The Korean Communications
in Statistics Vol. 7, No. 2, 2000
pp. 563-573

Nonparametric Test for Ordered Alternatives on Multiple Ranked-Set Samples

Dong Hee Kim[1], Hyun Gee Kim[2] and Hae Kyung Park[3]

Abstract

In this thesis, we propose the test statistic for ordered alternatives on $c-$sample ranked set samples(RSS). The proposed test statistic J_{RSS} is Jonckheere type statistic using the median of the $i-$th samples in each cycle. We obtained the asymptotic property of the proposed test statistic and the asymptotic relative efficiencies of the proposed test statistic with respect to J_{SRS} which Jonckheere type statistic on simple random samples(SRS). From the simulation works, J_{RSS} is superior to J_{SRS}. We compared the empirical powers of J_{RSS} with respect to U_{RSS} on ranked set sample and U_{SRS} on simple random sample using all samples, which are proposed by Kim, Kim and Lee(1999). The powers of J_{RSS} and U_{RSS} are nearly the same values when entire sample size is large. J_{RSS} is superior to U_{RSS}. J_{RSS} is simpler than U_{RSS} on calculating process.

Keywords : Ranked-set samples, Ordered alternatives, Asymptotic relative efficiency

1. Introduction

Ranked set sampling is a widely useful sampling method for situations in which the variable of interest is much easier7 to order than to measure. It is useful when the measurements are destructive or too expensive in data collection.

The first application of the RSS procedure was made by McIntyre(1952), in relation to estimating pasture yields. Dell and Clutter(1972) investigated the effect of imperfect judgement rankings on estimation of the population mean. Stokes(1980) studied the ranked set sample with concomitant variable. Stokes and Sager(1988) first considered nonparametric inference for the ranked set sample. They studied the properties of the empirical distribution function. Bohn

1) Professor, Department of Statistics, (Research Institute of Computer, Information and Communication), Pusan National University, Pusan, 609-735, Korea.
E-mail : heekim@hyowon.cc.pusan.ac.kr

2) Department of Statistics, Pusan National University, Pusan, 609-735, Korea

3) Department of Statistics, Pusan National University, Pusan, 609-735, Korea

and Wolfe(1992, 1994) considered perfect and imperfect ranking of two-sampling procedures using Mann-Whitney-Wilcoxon statistics. Hettmansperger(1995), Koti and Babu(1996) studied the sign test for ranked set sample. Kim, Kim and Lee(1999) studied nonparametric test for ordered alternatives on multiple ranked-set samples, but their test statistic is complicate to calculate, because it uses all samples. Kim, Kim and Kim(1999) studied Page type test for ordered alternatives on multiple ranked set samples. They considered the statistic using the median of each cycle. Their statistic is simple to calculate.

In this thesis, we consider c-sample nonparametric testing problem for the ordered alternatives using Jonckheere(1954) type statistic on ranked set samples. This statistic uses the median of the r-th order statistics in cycles. So this test is more convenient than test using all samples. We obtain the asymptotic relative efficiencies of the proposed test statistic with respect to Jonckheere type statistic on simple random sample using the median of r-th order statistics in cycles.

From the simulation work, we compare the empirical powers of the proposed test statistic with Jonckheere type statistic on SRS. And we compare the empirical powers of the proposed test statistic with respect to Mann-Whitney type U-statistic on RSS and on SRS using all samples, respectively, which are proposed by Kim, Kim and Lee(1999). The underlying distributions which we consider are uniform, normal, double exponential, logistic and Cauchy distributions.

In Chapter 2, we propose the test statistic. In Chapter 3, we deal with asymptotic properties of the proposed test statistic. Simulation design and results under underlying distributions are given in Chapter 4. Finally, Chapter 5 provides some conclusions.

2. The proposed test statistic

Let $X_{j(1)1}, \cdots, X_{j(1)n_j}, \cdots, X_{j(k_j)1}, \cdots, X_{j(k_j)n_j}$ be a ranked-set sample of size $n_j k_j$ from a continuous distribution with cdf $F_j(x) = F(x-\theta_j)$ and pdf $f_j(x) = f(x-\theta_j)$, $j = 1, \cdots$ where θ_j is a location parameter of the j-th population, and let k_j and n_j be the sample size and the cycle size, respectively. We get independent $n_j k_j$ observations from $n_j k_j^2$ preranking sample observations.

Let $\widetilde{X}_{j(r)}$ be a median of $\{X_{j(r)1}, \cdots, X_{j(r)n_j}\}$, $r = 1, \cdots, k_j$, $j = 1, \cdots$ i.e. $\widetilde{X}_{j(r)}$ is a median of the r-th order statistics in cycles. The sample structure is on Figure 2.1. We can assume that $\widetilde{X}_{j(r)}$ has a continuous distribution with cdf $F_{(m_j:r)}(x)$ and pdf $f_{(m_j:r)}(x)$, $m_j = (n_j+1)/2$, n_j is odd. i.e.

$$F_{(m_j:r)}(x) = \sum_{u=m_j}^{n_j} \binom{n_j}{u} [F_{(r)}(x)]^{u} [1-F_{(r)}(x)]^{(n_j-u)},$$

$$f_{(m_j:r)}(x) = \frac{n_j!}{(m_{j-1})!(n_j-m_j)!}[F_{(r)}(x)]^{(m_j-1)}[1-F_{(r)}(x)]^{(n_j-m_j)} f_{(r)}(x),$$

where $F_{(r)}(x)$ is a cdf of the r-th order statistic from a distribution $F(x)$ and $f_{(r)}(x)$ is its pdf.

$$\begin{array}{cccccc}
\text{population} & \text{cycle} & & \text{pdf} & & \\
1 & \begin{pmatrix} 1 & 2 & \cdots & n_1 \\ X_{1(1)1} & X_{1(1)2} & \cdots & X_{1(1)n_1} \\ X_{1(2)1} & X_{1(2)2} & \cdots & X_{1(2)n_1} \\ \vdots & \vdots & \vdots & \vdots \\ X_{1(k_1)1} & X_{1(k_1)2} & \cdots & X_{1(k_1)n_1} \end{pmatrix} & \begin{matrix} \\ \sim \\ \sim \\ \vdots \\ \sim \end{matrix} & \begin{matrix} \\ f_{(1)} \\ f_{(2)} \\ \vdots \\ f_{(k_1)} \end{matrix} & \begin{matrix} \\ \rightarrow \\ \rightarrow \\ \vdots \\ \rightarrow \end{matrix} & \begin{matrix} \\ \hat{X}_{1(1)} \\ \hat{X}_{1(2)} \\ \vdots \\ \hat{X}_{1(k_1)} \end{matrix} \\
2 & \begin{pmatrix} 1 & 2 & \cdots & n_2 \\ X_{2(1)1} & X_{2(1)2} & \cdots & X_{2(1)n_2} \\ X_{2(2)1} & X_{2(2)2} & \cdots & X_{2(2)n_2} \\ \vdots & \vdots & \vdots & \vdots \\ X_{2(k_2)1} & X_{2(k_2)2} & \cdots & X_{2(k_2)n_2} \end{pmatrix} & \begin{matrix} \\ \sim \\ \sim \\ \vdots \\ \sim \end{matrix} & \begin{matrix} \\ f_{(1)} \\ f_{(2)} \\ \vdots \\ f_{(k_2)} \end{matrix} & \begin{matrix} \\ \rightarrow \\ \rightarrow \\ \vdots \\ \rightarrow \end{matrix} & \begin{matrix} \\ \hat{X}_{2(1)} \\ \hat{X}_{2(2)} \\ \vdots \\ \hat{X}_{2(k_2)} \end{matrix} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
c & \begin{pmatrix} 1 & 2 & \cdots & n_c \\ X_{c(1)1} & X_{c(1)2} & \cdots & X_{c(1)n_c} \\ X_{c(2)1} & X_{c(2)2} & \cdots & X_{c(2)n_c} \\ \vdots & \vdots & \vdots & \vdots \\ X_{c(k_c)1} & X_{c(k_c)2} & \cdots & X_{c(k_c)n_c} \end{pmatrix} & \begin{matrix} \\ \sim \\ \sim \\ \vdots \\ \sim \end{matrix} & \begin{matrix} \\ f_{(1)} \\ f_{(2)} \\ \vdots \\ f_{(k_c)} \end{matrix} & \begin{matrix} \\ \rightarrow \\ \rightarrow \\ \vdots \\ \rightarrow \end{matrix} & \begin{matrix} \\ \hat{X}_{c(1)} \\ \hat{X}_{c(2)} \\ \vdots \\ \hat{X}_{c(k_c)} \end{matrix}
\end{array}$$

Figure 2.1 Multiple Ranked Set Sample Structure.

We consider the testing problem for testing $H_0 : \theta_1 = \cdots = \theta_c (= \theta_0)$ against $H_1 : \theta_1 \le \cdots \le \theta_c$ with at least one strict inequality.

Our proposed test statistic based on RSS is

$$J_{RSS} = \sum_{j=1}^{c-1} \sum_{j'=j+1}^{c} \sum_{r=1}^{k_j} \sum_{r'=1}^{k_{j'}} \Psi(\hat{X}_{j'(r')} - \hat{X}_{j(r)})$$

where $\Psi(t) = 1, 0$ as $t > 0$, ≤ 0.

The proposed test statistic J_{RSS} uses a median of r-th order statistics in cycles. So this test statistic is simpler than the test statistic using all samples on RSS for $c-$sample ordered alternatives. Under the ordered alternatives H_1, the test statistic J_{RSS} tends to have large values, so we reject H_0 for large values of J_{RSS}.

3. Asymptotic property

To obtain the asymptotic properties of the proposed statistic J_{RSS}, we calculate the mean and the variance under H_0. When cycle size is $n_j = n$ and sample size is $k_j = k$, $j = 1, \cdots, c$ and where specially n and k are odd, $m = (n+1)/2$ the mean of the test statistic J_{RSS} is

$$\begin{aligned} E_0(J_{RSS}) &= \sum_{j=1}^{c-1} \sum_{j'=j+1}^{c} \sum_{r=1}^{k} \sum_{r'=1}^{k} P(\tilde{X}_{j'(r')} > \tilde{X}_{j(r)}) \\ &= \sum_{j=1}^{c-1} \sum_{j'=j+1}^{c} \sum_{r=1}^{k} \sum_{r'=1}^{k} \int_{-\infty}^{\infty} F_{(m:r)}(x)\, dF_{(m:r')}(x) \\ &= \frac{k^2 c(c-1)}{2} \end{aligned}$$

where $F_{(m:r)}(x) = \sum_{u=m}^{n} \binom{n}{u} [F_{(r)}(x)]^u [1 - F_{(r)}(x)]^{(n-u)} F_{(r)}(x)$ is cdf of the $r-$th order statistic from a distribution with $F(x)$.

The variance for the test statistic J_{RSS} is given by

$$\begin{aligned} &Var_0(J_{RSS}) \\ &= \sum_{j=1}^{c-1} \sum_{j'=j+1}^{c} \Big[\sum_{r=1}^{k} \sum_{r'=1}^{k} [E[\Psi(\tilde{X}_{j'(r')} - \tilde{X}_{j(r)})]^2 - [E[\Psi(\tilde{X}_{j'(r')} - \tilde{X}_{j(r)})]]^2] \\ &\quad + \sum_{r_1 \neq r_2}^{k} \sum_{r'=1}^{k} [E[\Psi(\tilde{X}_{j'(r')} - \tilde{X}_{j(r_1)}) \Psi(\tilde{X}_{j'(r')} - \tilde{X}_{j(r_2)})] \\ &\qquad - E[\Psi(\tilde{X}_{j'(r')} - \tilde{X}_{j(r_1)})] E[\Psi(\tilde{X}_{j'(r')} - \tilde{X}_{j(r_2)})]] \\ &\quad + \sum_{r=1}^{k} \sum_{r_1 \neq r_2}^{k} [E[\Psi(\tilde{X}_{j'(r_1)} - \tilde{X}_{j(r)}) \Psi(\tilde{X}_{j'(r_2)} - \tilde{X}_{j(r)})] \\ &\qquad - E[\Psi(\tilde{X}_{j'(r_1)} - \tilde{X}_{j(r)})] E[\Psi(\tilde{X}_{j'(r_2)} - \tilde{X}_{j(r)})]] \Big] \end{aligned}$$

$$= \frac{c(c-1)}{36} \left[9k^2 + 4k^3(c+1) + 18 \sum_{r=1}^{k} \sum_{r'=1}^{k} \left[\int_{-\infty}^{\infty} F_{(m:r)}(x)\, dF_{(m:r')}(x) \right]^2 - 36 \sum_{r=1}^{k} \sum_{r'=1}^{k} \int_{-\infty}^{\infty} F_{(m:r)}(x)^2\, dF_{(m:r')}(x) - 12(c+1) \sum_{r_1 \neq r_2} \sum_{r'=1}^{k} \int_{-\infty}^{\infty} F_{(m:r_1)}(x)\, dF_{(m:r')}(x) \int_{-\infty}^{\infty} F_{(m:r_2)}(x)\, dF_{(m:r')}(x) \right],$$

For the detailed derivation, see Appendix.

Since the explicit form of the variance is somewhat complicated to calculate, we have to use the computer.

Table 3.1 Variance of J_{RSS} for some specified c, k, n.

c	k	n	variance	c	k	n	variance
3	3	3	6.673	5	3	3	31.570
3	3	5	5.020	5	3	5	23.200
3	5	3	17.768	5	5	3	83.997
3	5	5	12.511	5	5	5	58.547

The asymptotic normality of the proposed test statistic can be obtained by the central limit theorem.

Theorem 3.1 Under $H_0: \theta_1 = \cdots = \theta_c (= \theta_0)$, the sample sizes in each cycle are all equal (i.e. $k_j = k$, $j = 1, \cdots$), the limiting ($k \to \infty$) null distribution of

$$\frac{J_{RSS} - E_0(J_{RSS})}{\sqrt{Var_0(J_{RSS})}}$$

is standard normal.

To obtain the asymptotic relative efficiency, we consider a sequence of translation alternatives of the form, $H_{1k}: \theta_j = \theta_0 + j\triangle$, $j = 1, \cdots$ where $\triangle = \theta/\sqrt{k}$. We compare the asymptotic relative efficiencies of the our proposed test statistic with respect to the J_{SRS} based on SRS. Under the sequence of translation alternatives, the efficacy of the J_{RSS} is obtained as follows.

$$E_{\triangle}(J_{RSS}) = \sum_{j=1}^{c-1} \sum_{j'=j+1}^{c} \sum_{r=1}^{k} \sum_{r'=1}^{k} \int_{-\infty}^{\infty} F_{(m:r)}(t + (j' - j)\triangle) f_{(m:r')}(t)\, dt$$

The derivative of $E_{\triangle}(J_{RSS})$ evaluated at $\triangle=0$ is

$$\begin{aligned}
&\frac{\partial}{\partial\triangle}E_{\triangle}(J_{RSS})|_{\triangle=0}\\
&=\sum_{j=1}^{c-1}\sum_{j'=j+1}^{c}(j'-j)\sum_{r=1}^{k}\sum_{r'=1}^{k}\int_{-\infty}^{\infty}f_{(m:r)}(t+(j'-j)\triangle)f_{(m:r')}(t)dt|_{\triangle=0}\\
&=\sum_{j=1}^{c-1}\sum_{j'=j+1}^{c}(j'-j)\sum_{r=1}^{k}\sum_{r'=1}^{k}\int_{-\infty}^{\infty}\left(\frac{n!}{(m-1)!(n-m)!}\right)^2\frac{k!}{(r-1)!(k-r)!}\\
&\quad\times\frac{k!}{(r'-1)!(k-r')!}[F_{(r)}(t)]^{(m-1)}[1-F_{(r)}(t)]^{(n-m)}[F_{(r')}(t)]^{(m-1)}\\
&\quad\times[1-F_{(r')}(t)]^{(n-m)}[F(t)]^{(r+r'-2)}[1-F(t)]^{(2k-r-r')}f^2(t)dt.
\end{aligned}$$

By the definition 5.2.14 of Randles and Wolfe(1979), we have the following efficacy of J_{RSS},

$$eff(J_{RSS})=\lim_{k\to\infty}\frac{\frac{\partial}{\partial\triangle}E_{\triangle}(J_{RSS})|_{\triangle=0}}{\sqrt{kVar_{0(J_{RSS})}}}\quad.$$

Let J_{SRS} be the statistic based on SRS, which has the same structure of J_{SRS}.

$$J_{SRS}=\sum_{j=1}^{c-1}\sum_{j'=j+1}^{c}\sum_{r=1}^{k}\sum_{r'=1}^{k}\Psi(\tilde{X}_{j'r'}-\tilde{X}_{jr})$$

$$\begin{aligned}
\frac{\partial}{\partial\triangle}E_{\triangle}(J_{SRS})|_{\triangle=0}=\sum_{j=1}^{c-1}\sum_{j'=j+1}^{c}(j'-j)k^2\int_{-\infty}^{\infty}\left(\frac{n!}{(m-1)!(n-m)!}\right)^2\\
\times[F(t)]^{2(m-1)}[1-F(t)]^{2(n-m)}f^2(t)dt.
\end{aligned}$$

The efficacy of J_{SRS} is given by

$$eff(J_{SRS})=\lim_{k\to\infty}\frac{\frac{\partial}{\partial\triangle}E_{\triangle}(J_{SRS})|_{\triangle=0}}{\sqrt{kVar_0(J_{SRS})}}\quad,$$

where $Var_0(J_{SRS})=ck^2(c-1)(2kc+2k+3)/72$.

Then, the ARE of J_{RSS} with respect to J_{SRS} is

$$ARE(J_{RSS},J_{SRS})=\frac{eff^2(J_{RSS})}{eff^2(J_{SRS})}\quad.$$

In the special case of $c=3$, $n=3$, $k=3$, when the underlying distribution is uniform(0,1), we can obtain $ARE(J_{RSS}, J_{SRS}) = 2.345$.

4. Power Comparison

This simulation is designed to compare the powers of J_{RSS}, J_{SRS}, U_{RSS} and U_{SRS}.The statistic U_{RSS}, proposed by Kim, Kim and Lee(1999) is of the form

$$\begin{aligned} U_{RSS} &= U(X_{11}, \cdots, X_{1n_1}; \cdots; X_{c1}, \cdots, X_{cn_c}) \\ &= \frac{1}{n_1 \cdots n_c} \sum_{\beta_1=1}^{n_1} \cdots \sum_{\beta_c=1}^{n_c} \nu(X_{1\beta_1}; \cdots; X_{c\beta_c}) \\ &= \sum_{j=1}^{c-1} \sum_{j'=j+1}^{c} \frac{\nu_{jj'}}{n_j n_{j'}} \end{aligned}$$

where $\nu_{jj'} = \sum_{r=1}^{k_j} \sum_{i=1}^{n_j} \sum_{r'=1}^{k_{j'}} \sum_{i'=1}^{n_{j'}} \Psi(X_{j'(r')i'} - X_{j(r)i})$.

The statistic U_{SRS}, as a counterpart of U_{RSS} is of the form

$$U_{SRS} = \sum_{j=1}^{c-1} \sum_{j'=j+1}^{c} \frac{\nu_{jj'}}{n_j n_{j'}}$$

where $\nu_{jj'} = \sum_{r=1}^{k_j} \sum_{i=1}^{n_j} \sum_{r'=1}^{k_{j'}} \sum_{i'=1}^{n_{j'}} \Psi(X_{j'r'i'} - X_{jri})$.

The powers are obtained from the uniform(0, 1), normal(with variance 1), double exponential(with scale parameter 1), logistic(with scale parameter 1) and Cauchy(with scale parameter 1) distributions on which these simulations are based. We cosider the design with the sample size $k=3$,the cycle size $n=3$,the population size $c=3$,location parameter θhas 0.0(0.5)2.0 and $k=5$, $n=5$, $c=5$, θ has 0.0(0.1)0.5. The results of this simulation have 10,000 replications. When the number of times is divided by 10,000, it denotes the empirical power at $\theta = 0.0$ and is the empirical significance level.

The IMSL(International Mathematics and Statistical Library) procedures RNUN, RNNOA, and RNCHY are used to generate the required uniform, normal and Cauchy pseudo-random variates, respectively. And the double exponential and logistic random variates are generated by using the probability integral transformation.

From the results of simulation on Table 4.1 - Table 4.2, we know that J_{RSS} is superior to

J_{SRS}, because J_{RSS} has more information than J_{SRS} in sampling process. The powers of J_{RSS} and U_{RSS} have not great difference even though the powers of J_{RSS} are small. The difference of powers of J_{RSS} and U_{RSS} decrease as the entire sample size increases. In the case of $c=5$, $k=5$, $n=5$, the powers of J_{RSS} and U_{RSS} have nearly the same values.

Table 4.1 Empirical Powers of Tests (c = 3, k = 3 n = 3, α = 0.05)

distribution	statistic	$\theta=$ 0.0	0.5	1.0	1.5	2.0
Uniform	U_{RSS}	0.0546	0.2451	0.5912	0.8586	0.9741
	U_{SRS}	0.0528	0.1755	0.3800	0.6275	0.8332
	J_{SRS}	0.0589	0.1306	0.2465	0.4158	0.5718
	J_{RSS}	0.0579	0.1870	0.4204	0.6845	0.8666
Normal	U_{RSS}	0.0546	0.2479	0.6039	0.8811	0.9820
	U_{SRS}	0.0565	0.1753	0.4033	0.6528	0.8643
	J_{SRS}	0.0628	0.1534	0.3184	0.5314	0.7160
	J_{RSS}	0.0586	0.2180	0.4791	0.7433	0.9141
D·E	U_{RSS}	0.0495	0.3215	0.7291	0.9461	0.9925
	U_{SRS}	0.0491	0.2249	0.5070	0.7726	0.9246
	J_{SRS}	0.0598	0.2061	0.4493	0.6876	0.8509
	J_{RSS}	0.0593	0.2707	0.5943	0.8485	0.9563
Logistic	U_{RSS}	0.0538	0.2707	0.6397	0.9058	0.9866
	U_{SRS}	0.0568	0.1917	0.4312	0.7033	0.8849
	J_{SRS}	0.0585	0.1740	0.3487	0.5811	0.7698
	J_{RSS}	0.0582	0.2247	0.5282	0.7850	0.9307
Cauchy	U_{RSS}	0.0512	0.2621	0.5781	0.8243	0.9381
	U_{SRS}	0.0537	0.1768	0.3903	0.6065	0.7733
	J_{SRS}	0.0639	0.1887	0.3928	0.5810	0.7411
	J_{RSS}	0.0596	0.2222	0.4752	0.7008	0.8356

Table 4.2 Empirical Powers of Tests (c = 5, k = 5 n = 5, α = 0.05)

distribution	ststistic	θ = 0.0	0.1	0.2	0.3	0.4	0.5
Uniform	U_{RSS}	0.0374	0.1878	0.4932	0.7918	0.9529	0.9944
	U_{SRS}	0.0354	0.1006	0.2213	0.3852	0.5879	0.7449
	J_{SRS}	0.0483	0.0931	0.1585	0.2509	0.3596	0.4811
	J_{RSS}	0.0497	0.1605	0.3764	0.6299	0.8304	0.9473
Normal	U_{RSS}	0.0395	0.1710	0.4789	0.8008	0.9599	0.9951
	U_{SRS}	0.0339	0.0923	0.2120	0.3871	0.5934	0.7638
	J_{SRS}	0.0506	0.1097	0.2088	0.3387	0.4963	0.6608
	J_{RSS}	0.0500	0.1727	0.4024	0.6758	0.8709	0.9659
D · E	U_{RSS}	0.0354	0.2430	0.6464	0.9273	0.9941	0.9997
	U_{SRS}	0.0345	0.1218	0.2965	0.5254	0.7501	0.9067
	J_{SRS}	0.0499	0.1458	0.3165	0.5452	0.7503	0.8786
	J_{RSS}	0.0492	0.2137	0.5317	0.8141	0.9571	0.9930
Logistic	U_{RSS}	0.0347	0.1994	0.5324	0.8498	0.9742	0.9985
	U_{SRS}	0.0330	0.1042	0.2337	0.4235	0.6450	0.8216
	J_{SRS}	0.0490	0.1169	0.2266	0.3998	0.5743	0.7402
	J_{RSS}	0.0499	0.1867	0.4385	0.7280	0.8999	0.9791
Cauchy	U_{RSS}	0.0368	0.1837	0.4957	0.8115	0.9546	0.9926
	U_{SRS}	0.0369	0.0958	0.2212	0.3986	0.6006	0.7617
	J_{SRS}	0.0494	0.1302	0.2808	0.4731	0.6689	0.8070
	J_{RSS}	0.0498	0.1737	0.4194	0.6822	0.8659	0.9523

5. Conclusion

In this thesis we consider c-sample location problem for the ordered alternative using RSS method. The proposed test statistic J_{RSS} is simple because it uses a median of the r-th order statistics in cycles.

Through the ARE of J_{RSS} based on RSS with respect to J_{SRS} based on SRS and empirical powers of J_{RSS} and J_{SRS}, J_{RSS} is superior to J_{SRS}.

From the results of simulation, we know that J_{RSS} is superior to U_{SRS} using all samples in the case that the sample size in each cycle is greater than or equal to the size of cycle. J_{RSS} is slightly inferior to U_{RSS} using all samples. But the powers of J_{RSS} and U_{RSS} are nearly the same values when the entire sample size is large.

So we are able to use conveniently J_{RSS} because J_{RSS} is superior to U_{SRS} on empirical powers and is simpler than U_{RSS} on calculating process.

For further research, we can apply the idea in this thesis to testing the umbrella alternatives on RSS.

Appendix

A.1 Variance of J_{RSS}

$$
\begin{aligned}
Var_0(J_{RSS}) &= \sum_{j=1}^{c-1}\sum_{j'=j+1}^{c}\Big[\sum_{r=1}^{k}\sum_{r'=1}^{k}\big[E[\Psi(\tilde{X}_{j'(r')}-\tilde{X}_{j(r)})]^2-[E[\Psi(\tilde{X}_{j'(r')}-\tilde{X}_{j(r)})]]^2\big] \\
&\quad+\sum_{r_1\neq r_2}^{k}\sum_{r'=1}^{k}\big[E[\Psi(\tilde{X}_{j'(r')}-\tilde{X}_{j(r_1)})\Psi(\tilde{X}_{j'(r')}-\tilde{X}_{j(r_2)})] \\
&\qquad-E[\Psi(\tilde{X}_{j'(r')}-\tilde{X}_{j(r_1)})]E[\Psi(\tilde{X}_{j'(r')}-\tilde{X}_{j(r_2)})]\big] \\
&\quad+\sum_{r=1}^{k}\sum_{r_1\neq r_2}^{k}\big[E[\Psi(\tilde{X}_{j'(r_1)}-\tilde{X}_{j(r)})\Psi(\tilde{X}_{j'(r_2)}-\tilde{X}_{j(r)})] \\
&\qquad-E[\Psi(\tilde{X}_{j'(r_1)}-\tilde{X}_{j(r)})]E[\Psi(\tilde{X}_{j'(r_2)}-\tilde{X}_{j(r)})]\big]\Big] \\
&= \sum_{j=1}^{c-1}\sum_{j'=j+1}^{c}\Big[\sum_{r=1}^{k}\sum_{r'=1}^{k}\big[P(\tilde{X}_{j'(r')}>\tilde{X}_{j(r)})-[P(\tilde{X}_{j'(r')}>\tilde{X}_{j(r)})]^2\big] \\
&\quad+\sum_{r_1\neq r_2}^{k}\sum_{r'=1}^{k}\big[P[\tilde{X}_{j'(r')}>\max(\tilde{X}_{j(r_1)},\tilde{X}_{j(r_2)})] \\
&\qquad-P(\tilde{X}_{j'(r')}>\tilde{X}_{j(r_1)})P(\tilde{X}_{j'(r')}>\tilde{X}_{j(r_2)})\big] \\
&\quad+\sum_{r=1}^{k}\sum_{r_1\neq r_2}^{k}\big[P[\min(\tilde{X}_{j'(r_1)},\tilde{X}_{j'(r_2)})>\tilde{X}_{j(r)}] \\
&\qquad-P(\tilde{X}_{j'(r_1)}>\tilde{X}_{j(r)})P(\tilde{X}_{j'(r_2)}>\tilde{X}_{j(r)})\big]\Big] \\
&= \sum_{j=1}^{c-1}\sum_{j'=j+1}^{c}\Big[\sum_{r=1}^{k}\sum_{r'=1}^{k}\frac{1}{2}-\sum_{r=1}^{k}\sum_{r'=1}^{k}\Big[\int_{-\infty}^{\infty}F_{(m:r)}(x)\,dF_{(m:r')}(x)\Big]^2 \\
&\quad+\sum_{r_1\neq r_2}^{k}\sum_{r'=1}^{k}\int_{-\infty}^{\infty}F_{(m:r_1)}(x)F_{(m:r_2)}(x)\,dF_{(m:r')}(x) \\
&\quad-\sum_{r_1\neq r_2}^{k}\sum_{r'=1}^{k}\int_{-\infty}^{\infty}F_{(m:r_1)}(x)\,dF_{(m:r')}(x)\int_{-\infty}^{\infty}F_{(m:r_2)}(x)\,dF_{(m:r')}(x) \\
&\quad+\sum_{r=1}^{k}\sum_{r_1\neq r_2}^{k}\int_{-\infty}^{\infty}F_{(m:r_1')}(x)F_{(m:r_2')}(x)\,dF_{(m:r)}(x)
\end{aligned}
$$

$$- \sum_{r=1}^{k} \sum_{r_1 \neq r_2}^{k} \int_{-\infty}^{\infty} F_{(m:r_1)}(x)\, dF_{(m:r)}(x) \int_{-\infty}^{\infty} F_{(m:r_2)}(x)\, dF_{(m:r)}(x) \ ]$$

$$= \frac{c(c-1)}{36} [\ 9k^2 + 4k^3(c+1) + 18 \sum_{r=1}^{k} \sum_{r'=1}^{k} [\int_{-\infty}^{\infty} F_{(m:r)}(x)\, dF_{(m:r')}(x)]^2$$

$$-36 \sum_{r=1}^{k} \sum_{r'=1}^{k} \int_{-\infty}^{\infty} F_{(m:r)}(x)^2\, dF_{(m:r')}(x)$$

$$-12(c+1) \sum_{r_1 \neq r_2}^{k} \sum_{r'=1}^{k} \int_{-\infty}^{\infty} F_{(m:r_1)}(x)\, dF_{(m:r')}(x) \int_{-\infty}^{\infty} F_{(m:r_2)}(x)\, dF_{(m:r')}(x) \ ] \ .$$

References

[1] Bohn, L.L. and Wolfe, D.A.(1992). Nonparametric two-sample procedures for ranked-set samples data, *Journal of the American Statistical Association* **87(418)**, 552-561.

[2] Bohn, L.L. and Wolfe, D.A.(1994). The effect of imperfect judgement rankings on properties of procedures based on the ranked-set samples analog of Mann-Whitney-Wilcoxon statistic, *Journal of the American Statistical Association* **89(425)**, 168-176.

[3] Dell, T.R. and Clutter, J.L.(1972). Ranked-set sampling theory with order statistics background, *Biometrics*, **28**, 545-553.

[4] Hettmansperger, T.P.(1995). The ranked-set sample sign test, *Journal of the Nonparametric Statistics*, **4**, 271-282.

[5] Jonckheere, A.P.(194). A distribution-free k-sample test against ordered alternatives, *Biometrika*, **41**, 271-282.

[6] Kim, D.H., Kim, Y.C. and Kim, H.G.(1999). Page type test for ordered alternatives on multiple ranked set sample, *The Korean Communications in Statistics*, **6(2)**,479-486

[7] Kim, D.H., Kim, Y.C. and Lee, G.B.(1999). Nonparametric c-sample test for ordered alternatives on ranked-set samples, *Journal of the Korean Data Analysis,* **1(2)**,99-109

[8] Koti, M.K. and Babu, G.J.(1996). Sign test for ranked-set sample sign test, *Communications in Statistics - Theory and Methods*, **25(7)**, 1617-1630.

[9] McIntyre, G.A.(1952). A method of unbiased selective sampling using ranked sets, *Australian Journal of Agricultural Research*, **3**, 385-390.

[10] Randles, R.H. and Wolfe, D.A.(1979). *Introduction to the theory of nonparametric statistics,* New York : John Wiley.

[11] Stokes, S.L. and Sager, T.W.(1988). Characterization of a ranked-set sample with application to estimating distribution function, *Journal of the American Statistical Association*, **83**, 374-381.