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On Testing Equality of Matrix Intraclass Covariance Matrices of K Multivariate Normal Populations

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Abstract

We propose a criterion for testing homogeneity of matrix intraclass covariance matrices of K multivariate normal populations. It is based on a variable transformation, intended to propose and develop a likelihood ratio criterion that makes use of properties of eigen structures of the matrix intraclass covariance matrices. The criterion then leads to a simple test that uses an asymptotic distribution obtained from Box's (1949) theorem for the general asymptotic expansion of random variables.

Key Words: Matrix Intraclass Covariance Matrix; Homogeneity Test Crite-rion; Modified Likelihood Ratio Statistic; Box's Approximation.

1. Introduction

Suppose that $X_1(i), \ldots, X_{N_i}(i)$ denote N_i independent $p-$variate observations from $N_p(\theta_i, \Omega_i)$, where Ω_i denotes the positive definite, symmetric, $p\times p(p=q\ell)$ matrix intraclass covariance matrix

$$\begin{bmatrix} \Sigma_i & \Lambda_i & \ldots & \Lambda_i \\ \Lambda_i & \Sigma_i & \ldots & \Lambda_i \\ \cdot & \ldots & \ldots & \cdot \\ \Lambda_i & \ldots & \Lambda_i & \Sigma_i \end{bmatrix} \quad (1)$$

in which $\Sigma_i : (q\times q) > 0$ denotes a positive definite symmetric matrix of order q which is repeated as a block along the principal diagonal, and $\Lambda_i : (q\times q) > 0$ denotes a matrix of order q which is repeated as a block in all of the off-diagonal entries. This matrix arises in various contexts in statistical analysis, such as in pooling of times series and cross sectional data (Wallace and Hussian, 1969, p. 58); group decision making (Press, 1978); Model 2 MANOVA (multivariate analysis of variance); and in the growth curves and repeated measures models (Kshirsagar and Smith, 1995; Krzanowski and Marriott, 1995).

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For example, the Model 2 (variance components model) often arises in agricultural applications. Suppose a horticulturist wants to determine how a certain treatment has affected the nitrogen and some other components in the foliage of the trees in an orchard. Because he can not examine each and every leaf on every tree, he selects a random group of trees to study. Then he selects a random set of leaves from each selected tree.

Let $Y_{\alpha\beta}(i)$ denote $q\times 1$ observed nitrogen content and other components of β th leaf from the α th tree in i th population, and assume a model of the form

$$Y_{\alpha\beta}(i)=\mu(i)+a_{\alpha}(i)+b_{\alpha\beta}(i), \tag{2}$$

where $\mu(i)$ is a $q\times 1$ constant vector, $a_{\alpha}(i)\sim N_q(0,\Lambda_i)$, $b_{\alpha\beta}(i)\sim N_q(0,\Lambda_i^*)$ and all random vectors $a_{\alpha}(i), b_{\alpha\beta}(i)$, for all $\alpha=1,\cdots,\ell$, $\beta=1,\cdots,N_i$, are independent. It is straightforward to check that $Y_{\alpha\beta}(i)$ and $Y_{\alpha\beta^*}(i)$ are correlated even when $\beta\neq\beta^*$. Specifically, $cov(Y_{\alpha\beta}(i), Y_{\alpha\beta^*}(i))$ is Λ_i for $\beta\neq\beta^*$ and it is $\Lambda_i+\Lambda_i^*$ for $\beta=\beta^*$. Thus, the covariance matrix of $Y_{\alpha\beta}(i)$'s is equal, $\Sigma_i=\Lambda_i+\Lambda_i^*$, along the diagonal, and is Λ_i in all of the off-diagonal elements. Thus the associate covariance matrix of the $Y_{\alpha\beta}(i)$'s become a matrix intraclass covariance matrix. Other examples are the growth curve and repeated measures models that are designed so that responses on the same experimental units are observed at each repetition and experiments of this type have broad application, especially in the life and social sciences (see e.g. Kshirsagar and Smith 1995, Krzanowski and Marriott 1995, Timm 1980 and Potthoff and Roy 1964).

Usual likelihood ratio statistic, λ_H, for testing the hypothesis $H:\Omega_1=\cdots=\Omega_K$, can be obtained by generalizing Rencher (1995). The likelihood ratio, however, does not reduce to a standard statistic, and we resort to an approximation for its distribution. An asymptotic approximation to function of the likelihood ratio statistic has not been seen yet. This is due to complexity of the sampling distribution involved in it. Although usual Wilks (1962) approximation, $-2\ln\lambda_H\sim\chi^2_{(q(q+1)(K-1))}$, is available for testing H, however in small samples with large p, it may tend to have the actual significance level greater than the nominal significance level (cf. Greenstreet and Conner 1974). In this paper it is shown that under certain transformation of $X_j(i)$'s, an alternative of Wilks approximation can be derived. This result is obtained by observing that H is equivalent to the hypothesis that $\Sigma_1=\cdots=\Sigma_K$ and $\Lambda_1=\cdots=\Lambda_K$.

2. Preliminaries

Suppose $\Omega_i>0$, $i=1,\cdots,K$, are the $p(=\ell q)$th-order matrix intraclass covariance matrices each having common diagonal and off-diagonal matrices Σ_i and Λ_i, respectively. Then it can be expressed as

$$\Omega_i = I_\ell \otimes \Sigma_i + (e_\ell e'_\ell - I_\ell) \otimes \Lambda_i, \quad i=1,\dots,K, \tag{3}$$

where I_ℓ denotes the identity matrix of order ℓ, and e_ℓ denotes the $\ell \times 1$ vector of ones.

Lemma 2.1. Suppose $\Gamma \equiv \Gamma_0 \otimes I_q$, $\Gamma\Gamma' = I_{\ell q}$, and Γ_0 is any ℓ-dimensional orthogonal matrix whose first row has equal elements such as the Helmert matrix.
Then by use of the transformation $\Omega_i^* = \Gamma \Omega_i \Gamma'$, the matrix Ω_i $i=1,\dots,K$, defined in (3) may be reduced to the block diagonal form

$$\Omega_i^* = \begin{bmatrix} \Sigma_i(1) & 0 & 0 & \cdots & 0 \\ 0 & \Sigma_i(2) & 0 & \cdots & 0 \\ 0 & \cdot & \cdot & \cdots & 0 \\ 0 & \cdot & 0 & \cdots & \Sigma_i(2) \end{bmatrix}, \tag{4}$$

where $\Sigma_i(1) > 0$, $\Sigma_i(2) > 0$, and $\Sigma_i(1) = \Sigma_i + (\ell - 1)\Lambda_i, \Sigma_i(2) = \Sigma_i - \Lambda_i$.

Proof. Write Ω_i as in (3) and pre and post multiply it by $\Gamma_0 \otimes I_q$. This gives

$$\begin{aligned} &(\Gamma_0 \otimes I_q)(I_\ell \otimes \Sigma_i + (e_\ell e'_\ell - I_\ell) \otimes \Lambda_i)(\Gamma_0 \otimes I_q)' \\ &= (\Gamma_0 \otimes \Sigma_i + \Gamma_0(e_\ell e'_\ell - I_\ell) \otimes \Lambda_i)(\Gamma'_0 \otimes I'_q) \\ &= (I_\ell \otimes \Sigma_i + \Gamma_0(e_\ell e'_\ell - I_\ell)\Gamma'_0 \otimes \Lambda_i) \\ &= (I_\ell \otimes (\Sigma_i - \Lambda_i) + \ell\, I_\ell^0 \otimes \Lambda_i), \end{aligned}$$

where I_ℓ^0 denotes $\ell \times \ell$ matrix whose principal element is one and all the elements are zeros. Since $\Omega_i > 0$, the latent roots of $\Omega_i, i=1,\dots,K$ are all positive and they are found from the characteristic equation

$$|\Omega_i - \lambda I_{\ell q}| = |\Gamma \Omega_i \Gamma' - \lambda \Gamma I_{\ell q} \Gamma'| = |\Omega_i^* - \lambda I_{\ell q}| = 0,$$

so that the latent roots of Ω_i and Ω_i^* are equivalent. Moreover, we see that

$$|\Omega_i^* - \lambda I_{\ell q}| = |\Sigma_i + (\ell - 1)\Lambda_i - \lambda I_q| |\Sigma_i - \Lambda_i - \lambda I_q|^{\ell - 1} = 0.$$

This relation yields that all the latent roots of $\Sigma_i + (\ell - 1)\Lambda_i$ and $\Sigma_i - \Lambda_i$, are positive, and hence those corresponding matrices are positive definite matrices, i.e. $\Sigma_i(1) > 0$ and $\Sigma_i(2) > 0$ for $i=1,\dots,K$.

Lemma 2.2. Let $\Omega_i > 0$, $i=1,\dots,K$, be the pth-order matrix intraclass covariance matrices defined in (3) and let a $q \times p$ matrix $\Delta_1 = A\Gamma$, where $A = [I_q; I_q; \cdots; I_q]$, a $q \times \ell q$

matrix. Then a necessary and sufficient condition for the equality of the common diagonal elements in $\Omega_i's$, i.e. $\Sigma_1 = \cdots = \Sigma_K$, is

$$\Delta_1\Omega_i\Delta_1' = \Delta_1\Omega_j\Delta_1, \quad \text{for } i,j=1,\cdots,K;\ i\neq j, \tag{5}$$

Proof. If $\Sigma_i = \Sigma_j$ for any $i\neq j$, $\Delta_1\Omega_i\Delta_1' - \Delta_1\Omega_j\Delta_1' = A\Omega_i^*A' - A\Omega_j^*A' = \{\Sigma_i(1)+(\ell-1)\Sigma_i(2)\} - \{\Sigma_j(1)+(\ell-1)\Sigma_j(2)\} = \ell(\Sigma_i - \Sigma_j)$. Thus (5) is sufficient condition for $\Sigma_i = \Sigma_j$. The necessity follows from the fact that if $\Delta_1\Omega_i\Delta_1' = \Delta_1\Omega_j\Delta_1'$ for $i,j=1,\cdots,K;i\neq j$.

$$\Delta_1\Omega_i\Delta_1' - \Delta_1\Omega_j\Delta_1' = A(\Omega_i^* - \Omega_j^*)A' = 0. \tag{6}$$

This yields $\ell(\Sigma_i - \Sigma_j) = 0$, and hence $\Sigma_i = \Sigma_j$ for all $i\neq j$.

Lemma 2.3. For $\ell > 2$ and $\Omega_i > 0$, $i=1,\ldots,K$, the pth-order matrix intraclass covariance matrices, let a $q\times p$ matrix $\Delta_2 = C\Gamma$, where $C=[O;I_q;\cdots;I_q;-(\ell-2)I_q]$, a $q\times \ell q$ matrix. Then, for given $\Sigma_1 = \cdots = \Sigma_K$, a necessary and sufficient condition for $\Lambda_1 = \cdots = \Lambda_K$, equality of off-diagonal elements of $\Omega_i's$, is

$$\Delta_2\Omega_i\Delta_2' = \Delta_2\Omega_j\Delta_2', \text{ for } i,j=1,\cdots,K;\ i\neq j, \tag{7}$$

Proof. If $\Lambda_i = \Lambda_j$ and $\Sigma_i = \Sigma_j$ for any $i\neq j$, $\Delta_2\Omega_i\Delta_2' - \Delta_2\Omega_j\Delta_2' = C(\Omega_i^* - \Omega_j^*)C' = 0$. The necessity follows from the fact that if $\Delta_2\Omega_i\Delta_2' = \Delta_2\Omega_j\Delta_2'$, for $i,j=1,\cdots,K;i\neq j$,

$$\Delta_2\Omega_i\Delta_2' - \Delta_2\Omega_j\Delta_2' = C(\Omega_i^* - \Omega_j^*)C' = 0. \tag{8}$$

This yields $\Lambda_i - \Lambda_j = 0$, and hence $\Lambda_i = \Lambda_j$ for all $i\neq j$.

The above two lemmas give the following theorem.

Theorem 2.1. Let $\Omega_i > 0$, $i=1,\ldots,K$, are the pth-order matrix intraclass covariance matrices with $\ell > 2$. Then a necessary and sufficient condition that $\Omega_1 = \cdots = \Omega_K$ is

$$D\Omega_iD' = D\Omega_jD' \quad \text{for } i,j=1,\cdots,K;\ i\neq j, \tag{9}$$

where $D=[\Delta_1'\Delta_2']'$ is a $2q\times p$ matrix, where $\Delta_1 = A\Gamma$, with $A=[I_q;I_q;\cdots;I_q]$ and $\Delta_2 = C\Gamma$, with $C=[O;I_q;\cdots;I_q;-(\ell-2)I_q]$.

Proof. It is straightforward to see that if $\Delta_1\Omega_i\Delta_1' = \Delta_1\Omega_j\Delta_1'$ and $\Delta_2\Omega_i\Delta_2' = \Delta_2\Omega_j\Delta_2'$ hold, $\Delta_1\Omega_i\Delta_2' = \Delta_1\Omega_j\Delta_2' = 0$ for all $i\neq j$. Thus the result follows from Lemma 2.2 and 2.3.

3. Homogeneity Test

3.1 Test Criterion

Let $X_1(i),\dots,X_{N_i}(i)$ denote N_i independent p-variate $(p>2)$ observations from $N_p(\theta_i,\Omega_i)$, with Ω_i having the pattern in (1). This pattern of equal variance and equal covariance matrices is referred to as matrix intraclass covariance model. If it is desired to test the hypothesis $H:\Omega_1=\cdots=\Omega_K$ based upon $X_j(i)'s$, it is not easy to use the standard Box M-test (cf. Box 1949, Anderson 1984). This is due to complex distribution obtained from the likelihood ratio criterion for H so that a test for H has not been seen yet. However the following theorem enables us to test the hypothesis H. Using the same notations as the previous section, if we let independent $2q\times 1$ variate $Y_j(i)=DX_j(i)$ then its distribution is $N_{2q}(\mu_i,\Psi_i), i=1,\dots,K, ;j=1,\dots,N_i$, where $\mu_i=D\theta_i$ and $\Psi_i=D\Omega_iD'$. Therefore, Theorem 2.1 gives that the hypothesis $H:\Omega_1=\cdots=\Omega_K$ defined above is equivalent to the homogeneity of population diagonal covariance matrices of $Y_j(i)'s$, i.e. $H:\Psi_1=\cdots=\Psi_K$, where $\Psi_i=diagonal\{\Delta_1\Omega_i\Delta_1',\Delta_2\Omega_i\Delta_2'\}$ $i=1,\dots,K$, are the $2q\times 2q$ block diagonal matrices. In this transformation the hypothesis H is a combination of the hypothesis H_1: First block diagonal matrix of $\Psi_i's$ are equal ($\Delta_1\Omega_1\Delta_1'=\cdots=\Delta_1\Omega_K\Delta_1'$) and H_2: Second block diagonal matrix of $\Psi_i's$ are equal given that H_1 is true.

Let $Y_j(i)=(Y_{1j}(i)',Y_{2j}(i)')'$ and let the mean vector $D\theta_i=(\mu_1(i)',\mu_2(i)')'$, $i=1,\dots,K$, $j=1,\dots,N_i$, so that $Y_{1j}(i)$ be an observation from the ith q-variate normal population $N_q(\mu_1(i),\Delta_1\Omega_i\Delta_1')$. Suppose we wish to test the hypothesis

$$H_1:\Delta_1\Omega_1\Delta_1'=\cdots=\Delta_1\Omega_K\Delta_1',$$

which is equivalent to the hypothesis of equality of diagonal matrices in the matrix intraclass covariance matrix in $\Omega_i's$, i.e. $\Sigma_1=\cdots=\Sigma_K$, by Lemma 2.2. For the several sample case, various procedures have been proposed (cf. Rencher 1995). The likelihood ratio criterion obtained from the marginal distributions of $Y_{1j}(i)'s$ is

$$\lambda_{H_1}=\left(\prod_{i=1}^{K}|V_{11}(i)|^{N_i/2}/|V_{11}|^{N/2}\right)\left(N^{qN/2}/\prod_{i=1}^{K}N_i^{qN_i/2}\right), \tag{10}$$

where $V_{11}(1),\dots,V_{11}(K)$ are independent mean corrected sum of square cross product matrices of $Y_{1j}(i)'s$, $V_{11}=\sum_{i=1}^{K}V_{11}(i)$ and $N=\sum_{i=1}^{K}N_i$.

Now we consider testing the equality of variance matrices of $Y_{2j}(i)'s$ when we assume variance matrices of $Y_{1j}(i)'s$ are the same; that is, we consider to test

$$H_2: \Delta_2\Omega_1\Delta_2' = \cdots = \Delta_2\Omega_K\Delta_2' \text{ given } \Delta_1\Omega_1\Delta_1' = \cdots = \Delta_1\Omega_K\Delta_1'.$$

By use of Lemma 2.3, we can see that the hypothesis is equivalent to that of homogeneity in off-diagonal matrices of $\Omega_i's$, that is $\Lambda_1 = \cdots = \Lambda_K$. From the joint distribution of $Y_{1j}(i)'s$ and $Y_{2j}(i)'s$, we can easily derive that the likelihood ratio criterion for testing H_2 that is

$$\lambda_{H_2} = \left(\prod_{i=1}^{K} |V_{22}(i)|^{N_i/2} / |V_{22}|^{N/2} \right) \left(N^{qN/2} / \prod_{i=1}^{K} N_i^{qN_i/2} \right), \tag{11}$$

where $V_{22}(1), \ldots, V_{22}(K)$ are independent mean corrected sum of squares of $Y_{2j}(i)'s$ and $V_{22} = \sum_{i=1}^{K} V_{22}(i)$.

Lemma 3.1 (Anderson 1984). Let Z be an observation vector on a random vector with density $f(Y, \delta, \psi)$, where δ and ψ are vectors of variances and means in spaces Ω and Ψ. Let H_A be the hypothesis $\delta \in \Omega_a \subset \Omega$, let H_B be the hypothesis $\delta \in \Omega_b \subset \Omega_a$, given $\delta \in \Omega_a$. Then the likelihood ratio of the hypothesis, H_{AB}, that $\delta \in \Omega_b$, given $\delta \in \Omega$ is

$$\lambda_{AB} = \frac{\max_{\delta \in \Omega_b, \psi \in \Psi} f(z, \delta, \psi)}{\max_{\delta \in \Omega, \psi \in \Psi} f(z, \delta, \psi)} = \lambda_A \times \lambda_B,$$

where

$$\lambda_A = \left(\frac{\max_{\delta \in \Omega_a, \psi \in \Psi} f(z, \delta, \psi)}{\max_{\delta \in \Omega, \psi \in \Psi} f(z, \delta, \psi)} \right) \text{ and } \lambda_B = \left(\frac{\max_{\delta \in \Omega_b, \psi \in \Psi} f(z, \delta, \psi)}{\max_{\delta \in \Omega_a, \psi \in \Psi} f(z, \delta, \psi)} \right).$$

From Lemma 3.1, we see that the likelihood criterion for the hypothesis H is the product of the likelihood criteria for H_1 and H_2, so that

$$\begin{aligned} \lambda_H &= \lambda_{H_1} \lambda_{H_2} \\ &= \left(N^{N/2} / \prod_{i=1}^{K} N_i^{N_i/2} \right)^{2q} \left(\frac{\prod_{i=1}^{K} |V_{11}(i)|^{N_i/2}}{|V_{11}|^{N/2}} \right) \left(\frac{\prod_{i=1}^{K} |V_{22}(i)|^{N_i/2}}{|V_{22}|^{N/2}} \right). \end{aligned}$$

Note that λ_{H_1} and λ_{H_2} are invariant with respect to changing the value of I_q to $aI_q (a \neq 0)$ in Δ_1 and Δ_2.

3.2. Distribution of the Criterion

In this section we shall consider the distribution of a test criterion. The test criterion considered is obtained from Box's approximation (Box, 1949) to the likelihood ratio statistic modified for unbiased estimates (replacing maximum likelihood estimates by unbiased estimates). The modified likelihood ratio statistic is defined by

$$\lambda_H^* = \lambda_{H_1}^* \lambda_{H_2}^*$$
$$= \left(\nu^{\nu/2} / \prod_{i=1}^{K} \nu_i^{\nu_i/2} \right)^{2q} \left(\frac{\prod_{i=1}^{K} |V_{11}(i)|^{\nu_i/2}}{|V_{11}|^{\nu/2}} \right) \left(\frac{\prod_{i=1}^{K} |V_{22}(i)|^{\nu_i/2}}{|V_{22}|^{\nu/2}} \right),$$

where $\nu_i = N_i - 1$ and $\nu = \sum_{i=1}^{K} \nu_i$. The reason for the modification is as follows: Bartlett (1937) gave an intuitive argument of the use of the modified likelihood ratio statistic in place of the usual likelihood ratio statistic and Perlman (1980) showed that likelihood test based on the modified statistic is unbiased. Exact distribution of λ_H^* is not available. Box (1949) suggested a general asymptotic expansion of the distribution of a random variable whose moments are certain functions of gamma functions. By use of the expansion, we will derive asymptotic distribution of λ_H^* for testing the null hypothesis, H.

The hth moment of λ_H^* can be easily found by making use of the characterization of a distribution in Theorem 10.4.2 of Anderson (1984) and the independence of $\lambda_{H_1}^*$ and $\lambda_{H_2}^*$. This yields

$$E\lambda_H^{*h} = E\lambda_{H_1}^{*h} E\lambda_{H_2}^{*h}$$
$$= \Delta \left(\frac{\nu^{\nu/2}}{\prod_{i=1}^{K} \nu_i^{\nu_i/2}} \right)^{2qh} \left(\frac{\prod_{i=1}^{K} \prod_{j=1}^{q} \Gamma\{ (\nu_i(1+h)+1-j)/2 \}}{\prod_{j=1}^{q} \Gamma\{ (\nu(1+h)+1-j)/2 \}} \right)^2, \qquad (12)$$

where $\Delta = \left(\prod_{i=1}^{K} \prod_{j=1}^{q} \Gamma\{ (\nu_i+1-j)/2 \} / \prod_{j=1}^{q} \Gamma\{ (\nu+1-j)/2 \} \right)^2$. Since $0 \le \lambda_H^* \le 1$, the moments determine the distribution uniquely. Moreover, the moments of λ_H^* are certain functions of gamma functions so that we may apply Box's asymptotic expansion to the distribution of λ_H^*. By use of the expansion, we derive the distribution of λ_H^* for testing H as follows.

Theorem 3.1. A second order approximation to the distribution of λ_H^* under H is given by

$$P\{-2\rho \log \lambda_H^* \le t\} = P\{\chi^2(f) \le t\} + \omega_2 [P\{\chi^2(f+4) \le t\} - P\{\chi^2(f) \le t\}] + O(\nu^{-3}), \qquad (13)$$

where

$$f = q(q+1)(K-1), \quad \rho = 1 - \sum_{i=1}^{K} \left(\frac{1}{\nu_i} - \frac{1}{\nu} \right) \frac{2q^2+3q-1}{6(q+1)(K-1)},$$

and

$$\omega_2 = \frac{q(q+1)}{24\rho^2} \left[(q-1)(q+2) \left(\sum_{i=1}^{K} \frac{1}{\nu_i^2} - \frac{1}{\nu^2} \right) - 6(K-1)(1-\rho)^2 \right].$$

Proof. If we set

$$b = 2q, \quad y_\ell = \nu/2, \quad \eta_\ell = (1-j)/2; \quad \ell = j, q+j, \quad j = 1, \cdots, q,$$
$$a = 2qK, \quad x_k = \nu_i/2; \quad k = 2(i-1)q+1, \cdots, 2iq, \quad i = 1, \cdots, K,$$
$$\xi_k = (1-j)/2; \quad k = j, q+j, 2q+j, \cdots, 2(K-1)q+j, (2K-1)q+j, \quad j = 1, \cdots, q,$$

the hth moment of λ_H^* can be expressed as

$$E\lambda_H^{*h} = \Delta \left(\frac{\prod_{\ell=1}^{b} y_\ell^{y_\ell}}{\prod_{k=1}^{a} x_k^{x_k}} \right)^h \frac{\prod_{k=1}^{a} \Gamma\{x_k(1+h)+\xi_k\}}{\prod_{\ell=1}^{b} \Gamma\{y_\ell(1+h)+\eta_\ell\}},$$

where $\Delta = \prod_{k=1}^{a} \Gamma\{x_k + \xi_k\} / \prod_{\ell=1}^{b} \Gamma\{y_\ell + \eta_\ell\}$. Since $E\lambda_H^{*0} = 1$ and $\sum_{k=1}^{a} x_k = \sum_{\ell=1}^{b} y_\ell$, the random variable λ_H^{*h}, whose moments are certain functions of gamma functions, satisfies the conditions for Box's general theory of asymptotic expansions (cf. Anderson 1984, p. 311). Such that if we take a second order approximation to the distribution of $-2\rho \log \lambda_H^{*h}$, Box's theorem gives the result.

Therefore, if we set the observed value of $-2\rho \log \lambda_H^*$ to t, $1 - P\{-2\rho \log \lambda_H^* \leq t\}$ obtained from (13) gives an approximate p-value of the test for H.

4. CONCLUDING REMARKS

In this paper we have suggested a modified likelihood ratio criterion for testing H that matrix intraclass covariance matrices of K multivariate normal populations are equal. This is obtained by observing that H can be reduced to the statement in (9) and (9) can be thought of as a combination of the hypothesis H_1 and H_2 given H_1 is true. Based upon this consideration, an appropriate likelihood ratio criterion for testing H is derived and it is shown that the criterion leads to a simple test as well as to accurate asymptotic distribution via the general theorem by Box (1949). When H is true, the analytical ease with which result can be obtained from using the test criterion makes it attractive for use in analyzing the multivariate linear models. For example, the Model 2 MANOVA problem in Section 1, if the suggested test accepts H, it will be straigthtforward to analyze main effect vectors simply under a MANOVA model.

While the suggested test criterion is useful, it is recognized that criteria for some other types of hypotheses, for example, a criterion for testing K covariance matrices are equal and intraclass covariance matrices, is worthy to study. Further study pertaining to this criterion would be useful and is left for future research.

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