1.55
1.3	12.70	7.74	1.47
1.4	12.12	7.33	1.41
1.5	11.34	6.98	1.34

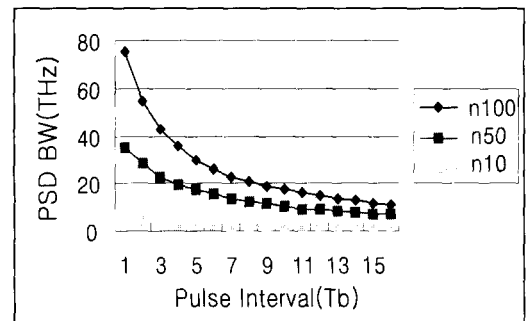


Fig. 9 Variation of Transmission Bandwidth According to the Pulse Intervals

## VI. Conclusion

We have proposed a novel method to analyze the spectrum of ultrashort pulse transmission system throughout this paper. It gives us more convenient and systematic procedures for investigating the characteristics of ultrashort pulses especially in the case of multiple widths or multiple types. According to the conventional methods, any type of ultrashort pulse would be approximated by a Gaussian, Sech, or Lorentz

profile etc. On the other hand, we can obtain the most adequate seed function for any other type of ultrashort pulse and match the FWHM of given ultrashort pulse by ordering the seed function using the method presented here.

Although we present the detail procedure only for the case of cosine pulse in this paper, it can be easily modified for any type and any speed of pulses. We got and saved the detail result for some useful type of pulses in order to support the background of the paper. In addition, applying the unipolar signaling scheme, we analyze PSD's and their bandwidth occupations to investigate the transmission characteristics according to their FWHM's and pulse intervals. These procedures are also easily extended to any other types of transmissions through the laser pulse channel.

We expect the proposed method and the results shown in Section V would be an excellent reference for the analysis of ultrahigh speed transmission system using laser short pulses. These results will play a significant role in the design of ultrashort pulse transmission system, which will be the subject of our future work.

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