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A note on Fuzzy Vietories Topology

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ABSTRACT

We introduce the concept of a fuzzy Vietories topology and we obtain some properties.

1. Preliminaries

Let $I = [0, 1]$ and $I_0 = (0, 1]$. For a set X. Let I^X be the collection of all the mappings from X into I. Each member of I^X, $A : X \rightarrow I$, is called a *fuzzy set* in X (*cf.*[6])). Let $F_p(X)$ denote the collection of all the *fuzzy points* in a set X[3-5].

Definition 1.1[3] A fuzzy point x_λ in a set X is said to be *quasi-coincident q-coincident*, in shorts) with a fuzzy set A in X, denoted by $x_\lambda q_A$, if $\lambda > A^c(x)$ or $\lambda + A(x) > 1$. A fuzzy set A is said to be *q-coincident* with a fuzzy set B, denoted by AqB, if there exists an $x \in X$ such that $A(x) > B^c(x)$ or $A(x)+B(x)>1$. In this case, we say that A and B are *q-coincident*.

Result 1.A[3] $A \subset B$ if and only if A and B^c are not q-coincident(denoted by $A\bar{q}B^c$). In particular, $x_\lambda \in A$ if and only if $x\lambda \bar{q} A^c$.

Result 1.B[4] Let $A, B \in I^X$. The followings are equivalent :

(a) $A \subset B$.

(b) $x_\lambda \subset B$ for all $x_\lambda \subset A$.

(c) $x_\lambda \in B$ for all $x_\lambda \in A$.

Definition 1.2[1] A subfamily $\mathcal{T}$ of I^X is called a *fuzzy topology* on X if $\mathcal{T}$ satisfies the following conditions:

(i) $\emptyset$, $X \in \mathcal{T}$

(ii) If $\{U_\alpha : \alpha \in A\} \subset \mathcal{T}$ then $\bigcup_{\alpha \in A} U_\alpha \in \mathcal{T}$ where A is an index set,

(iii) If $A, B \in \mathcal{T}$ then $A \cap B \in \mathcal{T}$

Each member of $\mathcal{T}$ is called a *fuzzy open set* in X and its complement a *fuzzy closed set* in X. The pair $(X, \mathcal{T})$ is called a *fuzzy topological space*(fts, in short)

Definition 1.3[3] For a fuzzy set A in a fts $(X, \mathcal{T})$, the *closure* $\bar{A}$ and the *interior*, $\mathring{A}$ of A are defined respectively, as

$$\bar{A} = \bigcap \{B : A \subset B, B^c \in \mathcal{T}\} \text{ and } \mathring{A} = \bigcup \{B : B \subset A, B \in \mathcal{T}\}.$$

Result 1.C[3] Let $(X, \mathcal{T})$ be a fts and let $A \in I^X$. Then:

(a) $x_\lambda \in \mathring{A}$ if and only if x_λ has a neighborhood contained in A.

(b) $x_\lambda \in \bar{A}$ if and only if for each q-neighborhood V of x_λ, VqA.

Result 1.D[3] Let $(X, \mathcal{T})$ be a fts and let $A \in I^X$. Then:

$\mathring{A} = (\overline{A^c})^c$ and $\bar{A} = (\mathring{A^c})^c$.

Definition 1.4[2] A fts $(X, \mathcal{T})$ is said to be:

(a) $\mathcal{T}_0$ if for any two distinct fuzzy points x_λ and y_μ:

(Case 1) When $x \neq y$ either x_λ has an open nbd which is not q-conincident with y_μ or y_μ has an open nbd which is not q-coincident with x_λ.

(Case 2) When $x = y$ and $\lambda < \mu$(say), then there exists a q-nbd V of y_μ which is not q-coincident with x_λ.

(b) $\mathcal{T}_1$, if for any two distinct fuzzy points x_λ and y_μ:

(Case 1) When $x \neq y$, x_λ has an open nbd which is not q-coincident with y_μ and y_μ has an open nbd which is not q-coincident with x_λ.

(Case 2) When $x = y$, and $\lambda < \mu$(say), then there exists a q-nbd V of y_μ such that $x_\lambda \bar{q} V$.

Result 1.E[2] A fts $(X, \mathcal{T})$ is $\mathcal{T}_1$ if and only if every singlton set is closed in X.

2. Definition of a fuzzy Vietories topology and fundamental properties

Notation. Let $(X, \mathcal{T})$ be a fts. Then:

(a) $I_0^X = \{E : E$ is nonempty and closed in $X\}$.

(b) $I_0^A = \{E \in I_0^X : E \subset A\}$, where $A \in I^X$.

From Notation, Result 1.B and Result 1.D, we obtain the following result :

Proposition 2.1 Let $(X, \mathcal{T})$ be a fuzzy $\mathcal{T}_1$-space. Then:

(a) $I_0^{A_0 \cap A_1} = I_0^{A_0} \cap I_0^{A_1}$ and generally $I_0^{\cap_\alpha A_\alpha} = \cap_\alpha I_0^{A_\alpha}$, where A_0, A_1, $A_\alpha \in I^X$.

(b) $A \subset B$ if and only if $I_0^A \subset I_0^B$ and hence $A=B$ if and only if $I_0^A = I_0^B$, where A, $B \in I^X$.

Proof. (a) $E \in I_0^{A_0 \cap A_1} \Leftrightarrow E \in I_0^X$ such that $E \subset A_0 \cap A_1$.

$\Leftrightarrow E \in I_0^X$ such that $E(x) \leq (A_0 \cap A_1)(x) = \min [A_0(x), A_1(x)]$, $\forall x \in X$.

$\Leftrightarrow E \in I_0^X$ such that $E(x) \leq A_0(x)$ and $E(x) \leq A_1(x)$, $\forall x \in X$.

$\Leftrightarrow E \in I_0^X$ such that $E \subset A_0$ and $E \subset A_1$.

$\Leftrightarrow E \in I_0^{A_0} \cap I_0^{A_1}$.

Now let $(A_\alpha)_{\alpha \in \Lambda}$ be a subfamily of I^X. Then: $E \in I_0^{\cap_{\alpha \in \Lambda} A_\alpha} \Leftrightarrow E \in I_0^X$ such that $E \subset \bigcap_{\alpha \in \Lambda} A_\alpha$.

$\Leftrightarrow E \in I_0^X$ such that $E(x) \leq (\bigcap_{\alpha \in \Lambda} A_\alpha)(x) = \inf_{\alpha \in \Lambda} A_\alpha(x)$, $\forall x \in X$.

$\Leftrightarrow E \in I_0^X$ such that $E \leq A_\alpha(x)$, $\forall x \in X$, $\alpha \in \Lambda$.

$\Leftrightarrow E \in I_0^X$ such that $E \subset A_\alpha$ for each $\alpha \in \Lambda$.

$\Leftrightarrow E \in I_0^{A_\alpha}$ for each $\alpha \in \Lambda$.

$\Leftrightarrow E \in \bigcap_{\alpha \in \Lambda} I_0^{A_\alpha}$.

(b)($\Rightarrow$): The necessary condition is obvious from Notation.

($\Leftarrow$): Suppose $I_0^A \subset I_0^B$. Let $x_\lambda \in A$. Since X is $\mathcal{T}_1$, by Result 1.E, $\{x_\lambda\} \in I_0^X$ and $\{x_\lambda\} \subset A$. Thus $\{x_\lambda\} \in I_0^A$. By the hypothesis, $\{x_\lambda\} \in I_0^B$ and thus $\{x_\lambda\} \subset B$. So $x_\lambda \in B$. Hence, by Result 1.B, $A \subset B$. ■

From Notation and Result 1.A, the following result is obvious:

Proposition 2.2 Let $(X, \mathcal{T})$ be a fts and let $A \in I^X$. Then

$$I_0^X - I_0^{A^c} = \{E \in I_0^X : EqA\}.$$

Lemma 2.3 Let $(X, \mathcal{T})$ be a fts and let $\mathfrak{Z}$ be a collection of all sets I_0^G and of all sets $I_0^X - I_0^{G^c}$, where $G \in \mathcal{T}$ Let $\mathcal{B}_e$ be the collection of all finite intersections of members of $\mathfrak{Z}$. Then for each $\mathbf{B} \in \mathcal{B}_e$,

$$\mathbf{B} = \{E \in I_0^X : E \subset A_0 \text{ and } EqA_i \text{ for each } i = 1, \cdots, n\},$$

where $A_i \in \mathcal{T}$ for each $i = 0, 1, \cdots, n$. In this case, $\mathbf{B}$ will be denoted as $<A_0, A_1, \cdots, A_n>_e$.

Proof. Let $\mathbf{B} \in \mathcal{B}_e$. Then there exist fuzzy open sets $A_0, A_1, \cdots, A_n$ in X such that $\mathbf{B} = I_0^{A_0} \cap (I_0^X - I_0^{A_1^c}) \cap \cdots \cap (I_0^X - I_0^{A_n^c})$. By Proposition 2.2, $\mathbf{B} = \{E \in I_0^X : E \subset A_0\} \cap \{E \in I_0^X : EqA_1\} \cap \cdots \cap \{E \in I_0^X : EqA_n\}$. Hence $\mathbf{B} = \{E \in I_0^X : E \subset A_0$ and EqA_i for each $i = 1, \cdots, n\}$.

From Lemma 2.3, we can easily obtain the following result:

Theorem 2.4 Let $(X, \mathcal{T})$ be a fts and let $\mathfrak{Z}$ be a collection of all sets I_0^G and of all set $I_0^X - I_0^{G^c}$, where $G \in \mathcal{T}$. Then there is a unique fuzzy topology $\mathcal{T}_e$(called the *fuzzy exponential topology*) on I_0^X such that $\mathfrak{Z}$ is a subbase for $\mathcal{T}_e$. In fact, $\mathcal{B}_e$ is a base for $\mathcal{T}_e$.

Definition 2.5 Let $(X, \mathcal{T})$ be a fts, Then the *fuzzy Vietories*(or *finite*)topology $\mathcal{T}_v$ on I_0^X is the generated by the collection of the forms $< U_1, \cdots, U_n >_v$ with $U_1, \cdots, U_n$ fuzzy open sets in X, where $< U_1, \cdots, U_n >_v = \{E \in I_0^X : E \subset \bigcup_{i=1}^n U_i$ and EqU_i for each $i = 1, \cdots, n\}$. The pair $(I_0^X, \mathcal{T}_v)$ is called a *fuzzy hyperspace with fuzzy Vietories topology*(*fuzzy hyperspace*, in short).

Theorem 2.6 The collection k $\mathcal{B}_v$ of the forms $< U_1, \cdots, U_n >_v$ with $< U_1, \cdots, U_n$ fuzzy open sets in X, forms a base for $\mathcal{T}_v$.

Proof. Since $I_0^X = < X >$ and $< X > \in \mathcal{B}_v$, $I_0^X = \bigcup \mathcal{B}_v$. Let $\mathcal{U} = <U_1, \cdots, U_n>_v$, $\mathcal{V} = <V_1, \cdots, V_m>_v$, $U = \bigcup_{i=1}^n U_i$ and $V = \bigcup_{j=1}^m U_j$. then clearly, U, $V \in \mathcal{T}$ Consider $<U_1 \cap V, \cdots, U_n \cdots V, V_1 \cap U, \cdots, V_m \cap U>_v$. Let $E \in <U_1 \cap V, \cdots, U_n \cap V, V_1 \cap U, \cdots, V_m \cap U>_v$. Then $E \subset [\bigcup_{i=1}^n (U_i \cap V)] \cup [\bigcup_{j=1}^n (V_j \cap U)]$, $Eq(U_i \cap V)$ for each $i = 1, \cdots, n$ and $Eq(V_m \cap U)$ for each $j = 1, \cdots, m$. Thus $E \subset U \cap V$, i.e, $E \subset U$ and $E \subset V$, $E \subset U_i$, $E \subset V_j$ for each $i = 1, \cdots, n$, and $j = 1, \cdots, m$. So $E \in \mathcal{U} \cap \mathcal{V}$. This completes the proof. ■

Theorem 2.7 $\mathcal{B}_e$ and $\mathcal{B}_v$ are equivalent. Hence $\mathcal{T}_v = \mathcal{T}_e$.

Proof. Let $<G_0, G_1, \cdots, G_n>_e \in \mathcal{B}_e$ and let $E \in <G_0, G_1, \cdots, G_n>_e$. Then $E \subset G_0$ and EqG_i for each $i = 1, \cdots, n$. Let $A_i = G_0 \cup G_i$ for each $i = 1, \cdots, n$. Then clearly A_i is open in X for each $i = 1, \cdots, n$ and thus $E \in <A_1, \cdots, A_n>_v$.

Now let $F \in <A_1, \cdots, A_n>_v$. Then $F \subset \bigcup_{i=1}^n A_i$ and FqA_i for each $i = 1, \cdots, n$. Let $G_0 = \bigcup_{i=1}^n A_i$ and let $G_i = G_0 \cap A_i$ for each $i = 1, \cdots, n$. Then G_i is open in X for each $i = 1, \cdots, n$ and $F \in <G_0, G_1, \cdots, G_n>_e$. By similar argument, for each $<A_0, \cdots, A_n>_v \in \mathcal{B}_v$ and each $E \in <A_1, \cdots, A_n>_v$, there exists a$<G_0, \cdots, G_n>_e \in \mathcal{B}_e$ such that $E \in <G_0, G_1, \cdots, G_n>_e \subset <A_1, \cdots, A_n>_v$. Hence $\mathcal{B}_e$ and $\mathcal{B}_v$ are equivalent. ■

3. Further results

Lemma 3.1 Let $(X, \mathcal{T})$ be a fuzzy $\mathcal{T}_1$-space and let $A \in I^X$. Then:

(a) $\overline{I_0^A} \subset I_0^{\bar{A}}\}$. But if $A(x) < 1/2$ for each $x \in X$, then $\overline{I_0^A} \subset I_0^{\bar{A}}$.

(b) $\widehat{I_0^A} = I_0^{\mathring{A}}$.

Proof. (a) By Proposition 2.1(b), $I_0^A \subset I_0^{\bar{A}}$. But $I_0^X -$

$I_0^{\bar{A}}=I_0^X-I_0^{(\bar{A}^c)^c}=\{E\in I_0^X:Eq(\bar{A}^c)\}$ by Result 1.D and Proposition 2.2. Thus by Lemma 2.3 and Theorem 2.7, $I_0^X-I_0^{\bar{A}}$ is open in $(I_0^X, \mathcal{T}_v)$. So $I_0^{\bar{A}}$ is closed in $(I_0^X, \mathcal{T}_v)$, and thus $\overline{I_0^A}\subset I_0^{\bar{A}}$. Now let $E\in I_0^{\bar{A}}$, i.e, $E\subset\bar{A}$. Let $<G_1, \cdots, G_n>_v$ be any base member for $\mathcal{T}_v$ containing E. Then $E\subset\bigcup_{i=1}^n G_i$ and EqG_i for each $i=1,\cdots,n$. Thus there is an $x_i\in X$ such that $E(x_i)+G_i(x_i)>1$. Let $E(x_i)=v$ for each $i=1,\cdots,n$. Then $v_i+G_i(x_i)>1$ and thus $x_{i,\mu_i}qG_i$ for each $i=1,\cdots,n$. Since $x_{i,\mu_i}\in\bar{A}$, AqG_i, by Result 1.C(b). Thus there is $y_i\in X$ such that $A(y_i)+G_i(y_i)>1$ for each $i=1,\cdots,n$. Let $A(y_i)=\lambda_i$ for each $i=1,\cdots,n$ and let $F=\{y_{1,\lambda_1},\cdots,y_{n,\lambda_n}\}$. Then $F\in I_0^X$, $F\subset A$ and FqG_i for each $i=1,\cdots,n$. On the other hand, since $A(x)<1/2$ for each $x\in X$, $A(y_i)=\lambda_i<1/2$ for each $i=1,\cdots,n$. Thus $G_i(y_i)>1/2>\lambda_i$ for each $i=1,\cdots,n$. So $F\subset\bigcup_{i=1}^n G_i$, and thus $F\in I_0^A\cap<G_1,\cdots,G_n>_v\neq\emptyset$. Hence $E\in\overline{I_0^A}$. i.e, $I_0^{\bar{A}}\subset\overline{I_0^A}$. Therefore $\overline{I_0^A}=I_0^{\bar{A}}$.

(b) By Lemma 2.3 and Theorem 2.7, $I_0^{\mathring{A}}$ is open in $(I_0^X\ \mathcal{T}_v)$. By Proposition

2.1(b), $I_0^{\mathring{A}}\subset I_0^A$. So $I_0^{\mathring{A}}\subset\mathring{\widehat{I_0^A}}$. Now let $E\notin I_0^{\mathring{A}}$. Then by Result 1.D, $E\grave{A}\,(\overline{A^c})^c$. Thus by Result 1.A, $Eq\overline{A^c}$. So there is a $y\in X$ such that $E(y)+(\overline{A^c})(y)>1$. Let $(\overline{A^c})(y)=\mu$. Then $y_\mu\in(\overline{A^c})$ and $y_\mu qE$. Let $<G_0,\cdots,G_n>_e$ be a base member for $\mathcal{T}_e$ containing E. Then $E\subset G_0$ and EqG_i for each $i=1,\cdots,n$. Thus $y_\mu qG_0$. Since $y_v\in(\overline{A^c})$, by Result 1.C(b), A^cqG_0. So there is an $x\in X$ such that $A^c(x)+G_0(x)>1$. Let $G_0(x)=\lambda$. Then clearly $x_\lambda\in G_0$ and $x_\lambda qA^c$. Let $F=E\cup\{x_\lambda\}$. Then $F\in I_0^X$ and FqA^c and thus $F\in I_0^X-I_0^A$. Moreover $F\subset G_0$ and FqG_i for each $=1,\cdots,n$. Thus $F\in<G_0,\cdots,G_n>_e$, So $F\in<G_0,\cdots,G_n>_e\cap(I_0^X-I_0^A)\neq\emptyset$, and thus

$E\in\overline{I_0^X-I_0^A}$. Hence, $E\notin\mathring{\widehat{I_0^A}}$ i.e., $\mathring{\widehat{I_0^A}}\subset I^{\mathring{A}}$. Therefore $\mathring{\widehat{I_0^A}}=I^{\mathring{A}}$.

From Lemma 2.3, Theorem 2.7 and Lemma 3.1, we obtain the following result:

Lemma 3.2 Let $(X, \mathcal{T})$ be a fuzzy $\mathcal{T}_1$-space, and let $A\in \underline{I}^X$.

(a) I_0^A and $I_0^X-I_0^{A^c}$ are open in I_0^X if and only if A is open in X.

(b) If A is closed in X, then I_0^A and $I_0^X-I_0^{A^c}$ are closed in I_0^X.

(b') If I_0^A and $I_0^X-I_0^{A^c}$ are closed in I_0^X and $A(x)<1/2$ for each $x\in X$, then A is closed in X.

Theorem 3.3 Let $(X, \mathcal{T})$ be a fuzzy $\mathcal{T}_3$-space and let $A\in I^X$. Then the set $\{E\in I_0^X: A\subset E\}$ is closed in $(I_0^X, \mathcal{T}_v)$.

Proof. Let $\mathcal{A}=\{E\in I_0^X: A\subset E\}$. Then:

$\mathcal{A}^c=\{E\in I_0^X: A\not\subset E\}$
$=\bigcup_{x_\lambda\in A}\{E\in I_0^X: E\subset\{x_\lambda\}^c\}$
$=\bigcup_{x_\lambda\in A}I_0^{\{x_\lambda\}^c}$.

Since X is $\mathcal{T}_1$, by Result 1.E, $\{x_\lambda\}$ is closed in $(X, \mathcal{T})$ for each $x_\lambda\in F_p(X)$. Thus $\{x_\lambda\}^c$ is open in $(X, \mathcal{T})$. So by Lemma 3.2(a), $I_0^{\{x_\lambda\}^c}$ is open in $(I_0^X, \mathcal{T}_v)$ and thus $\mathcal{A}^c$ is open in $(I_0^X, \mathcal{T}_v)$. Hence $\mathcal{A}$ is closed in (I_0^X, T_v) ■

Theorem 3.4 Let $(X, \mathcal{T})$ be a fts. Then:

(a) $(I_0^X, \mathcal{T}_v)$ is always $\mathcal{T}_0$.

(b) If X is $\mathcal{T}_1$, then I_0^X is $\mathcal{T}_1$. But the converse is false.

Proof. (a) Let A, $B\in I_0^X$ such that $A\neq B$. Let $x_\lambda\in A$, $x_\lambda\notin B$ and $U=B^c$. Then U is open in X, AqU, AqX, $A\subset U\cup X$ and $\bar{B}qU$. Thus $A\in<U, X>_v$ and $B\in<U, X>_v$. Hence $(I_0^X, \mathcal{T}_v)$ is $\mathcal{T}_0$.

(b) Let $\mathcal{K}\in I_0^X$. Then :

$\{\mathcal{K}\}=\{E\in I_0^X: E=\mathcal{K}\}$
$=\{E\in I_0^X: E\subset\mathcal{K}\}\cap\{E\in I_0^X: \mathcal{K}\subset E\}$.

Thus, by Lemma 3.2 and Theorem 3.3, $\{\mathcal{K}\}$ is closed in I_0^X. Hence I_0^X is $\mathcal{T}_1$.

Example 3.5 Let X be a finite set containing more than two points. Let the topology $\mathcal{T}$ on X be the fuzzy trivial topology. Then $I_0^X=\{X\}$. So I_0^X is $\mathcal{T}_1$. But $(X, \mathcal{T})$ is not $\mathcal{T}_1$.

Definition 3.6 A fuzzy set A in a fts X is said to be *dense* in X if $\bar{A}=X$. In particular, A is said to be *countably dense* in X if A is dense in X and $S(A)$ is countable. If X has a fuzzy countable dense set, we say that X is *fuzzy separable.*

Theorem 3.7 Let $\mathcal{F}(X)$ be the family of all the fuzzy finite sets in a fuzzy $\mathcal{T}_1$-space X. Then $\mathcal{F}(X)$ is dense in $(I_0^X, \mathcal{T}_v)$.

Proof. Let $E\in I_0^X$ and let $<G_1,\cdots,G_n>_v$ be any base member for $\mathcal{T}_v$ such that $E\in<G_1,\cdots,G_n>_v$. Then $E\cup_{i=1}^n G_i$ and EqG_i for each $i=1,\cdots,n$. Let x_i, $\lambda_i\in E$ and $\lambda_i+G_i(x_i)>1$ for each $i=1,\cdots,n$. Let $F=\{x_{1,\lambda_1},\cdots,x_{n,\lambda_n}\}$. Then clearly $F\in\mathcal{F}(X)\cap<G_1,\cdots,G_n>_v\neq\emptyset$ and thus $E\in\overline{\mathcal{F}(X)}$, i.e., $I_0^X\subset\overline{\mathcal{F}(X)}$. So $\overline{\mathcal{F}(X)}=I_0^X$. Hence $\mathcal{F}(X)$ is dense in I_0^X.

Theorem 3.8 X is fuzzy separable if and only if I_0^X is fuzzy separable.

Proof. $(\Longrightarrow)$: Suppose X is fuzzy separable. Let D be a fuzzy countable dense set in X and let $\mathcal{D}$ be the collection of finite subsets of D. Then clearly, $\mathcal{D}$ is countable. Let $<G_1, \cdots, G_n>_v$ be a base member for $\mathcal{T}_v$. Since D is dense in X, by Result 1.C(b), DqG_i for each $i=1, \cdots, n$. Let $x_{i,\lambda i} \in D$ and $\lambda_i + G_i(x_i) > 1$ for each $i=1, \cdots, n$. Let $E=\{x_{1,\lambda 1}, \cdots, x_{n,\lambda n}\}$. Then clearly, $E \in \mathcal{D} \cap <G_1, \cdots, G_n>_v$. Thus $\mathcal{D}$ is countable dense in I_0^X. Hence I_0^X is fuzzy separable.

($\Longleftarrow$): Suppose I_0^X is fuzzy separable. Let $\mathcal{D} = \{A_n : n \in \mathbf{Z}^+\}$ be a countable dense subset of I_0^X. For each $A_n \in \mathcal{D}$, choose a fuzzy point $a_{n,\lambda n} \in A_n$ and let $D = a_{n,\lambda n} : n \in \mathbf{Z}^+\}$. Now let U be a fuzzy open set in X. Then $<U>_v$ open in I_0^X. Thus there is and $A_n \in \mathcal{D} < U >_{\underline{v}}$. So $A_n \subset U$ and $A_n q U$. And thus $a_{n,\lambda n} q U$ and UqD. So $\overline{D} = X$. Hence X is fuzzy separable

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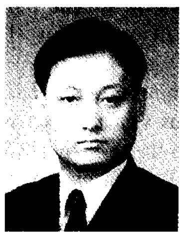

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