The Korean Communications
in Statistics Vol. 7, No. 2, 2000
pp. 525-531

Comparative studies for Bayes Reliability Estimators of Standby System with Imperfect Switch

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Abstract

Bayes estimators for reliability of a two-unit hot standby system with the imperfect switch based upon a complete sample of failure times observed from exponential distributions under squared error loss and some priors for failure rates are proposed, and mean squared errors of proposed several Bayes estimators for the system reliability are compared numerically each other through the Monte Carlo simulation.

Keywords : Standby System, Imperfect Switch, Bayes estimator

1. Introduction

The two-unit standby system configuration is a form of paralleling where only one component is in operation ; if the operating component fails, then another component is brought into operation, and the system configuration continues to function. Depending on failure characteristics, standby redundancy is classified into three types. Hot standby system, where each component has the same failure rate regardless of whether it is standby or in operation ; Cold standby system, where components do not fail when they are in standby ; Warm standby system, where a standby component can fail but has a smaller failure rate than the principal component. Here we shall consider Bayesian estimations for reliability of a two unit hot standby system with imperfect switch under squared error loss function and some priors for failure rates.

Osaki & Nakagawa(1971 & 1975) computed the reliability for a two unit standby redundant system with constant failure rate and studied properties of a two unit standby redundant system with imperfect switch. Subramanian & Ravichandran(1978), Goel & Gupta(1984) and Veklerov(1987) considered the two unit hot standby redundant system with an imperfect switch. Fujii & Sandoh(1984) studied the Bayes reliability assessment for a two unit hot standby redundant system with an imperfect switch.

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In this paper, we shall propose several Bayes estimators for reliability of a two unit hot standby system with the imperfect switch based upon a complete sample of failure times observed from exponential distributions under squared error loss function using noninformative, uniform, gamma priors for failure rates, and proposed Bayes estimators for hot standby system reliability are numerically compared each other in the sense of mean squared error by the Monte Carlo method in the small sample size.

2. Bayes Estimators for Reliability

In life testing, the most widely exploited model is an exponential model with following density function ;

$$f(x \mid \lambda) = \lambda e^{-\lambda x}, \quad 0 < x < \infty,$$

where λ is the failure rate.

In the context of the lifetime of industrial equipments and components, the probability that a given system will function for a specified period of "mission" time t_0 under the certain conditions is known as a system reliability. Problems of estimation for a system reliability play an important role in many practical reliability analysis. Here we shall consider Bayesian estimations for the two unit hot standby system reliability with imperfect switch, which can be considered as analogous to an on-line banking system. In the two-unit hot standby system with imperfect switch, we shall assume the following :

1. The system consists of two independent and identically distributed units and a switch.
2. One unit serves as a hot standby when the other is in use.
3. The failure times of both units in use and standby are independent and exponentially distributed with the failure rate λ_u.
4. The switch is instantaneous when the one in use fails.
5. The failure time of the switch is exponentially distributed with the failure rate λ_s.
6. The unit and the switch are independent.

In the two unit standby system with the imperfect switch, the reliability is given by

$$R(t_0) = e^{-\lambda_u t_0}\left(1 + \frac{\lambda_u}{\lambda_s}(1 - e^{-\lambda_s t_0})\right), \quad t_0 > 0 . \tag{2.1}$$

Most Bayes estimators of reliability derived so far were based upon the priors on the unknown parameters which are related to the reliability than the reliability itself. Hence we shall consider noninformative, uniform and gamma prior distributions on the failure rates λ_u for unit and the failure rate λ_s for switch. Also we shall use the squared error loss function to estimate the system reliability.

Let $T_{u1}, \cdots, T_{un}$ be a simple random sample from an exponential distribution with a failure rate λ_u and $T_u = \sum_{k=1}^{n} T_{uk}$ be the total test time for the unit. And let $T_{s1}, \cdots, T_{sn}$ be a simple random sample from a exponential distribution with failure rate λ_s and $T_s = \sum_{k=1}^{n} T_{sk}$ be the total test time for the switch.

Bayesian analysis with noninformative prior is very common when little or no prior information is available. From the likelihood function and Fisher's information(see Martz & Waller(1982)), the noninformative prior distributions for the unit failure rate Λ_u and switch failure rate Λ_s are given by

$$g(\lambda_u) \propto \frac{1}{\lambda_u} \quad \text{and} \quad g(\lambda_s) \propto \frac{1}{\lambda_s},$$

respectively. And we suppose that Λ_u and Λ_s are independent.

Then according to Bayes theorem, the joint posterior distribution for failure rates Λ_u and Λ_s given total test times $T_u = t_u$ and $T_s = t_s$ is

$$g(\lambda_u, \lambda_s \mid t_u, t_s) = \frac{(t_u \cdot t_s)^n}{\Gamma^2(n)} (\lambda_u \lambda_s)^{n-1} e^{-(\lambda_u t_u + \lambda_s t_s)}, \quad 0 < \lambda_u, \lambda_s < \infty. \tag{2.2}$$

where $\Gamma(a)$ is the gamma function.

Since the Bayes estimator of the system reliability is found by taking the expectation of reliability function (2.1) with respect to the posterior distribution (2.2), the Bayes estimator $\hat{R}_{NN}(t_0)$ for the two unit hot standby system reliability $R(t_0)$ with imperfect switch under squared error loss function using noninformative prior for failure rates is given by

$$\widehat{R_{NN}}(t_0) = \left(\frac{T_u}{T_u + t_0}\right)^n \left[1 + \frac{n \cdot T_s}{(n-1)(T_u + t_0)} \left\{1 - \left(\frac{T_s}{T_s + t_0}\right)^{n-1}\right\}\right]. \tag{2.3}$$

Next, suppose that prior distributions on failure rates Λ_u and Λ_s are independently and uniformly distributed so that

$$g(\lambda_u) = \frac{1}{\alpha}, \quad 0 < \lambda_u < \alpha \quad \text{and} \quad g(\lambda_s) = \frac{1}{\beta,} \quad 0 < \lambda_s < \beta,$$

denoted by $\Lambda_u \sim UNIF(0, \alpha)$ and $\Lambda_s \sim UNIF(0, \beta)$, respectively.

Then, the joint posterior distribution for failure rates Λ_u and Λ_s given total test times $T_u = t_u$ and $T_s = t_s$ is

$$g(\lambda_u, \lambda_s \mid t_u, t_s) = \frac{(t_u \cdot t_s)^{n+1}}{\Gamma(n+1, \alpha t_u)\Gamma(n+1, \beta t_s)} (\lambda_u \lambda_s)^n e^{-(\lambda_u t_u + \lambda_s t_s)}, \quad 0 < \lambda_u < \alpha, \ 0 < \lambda_s < \beta, \tag{2.4}$$

where $\Gamma(a, z)$ is the incomplete gamma function.
Therefore, under the squared error loss function and uniform priors for failure rates, the Bayes estimator $\hat{R}_{UU}(t_0)$ for reliability $R(t_0)$ of the two unit hot standby system with imperfect switch is given by

$$\widehat{R_{UU}}(t_0) = \left(\frac{T_u}{T_u+t_0}\right)^{n+1}\Bigg[\frac{\Gamma(n+1,\alpha(T_u+t_o))}{\Gamma(n+1,\alpha T_u)} + \frac{\Gamma(n+2,\alpha(T_u+t_o))}{(T_u+t_0)\Gamma(n+1,\alpha T_u)\Gamma(n+1,\beta T_s)} \times\left\{T_s\cdot\Gamma(n,\beta T_s)-\left(\frac{T_s}{T_s+t_0}\right)^{n+1}(T_s+t_0)\Gamma(n,\beta(T_s+t_0))\right\}\Bigg]. \quad (2.5)$$

The most widely used prior distribution for failure rate is the gamma distribution. The main reason for this acceptability is the mathematical tractability resulting from the fact that the gamma distribution is the nature conjugate prior distribution.

So, we suppose that the failure rate Λ_u for the unit has a $GAM(\theta_1, x_1)$ prior given by

$$g(\lambda_u) = \frac{1}{\theta^{x_1}\Gamma(x_1)}\lambda_u^{x_1-1}e^{-\lambda_u/\theta_1}, \quad 0<\lambda_u<\infty,$$

and the failure rate Λ_s for the switch has an $UNIF(0,\beta)$ prior distribution. And we suppose that Λ_u and Λ_s are independent. Then, the joint posterior distribution of failure rates Λ_u and Λ_s given total test times $T_u=t_u$ and $T_s=t_s$ is

$$g(\lambda_u,\lambda_s \mid t_u, t_s) = \frac{(1/\theta_1+t_s)^{n+x_1}}{\Gamma(n+x_1)\Gamma(n+1,\alpha t_u)}\lambda_u^{n+1}\lambda_s^{n+x_1-1} \times\exp\left\{-t_u\lambda_u-\left(\frac{1}{\theta_1}+t_s\right)\lambda_s\right\}, \quad 0<\lambda_u<\alpha,\ 0<\lambda_s<\infty. \quad (2.6)$$

Therefore, under the squared error loss function and gamma prior for failure rate of unit and uniform prior for failure rate of switch , the Bayes estimator $\widehat{R_{UG}}(t_0)$ for the two-unit hot standby system reliability $R(t_0)$ with imperfect switch is

$$\widehat{R_{UG}}(t_0) = \frac{1}{\Gamma(n+1,\alpha T_u)}\left(\frac{T_u}{T_u+t_0}\right)^{n+1}\Bigg[\Gamma(n+1,\alpha(T_u+t_o)) + \frac{(1/\theta_1+T_s)\cdot\Gamma(n+2,\alpha(T_u+t_0))}{(n+x_1-1)(T_u+t_0)}\left\{1-\left(\frac{1+\theta_1 T_s}{1+\theta_1(T_s+t_0)}\right)^{n+x_1-1}\right\}\Bigg]. \quad (2.7)$$

Now, we suppose that Λ_u has an $UNIF(0,\alpha)$ prior and prior distribution of failure rate

Λ_s has independently a $GAM(\theta_2, x_2)$ prior distribution.
According to Bayes theorem, the joint posterior distribution for failure rates Λ_u and Λ_s given total test times $T_u = t_u$ and $T_s = t_s$ is

$$g(\lambda_u, \lambda_s \mid t_u, t_s) = \frac{(1/\theta_2 + t_s)^{n+x_2}}{\Gamma(n+x_2)\Gamma(n+1, \alpha t_u)} \lambda_u^{n+1} \lambda_s^{n+x_2-1} \times \exp\left\{ -t_u\lambda_u - \left(\frac{1}{\theta_2} + t_s\right)\lambda_s \right\}, \quad 0 < \lambda_u < \alpha,\ 0 < \lambda_s < \infty. \tag{2.8}$$

Therefore, under the squared error loss function, the Bayes estimator $\widehat{R_{UG}}(t_0)$ for the two-unit hot standby system reliability $R(t_0)$ with imperfect switch is

$$\widehat{R_{UG}}(t_0) = \frac{1}{\Gamma(n+1, \alpha T_u)} \left(\frac{T_u}{T_u + t_0}\right)^{n+1} \Big[\Gamma(n+1, \alpha(T_u + t_o)) + \frac{(1/\theta_2 + T_s) \cdot \Gamma(n+2, \alpha(T_u + t_0))}{(n + x_2 - 1)(T_u + t_0)} \left\{ 1 - \left(\frac{1 + \theta_2 T_s}{1 + \theta_2 (T_s + t_0)} \right)^{n+x_2-1} \right\} \Big]. \tag{2.9}$$

Finally, we suppose that the prior distribution on failure rate Λ_u is a $GAM(\theta_1, x_1)$ and the prior distribution on failure rate Λ_s is a $GAM(\theta_2, x_2)$. And we suppose that Λ_u and Λ_s is independent. Then the joint posterior distribution of failure rates Λ_u and Λ_s given total test times $T_u = t_u$ and $T_s = t_s$ is

$$g(\lambda_u, \lambda_s \mid t_u, t_s) = \frac{(1/\theta_1 + t_u)^{n+x_1} (1/\theta_2 + t_s)^{n+x_2}}{\Gamma(n+x_1)\Gamma(n+x_2)} \lambda_u^{n+x_1-1} \lambda_s^{n+x_2-1} \times \exp\left\{ -\left(\frac{1}{\theta_1} + t_u\right)\lambda_u - \left(\frac{1}{\theta_2} + t_s\right)\lambda_s \right\}, \quad 0 < \lambda_u, \lambda_s < \infty. \tag{2.10}$$

Therefore, the Bayes estimator $\widehat{R_{GG}}(t_0)$ for the two unit hot standby system reliability $R(t_0)$ with imperfect switch under the squared error loss function is

$$\widehat{R_{GG}}(t_0) = \left(\frac{1 + \theta_1 T_u}{1 + \theta_1 (T_u + t_0)} \right)^{n+x_1} \Big[1 + \frac{(n+x_1)(1/\theta_2 + T_s)}{(n + x_2 - 1)(1/\theta_1 + T_u + t_0)} \times \left\{ 1 - \left(\frac{1 + \theta_2 T_s}{1 + \theta_2 (T_s + t_0)} \right)^{n+x_2-1} \right\} \Big]. \tag{2.11}$$

From results (2.3) through (2.11), Tables 1 through 4 show the simulated values for the mean squared errors(MSE) of the proposed Bayes estimators of reliability for the two-unit hot standby system with imperfect switch under the squared error loss function and noninformative, uniform and gamma priors for failure rates λ_u and λ_s when sample sizes n=5(5)30, system reliabilities $R(t_0) \simeq 0.1, 0.05$ and failure rates $\lambda_u = 0.2$ and $\lambda_s = 0.1$. From Tables, Bayes estimator $\hat{R}_{UU}(t_0)$ for reliability of the two-unit hot standby system with imperfect switch under uniform priors for failure rates λ_u and λ_s is more efficient than other Bayes estimator. For all the sample sizes, Bayes estimator $\hat{R}_{GG}(t_0)$ for reliability of the two-unit hot standby system with imperfect switch under noninformative priors for failure rates λ_u and λ_s is worse in a sense of MSE than other proposed Bayes estimators.

Table 1. Simulated MSE's for Bayes estimators for system reliability under noninformative prior on failure rates Λ_u and Λ_s ($\lambda_u = 0.2$, $\lambda_s = 0.1$).

$R(t_0)$ \ n	5	10	15	20	25	30
0.1	0.06013	0.03041	0.01981	0.01494	0.01168	0.00985
0.05	0.04213	0.01836	0.01066	0.00776	0.00593	0.00475

where simulations were repeated 5000 times

Table 2. Simulated MSE's for Bayes estimators for system reliability under uniform priors on failure rates Λ_u and Λ_s ($\lambda_u = 0.2$, $\lambda_s = 0.1$).

	$R(t_0)$ \ n	5	10	15	20	25	30
$\Lambda_u \sim$ UNIF(0,0.4)	0.1	0.00677	0.00465	0.00347	0.00284	0.00234	0.00204
$\Lambda_s \sim$ UNIF(0,0.2)	0.05	0.00435	0.00255	0.00170	0.00136	0.00110	0.00092

Table 3. Simulated MSE's for Bayes estimators for system reliability under gamma priors on failure rates Λ_u and Λ_s ($\lambda_u = 0.2$, $\lambda_s = 0.1$).

	$R(t_0)$ \ n	5	10	15	20	25	30
$\Lambda_u \sim$ GAM(0.4,0.5)	0.1	0.01290	0.00698	0.00466	0.00356	0.00281	0.00238
$\Lambda_s \sim$ GAM(0.2,0.5)	0.05	0.00886	0.00417	0.00249	0.00184	0.00142	0.00115
$\Lambda_u \sim$ GAM(0.2,1.0)	0.1	0.01118	0.00643	0.00440	0.00341	0.00271	0.00231
$\Lambda_s \sim$ GAM(0.1,1.0)	0.05	0.00755	0.00380	0.00234	0.00175	0.00137	0.00111
$\Lambda_u \sim$ GAM(0.1,2.0)	0.1	0.00860	0.00550	0.00394	0.00312	0.00252	0.00217
$\Lambda_s \sim$ GAM(0.05,2.0)	0.05	0.00563	0.00320	0.00207	0.00159	0.00126	0.00104

Table 4. Simulated MSE's for Bayes estimators for system reliability under uniform and gamma priors on failure rates Λ_u and Λ_s ($\lambda_u=0.2$, $\lambda_s=0.1$).

	$R(t_0)$ \ n	5	10	15	20	25	30
$\Lambda_u \sim$ UNIF(0,0.4)	0.1	0.00713	0.00486	0.00360	0.00293	0.00240	0.00209
$\Lambda_s \sim$ GAM(0.2,0.5)	0.05	0.00461	0.00269	0.00179	0.00142	0.00114	0.00095
$\Lambda_u \sim$ UNIF(0,0.4)	0.1	0.00712	0.00485	0.00359	0.00293	0.00240	0.00209
$\Lambda_s \sim$ GAM(0.1,1.0)	0.05	0.00460	0.00269	0.00179	0.00142	0.00114	0.00095
$\Lambda_u \sim$ UNIF(0,0.4)	0.1	0.00709	0.00484	0.00359	0.00292	0.00209	0.00209
$\Lambda_s \sim$ GAM(0.05,2.0)	0.05	0.00458	0.00268	0.00178	0.00142	0.00114	0.00095
$\Lambda_u \sim$ GAM(0.4,0.5)	0.1	0.01222	0.00663	0.00445	0.00343	0.00272	0.00231
$\Lambda_s \sim$ UNIF(0,0.2)	0.05	0.00838	0.00394	0.00237	0.00176	0.00137	0.00111
$\Lambda_u \sim$ GAM(0.2,1.0)	0.1	0.01056	0.00610	0.00420	0.00328	0.00262	0.00224
$\Lambda_s \sim$ UNIF(0,0.2)	0.05	0.00712	0.00359	0.00222	0.00168	0.00131	0.00107
$\Lambda_u \sim$ GAM(0.1,2.0)	0.1	0.00809	0.00522	0.00376	0.00301	0.00244	0.00211
$\Lambda_s \sim$ UNIF(0,0.2)	0.05	0.00530	0.00302	0.00196	0.00153	0.00122	0.00100

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