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Generalized Common Fixed Point Theorems on Menger PM-spaces

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ABSTRACT

More generalized common fixed point theorems for a sequence of fuzzy mappings to the nonexpansive case on Menger probabilistic metric spaces, which generalize recent results of Lee *et al.*[13], are obtained.

1. Introduction

There have been many results on fixed point theorems for fuzzy mappings, multi-valued mappings and single-valued mappings on probabilistic metric spaces including metric spaces, considered by Bose *et al.* [1], Butnariu [2], Chang *et al.* [4-6], Hadzic [8], Heilpern [9], Lee *et al.* [10-13], and others [3,7,17-19].

In 1996, Lee *et al.* [12] obtained a common fixed point theorem for a sequence of fuzzy mappings to the nonexpansive case on Menger probabilistic metric spaces under some equality-type condition.

In this paper we obtain more generalized common fixed theorems for a sequence of fuzzy mappings to the nonexpansive case on Menger probabilistic metric spaces, which generalize and improve the previous results of Lee *et al.* [12].

2. Preliminaries

In this section, we recall some topological properties and others of Menger probabilistic metric spaces in [14-16].

Definition 2.1 A probabilistic metric space (in short, a PM-space) is an ordered pair $(E, \mathcal{F})$, where E is a nonempty set and $\mathcal{F}$ is a mapping from $E\times E$ into D^+, where D^+ is the set of all distribution functions. We denote the distribution function $\mathcal{F}(x, y)$ by $F_{x,y}$ for each $x, y\in E$. The function $F_{x,y}$ is assumed to satisfy the following conditions;

(PM-1) $F_{x,y}(t)=1$ for all $t>0$ if and only if $x=y$,
(PM-2) $F_{x,y}(0)=0$,
(PM-3) $F_{x,y}(t)=F_{y,x}(t)$ for all $t\in \boldsymbol{R}$,
(PM-4) if $F_{x,y}(t_1)=1$ and $F_{y,z}(t_2)=1$, then $F_{x,z}(t_1+t_2)=1$.

Definition 2.2 A mapping $\Delta : [0,1]\times[0,1]\rightarrow[0,1]$ is called a t-norm if it satisfies the following conditions ; for any $a, b, c, d\in[0,1]$,

(T-1) $\Delta(a, 1)=a$,
(T-2) $\Delta(a, b)=\Delta(b, a)$,
(T-3) $\Delta(c, d)\geq\Delta(a, b)$ for $c\geq a$ and $d\geq b$,
(T-4) $\Delta(\Delta(a, b), c)=\Delta(a, \Delta(b, c))$.

Definition 2.3 A Menger PM-space is a triplet $(E, \mathcal{F}, \Delta)$, where $(E, \mathcal{F})$ is a PM-space and Δ is a t-norm satisfying the following triangle inequality

$F_{x,y}(t_1+t_2)\geq\Delta(F_{x,y}(t_1), F_{y,z}(t_2))$ for all $x, y, z\in E$ and $t_1, t_2\geq 0$.

Schweizer and Sklar have proved that if $(E, \mathcal{F}, \Delta)$ is a Menger PM-space with a continuous t-norm Δ, then $(E, \mathcal{F}, \Delta)$ is a Hausdorff topological space in the topology τ having

$$\beta=\{N_p(\varepsilon, \lambda): p\in E, \varepsilon, \lambda>0\}$$

as a basis, where

$$N_p(\varepsilon, \lambda)=\{x\in E: F_{x,p}(\varepsilon)>1-\lambda\}.$$

Definition 2.4 Let $(E, \mathcal{F}, \Delta)$be a Menger PM-space with a continuous t-norm Δ. Let $(x_n)_{n=1}^{\infty}$ be any sequence in E.

$(x_n)_{n=1}^{\infty}$ is said to be τ-convergent to $x\in E$(we write $x_n\xrightarrow{\tau} x$), if for any given $\varepsilon>0$ and $\lambda>0$, there exists a positive integer $N=N(\varepsilon, \lambda)$ such that $F_{x_n,x}(\varepsilon)>1-\lambda$ whenever $n\geq N$.

$(x_n)_{n=1}^{\infty}\subset E$ is called a τ-Cauchy sequence if for any $\varepsilon>0$ and $\lambda>0$, there exists a positive integer $N=N(\varepsilon, \lambda)$ such that $F_{x_n,x_m}(\varepsilon)>1-\lambda$, whenever $n, m\geq N$.

A Menger PM-space $(E, \mathcal{F}, \Delta)$ is said to be τ-complete if each τ-Cauchy sequence in E is τ-convergent to some point in E.

Definition 2.5 A function $\varphi: [0, +\infty)\rightarrow[0, +\infty)$ is said to satisfy the condition (Φ) if it is strictly increasing, left-continuous, $\varphi(0)=0$, $\lim_{t\rightarrow+\infty}\varphi(t)=+\infty$ and $\sum_{n=0}^{\infty}\varphi^n(t)<+\infty$ for all $t>0$, where $\varphi^n(t)$ is the n-th iterative of $\varphi(t)$.

Lemma 2.6 [6,14] Let a function $\varphi: [0, +\infty)\rightarrow[0, +\infty)$ satisfy the condition (Φ), then a function

ψ: $[0, +\infty) \rightarrow [0, +\infty)$ defined by

$$(2.1) \qquad \psi(t) = \begin{cases} 0, & t=0, \\ \inf\{s>0 : \varphi(s)>t\}, & t>0. \end{cases}$$

is continuous, nondecreasing, and satisfies the following assertions;

(i) $\varphi(t) < t$ for all $t > 0$,

(ii) $\varphi(\psi(t)) \leq t$ and $\psi(\varphi(t))=t$ for all $t \geq 0$,

(iii) $\psi(t) \geq t$ for all $t \geq 0$,

(iv) $\lim_{n \to \infty} \psi^n(t) = +\infty$ for all $t > 0$.

Definition 2.7 A t-norm Δ is called h-type if a family of functions $(\Delta^m(t))_{m=1}^{\infty}$ is equicontinuous at $t=1$, where

$$\Delta^1(t)=\Delta(t, t), \Delta^{m+1}(t)=\Delta(t, \Delta^m(t)), m=1, 2, \cdots, t \in [0, 1].$$

Obviously, Δ is an h-type t-norm if and only if for any $\lambda \in (0, 1)$, there exists $\delta(\lambda) \in (0, 1)$ such that $\Delta^m(t) > 1-\lambda$ for all $m \in N$, the set of natural numbers, when $t > \delta(\lambda)$.

Lemma 2.8 [6] Let $(E, \mathcal{F}, \Delta)$ be a Menger PM-space with an h-type t-norm Δ. If a sequence $(x_n)_{n=0}^{\infty}$ in Esatisfies the following condition;

$$F_{x_n, x_{n+1}}(t) \geq F_{x_0, x_1}(\psi^n(t)) t \geq 0,$$

where ψ is a function defined by (2.1), then $(x_n)_{n=0}^{\infty}$ is a τ-Cauchy sequence in E.

Lemma 2.9 A sequence $(x_n)_{n=0}^{\infty}$ converges to x in $(E, \mathcal{F}, \Delta)$ if and only if the sequence $(F_{x_n, x})_{n=1}^{\infty}$ converges to $F_{x,x} \in D^+$.

III. Common fixed point theorems

Throughout this section, $(E, \mathcal{F}, \Delta)$ is a τ-complete Menger PM-space with a left-continuous t-norm Δ of h-type. We always assume that $\hat{\Omega}$ is a family of all fuzzy sets A in E whose each α-level set $(A)_\alpha = \{x \in E \mid A(x) \geq \alpha\}$ for $\alpha \in (0, 1]$is a nonempty τ-closed set in E. We define a mapping $\hat{\mathcal{F}}$ on $\hat{\Omega} \times \hat{\Omega} \rightarrow D^+$ as follows (we also denote $\hat{\mathcal{F}}(A, B)$ by $\hat{F}_{A,B}$ and the value of $\hat{\mathcal{F}}(A, B)$ at $t \in \boldsymbol{R}$ by $\hat{F}_{A,B}(t)$);

$$\hat{F}_{A,B}(t) = \inf_{\alpha \in (0,1]} F_{(A)_\alpha, (B)_\alpha}(t), \ t \geq 0, \ A, \ B \in \hat{\Omega}, \text{ and}$$
$$\hat{F}_{\{x\},B}(t) = \inf_{\alpha \in (0,1]} F_{x,(B)_\alpha}(t), \ t \geq 0,$$

where

$$F_{(A)_\alpha,(B)_\alpha}(t) = \sup_{s<t} \Delta(\inf_{a \in (A)_\alpha} \sup_{b \in (B)_\alpha} F_{a,b}(s), \inf_{b \in (B)_\alpha} \sup_{a \in (A)_\alpha} F_{a,b}(s)), \text{ and}$$

$$F_{x,(B)_\alpha}(t) = \sup_{y \in (B)_\alpha} F_{x,y}(t), \ t \geq 0 \text{ for } \alpha \in [0, 1].$$

The last equality is called the probabilistic distance between a point xand a set $(B)_\alpha$. It is easily shown that $(\hat{\Omega}, \hat{\mathcal{F}}, \Delta)$ is a Menger PM-space. For $A, B \in \hat{\Omega}$, $A \subset B$ means $A(x) \leq B(x)$ for $x \in E$, and $\{x\} \subset Tx$ or $(Tx)(x)=1$ means that x is a fixed point of a fuzzy mapping T from E to $\hat{\Omega}$

Proposition 3.1 Let A and B be elements of $\hat{\Omega}$ Assume that the 1-level set $(B)_1$ of the fuzzy set B is compact. Then for any $\{x\} \subset A$, there exists $\{y\} \subset B$ such that $F_{x,y}(t) \geq \hat{F}_{A,B}(t)$, $t \geq 0$.

Proof.

$$\begin{aligned}
&\hat{F}_{A,B}(t) \\
&= \inf_{\alpha \in (0,1]} F_{(A)\alpha,(B)\alpha}(t) \\
&\leq F_{(A)_1,(B)_1}(t) \\
&= \sup_{s<t} \Delta\left(\inf_{a \in (A)_1} \sup_{b \in (B)_1} F_{a,b}(s), \inf_{b \in (B)_1} \sup_{a \in (A)_1} F_{a,b}(s)\right) \\
&\leq \sup_{s<t}\left(\inf_{a \in (A)_1} \sup_{b \in (B)_1} F_{a,b}(s)\right) \\
&\leq \sup_{b \in (B)_1} F_{x,b}(t)
\end{aligned}$$

Putting $k = \sup_{b \in (B)_1} F_{x,b}(t)$, $t \geq 0$, we obtain a sequence $(y_n)_{n=1}^{\infty}$ converging to $y \in (B)_1$ such that $k - 1/n < F_{x,y_n}(t) \leq k$. Letting $n \rightarrow \infty$, we have $k = F_{x,y}(t)$, $t \geq 0$, i.e.,

$$\sup_{b \in (B)_1} F_{x,b}(t) = F_{x,y}(t), \ t \geq 0.$$

This completes the proof.

Proposition 3.2 [13] Let A and B in $\hat{\Omega}$. Then the following hold;

(i) $\hat{F}_{\{x\},A}(t)=1$, $t>0$ only if $\{x\} \subset A$.

(ii) $F_{x,(A)\alpha}(t) \geq \hat{F}_{\{x\},A}(t)$, $t \geq 0$.

(iii) If $\{x\} \subset A$, then $\hat{F}_{\{x\},B}(t) \geq \hat{F}_{A,B}(t)$, $t \geq 0$.

Proposition 3.3 Let $B \in \hat{\Omega}$ and $\{y\} \subset B$. Then for $\{x\} \in \hat{\Omega}$,

$$\hat{F}_{\{x\},B}(t) \geq F_{x,y}(t), \ t \geq 0.$$

Proof. For any $\alpha \in (0, 1]$and $\{y\} \subset B$,

$$\begin{aligned}
F_{x,(B)\alpha}(t) &= \sup_{z \in (B)\alpha} F_{x,z}(t) \\
&\geq F_{x,y}(t), \ t \geq 0.
\end{aligned}$$

Hence we have

$$\begin{aligned}
\hat{F}_{\{x\},B}(t) &= \inf_{\alpha \in (0,1]} F_{x,(B)\alpha}(t) \\
&\geq F_{x,y}(t), \ t \geq 0.
\end{aligned}$$

Now we introduce the main theorem of this paper.

Theorem 3.4 Let $(T_i)_{i=1}^{\infty}$: $(E, \mathcal{F}, \Delta) \to (\hat{\Omega}, \hat{\mathcal{F}}, \Delta)$ be a sequence of fuzzy mappings and a function φ : $[0, +\infty) \to [0, +\infty)$ satisfy the condition (Φ). Assume that the 1-level set $(B)_1$ of the fuzzy set B is compact. Suppose that for any $x, y \in E$,

$$\hat{F}_{T_i x, T_j y}(\varphi(t)) \geq \min\{F_{x,y}(t), \hat{F}_{\{x\}, T_i x}(t), \hat{F}_{\{y\}, T_j y}(t)\}, \ t \geq 0.$$

Then there exists an $x_* \in E$ such that

$\{x_*\} \subset T_i x_*, \ i \in N.$

Proof. Define a function ψ: $[0, +\infty) \to [0, +\infty)$ by

$$\psi(t) = \begin{cases} 0, & t = 0, \\ \inf\{s > 0 : \varphi(s) > t\}, & t > 0. \end{cases}$$

Then by Proposition 3.1, for any given $x_0 \in E$ and any $\{x_1\} \subset T_1 x_0$, there exists $\{x_2\} \subset T_2 x_1$ such that

$$\begin{aligned} &F_{x_1, x_2}(\varphi(\psi(t))) \\ &\geq \hat{F}_{T_1 x_0, T_2 x_1}(\varphi(\psi(t))) \\ &\geq \min\{F_{x_0, x_1}(\psi(t)), \hat{F}_{\{x_0\}, T_1 x_0}(\psi(t)), \hat{F}_{\{x_1\}, T_2 x_1}(\psi(t))\}. \end{aligned}$$

Since F_{x_1, x_2} is nondecreasing, we obtain $F_{x_1, x_2}(t) \geq F_{x_1, x_2}(\varphi(\psi(t))$ from Lemma 2.6 (ii). Thus we have

$$\begin{aligned} F_{x_1, x_2}(t) &\geq \min\{F_{x_0, x_1}(\psi(t)), \hat{F}_{\{x_0\}, T_1 x_0}(\psi(t)), \\ &\qquad \hat{F}_{\{x_1\}, T_2 x_1}(\psi(t))\} \\ &\geq \min\{F_{x_0, x_1}(\psi(t)), F_{x_1, x_2}(\psi(t))\}, \end{aligned}$$

from the definition of the probabilistic distance between a point and a set and Proposition 3.3.

Since $F_{x_1, x_2}(t) \geq \min\{F_{x_0, x_1}(\psi(t)), F_{x_1, x_2}(\psi^m(t))\}$ for all $m \in N$ from Lemma 2.6 (iii), letting $m \to \infty$ we have

$$F_{x_1, x_2}(t) \geq F_{x_0, x_1}(\psi(t)), \ t \geq 0.$$

Similarly, there exists $\{x_3\} \subset T_3 x_2$ such that

$$F_{x_2, x_3}(t) \geq F_{x_1, x_2}(\psi(t)).$$

Continuing this process, we have a sequence $(x_n)_{n=0}^{\infty}$ such that

(i) $\{x_n\} \subset T_n x_{n-1}$,
(ii) $F_{x_n, x_{n+1}}(t) \geq F_{x_{n-1}, x_n}(\psi(t)), \ t \geq 0.$

Hence we have

$$F_{x_n, x_{n+1}}(t) \geq F_{x_{n-1}, x_n}(\psi(t)) \geq \cdots \geq F_{x_0, x_1}(\psi^n(t)), \ t \geq 0.$$

By Lemma 2.8., $(x_n)_{n=0}^{\infty}$ is a τ-Cauchy sequence in E. By the completeness of $(E, \mathcal{F}, \Delta)$, there exists an $x_* \in E$ such that $x_n \xrightarrow{\tau} x_*$.

Next, we prove that x_* satisfies $(T_i x_*)(x_*)=1$ for all $i \in N$, i.e., $\{x_*\}$ is a common fixed point of $(T_i)_{i=1}^{\infty}$. In fact, for $\{x_{n+1}\} \subset T_{n+1} x_n$, we have $\{z_i\} \subset T_i x_*$ for each fixed $i \in N$ such that

$$\begin{aligned} &F_{x_{n+1}, z_i}(t) \\ &\geq \hat{F}_{T_{n+1} x_n, T_i x_*}(t) \\ &\geq \min\{F_{x_n, x_*}(\psi(t)), \hat{F}_{\{x_n\}, T_{n+1} x_n}(\psi(t)), \hat{F}_{\{x_*\}, T_i x_*}(\psi(t))\} \\ &\geq \min\{F_{x_n, x_*}(\psi(t)), F_{x_n, x_{n+1}}(\psi(t)), \hat{F}_{\{x_*\}, T_i x_*}(\psi(t))\} \\ &\geq \min\{F_{x_n, x_*}(\psi(t)), F_{x_0, x_1}(\psi^n(t)), \hat{F}_{\{x_*\}, T_i x_*}(\psi(t))\}. \end{aligned}$$

Taking limit inferior in (3.1) we have by Lemma 2.6 and Lemma 2.9

(3.2) $$\varliminf_{n \to \infty} F_{x_{n+1}, z_i}(t) \geq \hat{F}_{\{x_*\}, T_i x_*}(\psi(t)).$$

On the other hand,

$$\begin{aligned} \hat{F}_{x_*, T_i x_*}(\psi(t)) &\geq F_{x_*, z_i}(\psi(t)) \\ &\geq \Delta\{F_{x_*, x_{n+1}}(\delta), F_{x_{n+1}, z_i}(\psi(t) - \delta)\}, \ \delta > 0. \end{aligned}$$

Taking limit superior we have

$$\hat{F}_{x_*, T_i x_*}(\psi(t)) \geq \varlimsup_{n \to \infty} F_{x_{n+1}, z_i}(\psi(t) - \delta), \ \delta > 0.$$

By the arbitrariness of $\delta > 0$, we have

$$\hat{F}_{\{x_*\}, T_i x_*}(\psi(t)) \geq \varlimsup_{n \to \infty} F_{x_{n+1}, z_i}(\psi(t)).$$

Combining (3.2) and (3.3) we have

$$\begin{aligned} \varliminf_{n \to \infty} F_{x_{n+1}, z_i}(\psi(t)) \geq \varliminf_{n \to \infty} F_{x_{n+1}, z_i}(t) &\geq \hat{F}_{\{x_*\}, T_i x_*}(\psi(t)) \\ &\geq \varlimsup_{n \to \infty} F_{x_{n+1}, z_i}(\psi(t)) \\ &\geq \varlimsup_{n \to \infty} F_{x_{n+1}, z_i}(t). \end{aligned}$$

Therefore

(3.4) $$\lim_{n \to \infty} F_{x_{n+1}, z_i}(\psi(t)) = \hat{F}_{\{x_*\}, T_i x_*}(\psi(t)), \text{ and}$$

(3.5) $$\lim_{n \to \infty} F_{x_{n+1}, z_i}(t) = \hat{F}_{\{x_*\}, T_i x_*}(\psi(t)).$$

By the arbitrariness of t, from (3.4) we have

$$\lim_{n \to \infty} F_{x_{n+1}, z_i}(t) = \hat{F}_{\{x_*\}, T_i x_*}(t), \ t \geq 0.$$

Therefore from (3.5) we have

$$\begin{aligned} \lim_{n \to \infty} F_{x_{n+1}, z_i}(t) &= \hat{F}_{\{x_*\}, T_i x_*}(\psi(t)) \\ &= \hat{F}_{\{x_*\}, T_i x_*}(\psi^2(t)) \\ &= \cdots\cdots \\ &= \hat{F}_{\{x_*\}, T_i x_*}(\psi^m(t)) \\ &\geq F_{x_*, z_i}(\psi^m(t)). \end{aligned}$$

Letting $m \to \infty$, we have

$F_{x_*, z_i}(t) = 1, \ t > 0.$

This shows that $x_* = z_i$ for all i. Hence we have $\{x_*\} \subset T_i x_*$ for all $i \in N$. This completes the proof.

The following theorem can be obtained from Theorem 3.4 as a corollary.

Theorem 3.5 Let $(T_i)_{i=1}^{\infty} : (E, \mathcal{F}, \Delta) \rightarrow (\hat{\Omega}, \hat{\mathcal{F}}, \Delta)$ be a sequence of fuzzy mappings. Assume that 1-level sets of a fuzzy set in $\hat{\Omega}$ are compact. Suppose that there exists a function $\varphi : [0, +\infty) \rightarrow [0, +\infty)$ satisfying the condition (Φ) such that for any $i, j \in N$ and any $x, y \in E$,

$$\hat{F}_{T_i x, T_j y}(\varphi(t)) \geq \min\{F_{x,y}(t), F_{x,(T_i x)_1}(t), F_{y,(T_j y)_1}(t)\}, \ t \geq 0.$$

Then there exists an $x_* \in E$ such that

$$\{x_*\} \subset T_i x_*, \ i \in N.$$

Proof. Since any closed subset of a compact set is compact, it is easily shown from Theorem 3.4 that this theorem holds.

In fact, Theorem 3.5 is a slight generalization and an improvement of the main result of Lee *et. al.* [12] by deleting some of it's conditions, which can be called as an equality-type condition.

References

[1] R. K. Bose and D. Sahani, *Fuzzy mappings and fixed point theorems*, Fuzzy Sets and Systems, 21, 53-58, 1987.

[2] D. Butnariu, *Fixed point theorems for fuzzy mappings*, Fuzzy Sets and Systems 7, 191-207, 1982.

[3] G. L. Cain and K. H. Kasriel, *Fixed and periodic point of local contraction mapping on probabilistic metric spaces*, Math. Systems Theory 9, 289-297, 1976.

[4] S. S. Chang, *Fixed point theorems for fuzzy mappings*, Fuzzy Sets and Systems 17, 181-187, 1985.

[5] S. S. Chang, *Fixed degree for fuzzy mappings and a generalization of Ky Fan's theorem*, Fuzzy Sets and Systems 24, 103-112, 1987.

[6] S. S. Chang, Y. J. Cho, S. M. Kang and J. X. Fan, *Common fixed point theorems for multi-valued mappings in Menger PM-spaces*, Math. Japonica 40, 289-293, 1994.

[7] J. X. Fan, *A note on fixed point theorem of Hadzic*, Fuzzy Sets and Systems 48, 391-395, 1992.

[8] O. Hadzic, *Fixed point theorems for multi-valued mappings in probabilistic metric spaces*, Mat. Vesnik 3, 125-133.

[9] S. Heilpern, *Fuzzy mappings and fixed point theorems*, J. Math. Anal. Appl. 83, 556-569, 1981.

[10] B. S. Lee and S. J. Cho, *A fixed point theorem for contractive-type fuzzy mappings*, Fuzzy Sets and Systems 61, 309-312, 1994.

[11] B. S. Lee and S. J. Cho, *Common fixed point theorem for sequences of fuzzy mappings*, Int. J. Math & Math. Sci. 17(3), 423-428, 1994.

[12] B. S. Lee, Y. J. Cho and J. S. Jung, *Fixed point theorems for fuzzy mappings and applications*, Comm. Korean Math. Soc. 11, 89-108, 1996.

[13] B. S. Lee, G. M. Lee and D. S. Kim, *Common fixed points of fuzzy mappings in Menger PM-spaces*, J. Fuzzy Math. 2, 859-870, 1994.

[14] B. Schweizer and A. Sklar, *Probabilistic Metric Spaces*, North-Holland, Amsterdam, 1983.

[15] B. Schweizer and A. Sklar, *Statistical metric spaces*, Pacific J. Math. 10, 313-334.

[16] B. Schweizer, A. Sklar and E. Thorp, *The metrization of statistical metric spaces*, Pacific J. Math. 10, 673-675, 1960.

[17] V. M. Sehgal and A. T. Bharucha-Reid, *Fixed points of contraction mappings on probabilistic metrc spaces*, Math. Systems Theory 6(2), 97-102, 1972.

[18] T. Som and R. N. Mukherjee, *Some fixed point theorems for fuzzy mappings*, Fuzzy Sets and Systems 33, 213-219, 1989.

[19] M. Stojakovic, *Fixed point theorems in probabilistic metric spaces*, Kobe J. Math. 2(1), 1-9, 1985.

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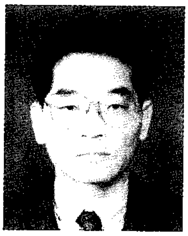

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