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Diagnostics for Weibull Regression Model with Censored Data[1]

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ABSTRACT

This paper discusses the local influence approach to the Weibull regression model with censored data. Diagnostics for the Weibull regression model are proposed and developed, when simultaneous perturbations of the response vector are allowed.

KEY WORDS : Weibull regression model, Observed information matrix, Direction of maximum curvature.

1. Introduction

Rather than omission approaches, Cook (1986) developed a general method for assessing the local influence of a model perturbation. A distinguishing feature of this method is in using log-likelihood contours to assess the local influence. Schwarzman (1991) provided a further justification for local-influence analysis by showing that the local influence approach of perturbations of the response is the same as the usual regression diagnostics in the fixed-effects model. Kim and Huggins (1998) applied and discussed the same method for use in autocorrelated regression models.

In recent years interests in diagnostic analysis have grown steadily when models are fit to censored data. A proportional hazards model possesses the property that different individuals have hazard functions which are proportional to one another. Proportional hazards models assume that concomitant variables have a multiplicative effect on the hazard function. Diagnostic analysis has been considered for both parametric proportional hazards models(Weissfeld and Schneider, 1990) and the proportional hazards regression model(Pettitt and Daud (1989), Weissfeld (1990)). Of course, they developed influence diagnostics for the proportional hazards model.

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This article discusses the effects of simultaneous perturbations of the response vector on all parameters for the Weibull regression model. These methods can also be applied to other parametric proportional hazards models for censored data such as the exponential, gamma, log-normal models. In Section 2 and Section 3, the model is formulated and diagnostics for the Weibull regression models are discussed. Some illustrative examples are considered in Section 4.

2. Local Influence Approach for the Weibull Regression Models

The Weibull distribution is useful in a great variety of applications. One reason for its popularity is that it has a great variety of shapes. When we let T_i denote the failure time of the ith observation, the Weibull probability density function is given by

$$f(t) = \lambda r(\lambda t)^{r-1} \exp\{-(\lambda t)^r\} \qquad t \geq 0 \text{ and } r, \lambda > 0$$

where r is a shape parameter and λ is a scale parameter, respectively. Furthermore, if T_i is distributed according to the Weibull distribution, $X_i = \log T_i$ has an extreme value distribution. (See Lawless (1982)).

In order to develop diagnostics for the Weibull regression models, we use the local influence approach proposed by Cook (1986). Cook presented a general method for assessing the local influence of minor perturbations of a statistical model. The method relies on a well-behaved likelihood and certain elementary ideas from differential geometry. To assess the influence of varying ω throughout Ω , Cook (1986) proposed the likelihood displacement

$$LD(\omega) = 2[L(\hat{\theta}) - L(\hat{\theta}_\omega)] \tag{2.1}$$

and the associated influence graph

$$a(\omega) = \begin{pmatrix} \omega \\ LD(\omega) \end{pmatrix} \tag{2.2}$$

where L denotes the log-likelihood corresponding to the postulated model, $\hat{\theta}$ and $\hat{\theta}_\omega$ are maximum likelihood estimates for θ in both the unperturbed and the perturbed models respectively, and θ is a $p \times 1$ vector of unknown parameters. Cook proposed to assess the local influence of ω at the postulated model ($\omega = \omega_o$) by the curvature

$$C_l = 2|\, l' \ddot{F} l \,| = 2|\, l' \Delta' \ddot{L}^{-1} \Delta l \,|$$

of the influence graph (2.2) in the direction $l \in R^q$, where l is a fixed nonzero vector of unit length, $\ddot{F}$ is the $q \times q$ matrix with (k, j)th element $\partial^2 L(\widehat{\theta_\omega}) / \partial \omega_k \partial \omega_j$, Δ is the $p \times q$ matrix with elements

$$\Delta_{ij} = \partial^2 L(\theta \mid \omega) / \partial \theta_i \partial \omega_j \mid_{\widehat{\theta}, \omega_o},$$

$L(\theta \mid \omega)$ denotes the log-likelihood corresponding to the perturbed model for a given ω in Ω, and $-\ddot{L}$ is the $p \times p$ observed information matrix for the postulated model($\omega = \omega_o$)

$$\ddot{L} = \partial^2 L(\theta) / \partial \theta_i \partial \theta_j \mid_{\widehat{\theta}}. \tag{2.3}$$

The vector ω_o is called the null vector or null perturbation. The direction of maximum curvature of the likelihood displacement surface is used as the main diagnostic tool in the local-influence method. When a subset θ_1 of $\theta = (\theta_1', \theta_2')'$ is of special interest, the corresponding likelihood displacement is defined as

$$LD(\omega) = 2[L(\widehat{\theta}) - L(\widehat{\theta_{1\omega}}, g(\widehat{\theta_{1\omega}}))] \tag{2.4}$$

where $g(\theta_1)$ is the function that maximizes $L(\theta_1, \theta_2)$ for each fixed θ_1 and $\widehat{\theta_{1\omega}}$ is determined from the partition $\widehat{\theta_\omega}' = (\widehat{\theta_{1\omega}}', \widehat{\theta_{2\omega}}')$.

3. Diagnostics for the Weibull regression models

Now suppose that the failure time t is perturbed according to

$$\tilde{t} = t + \omega = t + al,$$

where $\tilde{t} = (\tilde{t_1}, \cdots, \tilde{t_n})'$, $t = (t_1, \cdots, t_n)'$ and $l = (l_1, \cdots, l_n)'$ denotes a directional vector of unit length, and the quantity a measures the distance from t along the direction l. Therefore, this perturbation scheme produces an n-dimensional space, referred to as ω-space. The likelihood displacement $LD(\omega)$ is a function on the ω-space, and it forms a surface $\alpha(\omega)$ which is an $n+1$ dimensional space:

$$\alpha(\omega) = \begin{pmatrix} \omega \\ LD(\omega) \end{pmatrix}.$$

The hazard functions of the Weibull regression model is

$$h(t|x) = h_o(t)\exp(\beta' x) = \lambda r(\lambda t)^{r-1}\exp(\beta' x)$$

where $h_o(t)$ is a baseline hazard function, $(\beta_1, \cdots, \beta_p)'$ is a vector of regression parameters, and $x = (x_1, \cdots, x_p)'$ is a covariate vector. The effect of the covariates is to act multiplicatively on the Weibull hazard. Then the likelihood functions for the unperturbed and the perturbed models are given respectively by

$$l(\beta, \lambda, r) = \prod_{i=0}^{n} [f(t_i)]^{\delta_i}[S(t_i)]^{1-\delta_i} = \prod_{i=0}^{n} [\lambda r(\lambda t_i)^{r-1}\exp(\beta' x_i)]^{\delta_i} \; [\exp\{-(\lambda t_i)^r\}]^{\exp(\beta' x_i)}$$

$$l(\beta, \lambda, r|\omega) = \prod_{i=0}^{n} [f(\tilde{t}_i)]^{\delta_i}[S(\tilde{t}_i)]^{1-\delta_i} = \prod_{i=0}^{n} [\lambda r(\lambda \tilde{t}_i)^{r-1}\exp(\beta' x_i)]^{\delta_i} \; [\exp\{-(\lambda \tilde{t}_i)^r\}]^{\exp(\beta' x_i)}$$

where δ_i is an indicator variable taking the value 1 for uncensored, and 0 for censored observations.

If we let $L(\theta) = \log l(\theta)$ be the log-likelihood function of $\theta = (\beta', \lambda, r)'$, then the log-likelihood functions for the unperturbed and the perturbed model are given respectively by

$$L(\theta) = \sum_{i=0}^{n} [\delta_i\{r\log\lambda + \log r + (r-1)\log t_i + \beta' x_i\} + \exp(\beta' x_i)(-\lambda^r t_i^{\,r})] \tag{3.1}$$

$$L(\theta \mid \omega) = \sum_{i=0}^{n} [\delta_i\{r\log\lambda + \log r + (r-1)\log \tilde{t}_i + \beta' x_i\} + \exp(\beta' x_i)(-\lambda^r \tilde{t}_i^{\,r})] \tag{3.2}$$

And, Cook's(1986) likelihood displacement is defined as

$$LD(\omega) = 2[L(\hat{\theta}) - L(\hat{\theta}_\omega)]$$

where $\hat{\theta}$ and $\hat{\theta}_\omega$ are maximum likelihood estimates for $\theta = (\beta', \lambda, r)'$ in the unperturbed and the perturbed model, respectively. Differentiating (3.2) with respect to θ and ω, and evaluating at θ and $\omega = 0$, we find

$$\partial^2 L(\theta \mid \omega) / \partial \beta \partial \omega_j = -\exp(\beta' x_j) \lambda r (\lambda t_j)^{r-1} x_j$$
$$\partial^2 L(\theta \mid \omega) / \partial \lambda \partial \omega_j = -\exp(\beta' x_j) \cdot r^2 \lambda^{r-1} \cdot t_j^{r-1}$$
$$\partial^2 L(\theta \mid \omega) / \partial r \partial \omega_j = \delta_j / t_j - \exp(\beta' x_j) \lambda^r t_j^{r-1} \{1 + r \log(\lambda t_j)\} \tag{3.3}$$

where x_j is a covariate vector of the jth observation.

The observed information matrix, $-\ddot{L}$, may be found by taking minus the matrix of second derivatives of the log-likelihood function (3.1) with respect to θ and all derivatives are evaluated at the maximum likelihood estimates $\hat{\theta}$. The elements of this matrix are given by

$$\partial^2 L(\theta) / \partial \beta \partial \beta' = -\sum_{i=1} (\lambda t_i)^r \exp(\beta' x_i) x_i x_i'$$
$$\partial^2 L(\theta) / \partial \beta \partial \lambda = -\sum_{i=1} \{ r \lambda^{r-1} t_i^r \exp(\beta' x_i) x_i \}$$
$$\partial^2 L(\theta) / \partial \beta \partial r = -\sum_{i=1} \{ (\lambda t_i)^r \log(\lambda t_i) \exp(\beta' x_i) x_i \}$$
$$\partial^2 L(\theta) / \partial \lambda^2 = -(r/\lambda^2) \sum_{i=1} \{ \delta_i + (r-1)(\lambda t_i)^r \exp(\beta' x_i) \}$$
$$\partial^2 L(\theta) / \partial \lambda \partial r = (1/\lambda) \sum_{i=1} \{ \delta_i - \exp(\beta' x_i)(\lambda t_i)^r (1 + r \log(\lambda t_i)) \}$$
$$\partial^2 L(\theta) / \partial r^2 = -\sum_{i=1} \{ \delta_i / r^2 + \exp(\beta' x_i)(\lambda t_i)^r (\log(\lambda t_i))^2 \} \tag{3.4}$$

By that, the observed Hessian matrix $\ddot{F}$ takes the form

$$\ddot{F} = \Delta' \ddot{L}^{-1} \Delta. \tag{3.5}$$

Explicit analytic expressions of the direction of maximum curvature, however, can not be derived explicitly for (3.5). And, $c_{\max}$ corresponds to the maximum absolute eigenvalue of $\ddot{F}$ in (3.5) with corresponding eigenvector $l_{\max}$. Thus the maximum curvature occurs in the direction $l_{\max}$. Elements of $l_{\max}$ are examined individually with large values indicating observations which are possibly influential. In addition, it should be noted that the ith diagonal element of the Hessian matrix $\ddot{F}$ becomes the curvature of the likelihood displacement (2.1), when only a single observation, say the ith, is perturbed according to $\tilde{t}_i = t_i + \omega_i$.

4. Example

As a numerical illustration, we generate 30 samples of ω for the model

$$y = \log t = \alpha + \beta x + \sigma\omega,$$

where x is a regressor variable, α is an intercept term, regression coefficient β, scale parameter σ, and error term ω is standard extreme value distributed. The value of α was set to 0, and the values of β and σ were set to 1/2.1 and 1/2.1, respectively. The design matrix X and observation vector t are given in Table 1. We suppose there are three censored data; t_1, t_8 and t_{13}.

The diagnostics of local influence, $l_{\max}$, on all the parameters in Weibull regression model are given in Table 2, in case the response vector y is perturbed $\hat{y}$.

Next, consider the joint influence provided by $l_{\max}$, when both y_8 and y_{13} are perturbed as $\widetilde{y_8} = y_8 + \delta$ and $\widetilde{y_{13}} = y_{13} + \delta$, respectively. The diagnostics of local influence, $l_{\max}$, on all the parameters are given in Table 3 and Table 4, respectively, in case two or three response observations are perturbed.

For example, when two observations y_8 and y_{13} are perturbed as $\widetilde{y_8}$ and $\widetilde{y_{13}}$, respectively, we can find that the elements of $l_{\max}$ corresponding to $\widetilde{y_8}$ and $\widetilde{y_{13}}$ are the largest among those of $l_{\max}$. When there are only one or three perturbed observations, the same results can be obtained as the example given above (see Table 2 and Table 4 below).

Table 1. Design matrix X and observation vector t

Case	X	t
1	15.3333	4130.80
2	15.6667	1343.04
3	16.0000	2808.92
4	16.3333	2634.56
5	16.6667	3199.78
6	17.0000	2553.16
7	17.3333	6836.93
8	17.6667	9450.40
9	18.0000	8644.76
10	18.3333	8526.74
11	18.6667	4708.74
12	19.0000	20675.29
13	19.3333	26485.15
14	19.6667	15996.63
15	20.0000	11837.40
16	20.3333	13101.25
17	20.6667	18707.40
18	21.0000	54303.15
19	21.3333	18690.62
20	21.6667	44499.84
21	22.0000	90874.83
22	22.3333	51122.54
23	22.6667	72578.59
24	23.0000	50663.77
25	23.3333	48932.81
26	23.6667	273630.23
27	24.0000	72498.15
28	24.3333	121154.95
29	24.6667	151262.51
30	25.0000	260902.00

Table 2. Local influence of a single observation

cases	(1) $\widetilde{y_1} = y_1 + \delta$		(2) $\widetilde{y_8} = y_8 + \delta$		(3) $\widetilde{y_{13}} = y_{13} + \delta$	
	f_{ii}	l_{max}	f_{ii}	l_{max}	f_{ii}	l_{max}
1	-1.902E-6	0.9994522	-1.77E-8	0.0495059	-1.27E-8	0.3394813
2	-5.459E-9	-0.019796	-7.452E-9	-0.086795	-6.496E-9	0.365098
3	-5.918E-9	-0.013633	-6.261E-9	-0.045824	-5.25E-9	0.302368
4	-2.088E-9	-0.006819	-2.057E-9	-0.019128	-1.691E-9	0.1651818
5	-2.743E-9	-0.01106	-3.196E-9	-0.042482	-2.727E-9	0.2261837
6	-1.601E-9	-0.010772	-2.199E-9	-0.047326	-1.914E-9	0.1985573
7	-2.434E-9	-0.007361	-2.411E-9	-0.020607	-1.981E-9	0.1788377
8	-2.251E-9	-0.006091	-1.725E-7	0.9905703	-1.716E-9	0.1608819
9	-1.207E-9	-0.005809	-1.255E-9	-0.018432	-1.044E-9	0.1332802
10	-7.26E-10	-0.005204	-8.08E-10	-0.018728	-6.82E-10	0.111282
11	-3.24E-10	-0.005329	-4.81E-10	-0.024396	-4.2E-10	0.0946974
12	-8.48E-10	-0.003604	-7.94E-10	-0.007422	-6.4E-10	0.0976756
13	-6.47E-10	-0.002511	-5.74E-10	-0.002461	-3.368E-8	-0.649403
14	-2.11E-10	-0.002918	-2.4E-10	-0.010799	-2.03E-10	0.0613926
15	-1.01E-10	-0.002626	-1.36E-10	-0.011356	-1.18E-10	0.0491869
16	-7.15E-11	-0.002276	-9.83E-11	-0.009985	-8.54E-11	0.0421005
17	-5.95E-11	-0.001889	-7.64E-11	-0.007896	-6.59E-11	0.0363903
18	-1.4E-10	-0.002693	-1.37E-10	-0.004375	-1.12E-10	0.0424778
19	-9.29E-12	-0.000609	-1.05E-11	-0.002239	-8.92E-12	0.0128928
20	-3.53E-11	-0.001201	-4.02E-11	-0.004447	-3.41E-11	0.0252935
21	-5.89E-11	-0.001127	-5.8E-11	-0.003038	-4.78E-11	0.0280141
22	-1.51E-11	-0.000872	-1.6E-11	-0.003463	-1.56E-11	0.0174987
23	-1.39E-11	-0.000765	-1.67E-11	-0.002857	-1.36E-11	0.0160286
24	-5.9E-12	-0.000642	-7.99E-12	-0.002788	-6.92E-12	0.0119941
25	-3.88E-12	-0.000567	-5.62E-12	-0.002564	-4.9E-12	0.0102247
26	-2.08E-11	-0.000602	-1.96E-11	-0.001358	-1.61E-11	0.015972
27	-2.12E-12	-0.000407	-2.98E-12	-0.001815	-2.59E-12	0.0074157
28	-2.01E-12	-0.000341	-2.54E-12	-0.001407	-2.19E-12	0.006648
29	-1.56E-12	-0.000293	-1.94E-12	-0.001192	-1.67E-12	0.0057831
30	-1.9E-12	-0.000273	-2.13E-12	-0.000991	-1.8E-12	0.0058408

NOTE: δ = 1, f_{ii} = the i-th diagonal element of matrix $\ddot{F}$ and l_{max} corresponds to the maximum absolute eigenvalue of $\ddot{F}$.

Table 3. Local influence of two observations

cases	(1) $\widetilde{y_1}=y_1+\delta$ $\widetilde{y_8}=y_8+\delta$		(2) $\widetilde{y_1}=y_1+\delta$ $\widetilde{y_{13}}=y_{13}+\delta$		(3) $\widetilde{y_8}=y_8+\delta$ $\widetilde{y_{13}}=y_{13}+\delta$	
	f_{ii}	l_{max}	f_{ii}	l_{max}	f_{ii}	l_{max}
1	-1.608E-6	0.9908216	-1.538E-6	0.9967765	-1.056E-8	-0.042721
2	-3.901E-9	-0.025886	-3.145E-9	-0.028627	-5.067E-9	-0.126006
3	-3.973E-9	-0.020286	-3.057E-9	-0.023192	-4.209E-9	-0.089165
4	-1.377E-9	-0.010803	-1.046E-9	-0.012524	-1.365E-9	-0.045175
5	-1.886E-9	-0.015495	-1.48E-9	-0.017443	-2.162E-9	-0.071258
6	-1.146E-9	-0.014042	-9.27E-10	-0.015505	-1.491E-9	-0.068384
7	-1.607E-9	-0.011628	-1.226E-9	-0.013455	-1.595E-9	-0.048515
8	-4.673E-8	0.1272775	-.11E-9	-0.01196	-7.191E-8	0.8649552
9	-8.07E-10	-0.008782	-6.22E-10	-0.010056	-8.36E-10	-0.037843
10	-4.94E-10	-0.007481	-3.86E-10	-0.008464	-5.43E-10	-0.033547
11	-2.37E-10	-0.006764	-1.94E-10	-0.007409	-3.25E-10	-0.033608
12	-5.52E-10	-0.006116	-4.19E-10	-0.007168	-5.16E-10	-0.02392
13	-4.15E-10	-0.004715	-1.238E-8	0.0607836	-2.055E-8	0.4543693
14	-1.44E-10	-0.004133	-1.13E-10	-0.004656	-1.61E-10	-0.018704
15	-7.21E-11	-0.003443	-5.83E-11	-0.003802	-9.2E-11	-0.016623
16	-5.13E-11	-0.002959	-4.16E-11	-0.00326	-6.65E-11	-0.014384
17	-4.2E-11	-0.002519	-3.37E-11	-0.002794	-5.17E-11	-0.011975
18	-9.24E-11	-0.002693	-7.1E-11	-0.003112	-9.02E-11	-0.01098
19	-6.36E-12	-0.000863	-5.01E-12	-0.000971	-7.08E-12	-0.003891
20	-2.42E-11	-0.001694	-1.91E-11	-0.001905	-2.71E-11	-0.007662
21	-3.9E-11	-0.001778	-3.01E-11	-0.002047	-3.84E-11	-0.007304
22	-1.05E-11	-0.001191	-8.37E-12	-0.001328	-1.23E-11	-0.005533
23	-9.57E-12	-0.001074	-7.57E-12	-0.001205	-1.08E-11	-0.004868
24	-4.22E-12	-0.000837	-3.42E-12	-0.000922	-5.4E-12	-0.004048
25	-2.82E-12	-0.000723	-2.31E-12	-0.000792	-3.79E-12	-0.003567
26	-1.36E-11	-0.000989	-1.05E-11	-0.001146	-1.29E-11	-0.003902
27	-1.52E-12	-0.000522	-1.24E-12	-0.000573	-2.01E-12	-0.002557
28	-1.41E-12	-0.000456	-1.14E-12	-0.000505	-1.72E-12	-0.002152
29	-1.09E-12	-0.000395	-8.79E-13	-0.000438	-1.31E-12	-0.001851
30	-1.3E-12	-0.000387	-1.03E-12	-0.000435	-1.44E-12	-0.001734

NOTE: δ = 1, f_{ii} = the i-th diagonal element of matrix $\ddot{F}$ and l_{max} corresponds to the maximum absolute eigenvalue of $\ddot{F}$.

Table 4. Local influence of three observations

Cases	$\widetilde{y_1}=y_1+\delta$ $\widetilde{y_8}=y_8+\delta$ $\widetilde{y_{13}}=y_{13}+\delta$	
	f_{ii}	l_{max}
1	-1.369E-6	0.9899683
2	-2.735E-9	-0.029838
3	-2.601E-9	-0.024558
4	-8.83E-10	-0.013351
5	-1.269E-9	-0.018346
6	-8.05E-10	-0.016158
7	-1.033E-9	-0.014345
8	-3.034E-8	0.1172632
9	-5.27E-10	-0.010675
10	-3.29E-10	-0.008934
11	-1.7E-10	-0.007694
12	-3.5E-10	-0.007701
13	-9.723E-9	0.0577594
14	-9.67E-11	-0.004907
15	-5.05E-11	-0.003967
16	-3.61E-11	-0.003397
17	-2.91E-11	-0.002922
18	-5.95E-11	-0.003326
19	-4.27E-12	-0.001024
20	-1.63E-11	-0.002008
21	-2.52E-11	-0.002186
22	-7.17E-12	-0.001394
23	-6.45E-12	-0.001269
24	-2.96E-12	-0.000962
25	-2.01E-12	-0.000824
26	-8.75E-12	-0.00123
27	-1.08E-12	-0.000596
28	-9.79E-13	-0.000529
29	-7.54E-13	-0.000459
30	-8.74E-13	-0.000459

NOTE: $\delta = 1$, f_{ii} = the i-th diagonal element of matrix $\ddot{F}$ and l_{max} corresponds to the maximum absolute eigenvalue of $\ddot{F}$.

We consider another example, accelerated life test data of Nelson and Hahn (1972). They presented data on the number of hours to failure of motorettes operating under various temperatures. Hours to failure motorettes are given as a function of operating temperatures of 150° C, 170° C, 190° C, or 220° C. The data are presented in Table 5 and exhibit severe censoring, with only 17 out of 40 motorettes failing. The censoring here is Type I or time censoring; that is, censored survival times were observed only if failure had not occurred prior to termination of the study. A Weibull regression model with the single regressor variable, $z = 1000/(273.2 + \,^\circ C)$ may well be taken for failure time. Because all the 10 observations at temperature 150° C are censored(see Huh et al. (1991) and Kalbfleisch et al. (1980)), we can only consider the data set where all the 10 observations at temperature 150° C are not included. A Newton-Raphson iteration(See Kalbfleisch et al. (1980)) gives

$$\widehat{\alpha} = -11.891, \quad \widehat{\beta} = 9.038, \quad \widehat{\sigma} = .3613$$

where $\widehat{\alpha}$, $\widehat{\beta}$ and $\widehat{\sigma}$ are the estimators of intercept α, regression coefficient β and scale parameter σ, respectively.

The diagnostics of local influence, $l_{\max}$, on all the parameters are also given in Table 5, when the failure time t is perturbed $\tilde{t}$. Note that the ith diagonal element of the Hessian matrix $\ddot{F}$ becomes the curvature of the likelihood displacement (2.1), when only a single observation, say the ith, is perturbed according to $\tilde{t}_i = t_i + \omega_i$. The observations corresponding to dominating elements of $l_{\max}$ have the possibility of being perturbed ones. In Table 5, cases with higher influence are when life-times are 408 at temperature 190° C and life-times are 504 and 528 at temperature 220° C, respectively. These findings are consistent with the corresponding perturbed likelihood displacements, $2[L(\widehat{\theta}) - L(\widehat{\theta}(t_i + 100))]$ and $2[L(\widehat{\theta}) - L(\widehat{\theta}(t_i - 100))]$ where $L(\widehat{\theta})$ and $L(\widehat{\theta}(t_i + 100))$ are the log-likelihood corresponding to the postulated model and the log-likelihood corresponding to the postulated model perturbing t_i by adding 100, respectively. Also note that the absolute value of $l_{\max}$ is monotone, or decreasing and increasing at each level of temperatures, respectively. The proposed curvature diagnostics are considered to act quite well regardless of the masking effect that will go undetected with standard deletion diagnostics. In the computations of the $l_{\max}$ and other measures derived in this paper, SAS (SAS/IML) Release 6.12 was used.

Table 5. Diagnostics for Motorettes data when n = 30

Temperature ^{o}C	Lifetime (hours)	δ_i	No. of observation	diagonal of $\ddot{F}$	$l_{max}(100)$	L-(1000)	L+(1000)
170	1764	1	1	-1.45E-7	-5.294	1.5127	1.3923
170	2772	1	1	-6.22E-8	-3.465	.642	.6035
170	3444	1	1	-3.80E-8	-2.541	.3914	.3696
170	3542	1	1	-3.56E-8	-2.405	.3663	.3464
170	3780	1	1	-3.09E-8	-2.070	.3162	.3021
170	4860	1	1	-3.45E-8	-.340	.3347	.3557
170	5196	1	1	-4.70E-8	.290	.4531	.4866
170	5448	0	3	-9.57E-8	2.335	.9329	.9797
190	408	1	2	-2.39E-6	-22.022	29.7444	20.06
190	1344	1	2	-2.03E-7	-3.849	2.2567	1.8229
190	1440	1	1	-1.60E-8	-2.702	1.7832	1.42
190	1680	0	5	-8.51E-8	5.330	.721	1.0004
220	408	1	2	-2.31E-6	.012	30.3566	18.6345
220	504	1	3	-1.76E-6	18.627	18.3176	20.0509
220	528	0	5	-2.61E-6	39.556	17.8909	34.7867

Note : $\delta_i = 1$ if observation is uncensored and $\delta_i = 0$ if observation is censored, $l_{max}(100) = l_{max} \times 100$, and L-(1000) is 1000 times the difference between log-likelihood corresponding to the postulated model and the postulated model perturbing t_i by subtracting 100, respectively.

References

[1] Cook, R. D. (1986) Assessment of local analysis, *Journal of the Royal Statistical Society Series B*, 48, 133-169.

[2] Huh, M. H. & M. L. Park (1991) Survival Analysis, Seoul : Freedom Academy.

[3] Kalbfleisch, J. D. & R. L. Prentice (1980) The statistical Analysis of Failure Time Data, New York : Wiley.

[4] Kim, S. K. & R. Huggins (1998) Diagnostics for autocorrelated regression models, *Australian & New Zealand Journal of Statistics*, 40, 65-71.

[5] Lawless, J. F. (1982) Statistical Models and Methods for Lifetime Data, New York : Wiley.

[6] Nelson, W. B & G. J. Hahn (1972) Linear estimation of a regression relationship from censored data, part 1-simple methods and their applications, *Technometrics* 14, 247-276.

[7] Pettitt A. N. & I. Bin Daud (1989) Case-weighted measures of influence for Proportional Hazards Regression, *Applied Statistics.* 38, 51-67.

[8] Schwarzman, B. (1991) A Connection Between Local Influence Analysis and Residual Diagnostics, *Technometrics*, 103-4.

[9] Weissfeld, L. A. (1990) Influence diagnostics for the proportional hazards model, *Statistics & Probability Letters*, 10, 411-17.

[10] Weissfeld, L. A. & Helmut Schneider (1990) Influence diagnostics for the Weibull model fit to censored data, *Statistics & Probability Letters*, 9, 67-73.

[11] Weissfeld, L. A. & Helmut Schneider (1990) Influence diagnostics for the normal linear model with censored data, *Australian & New Zealand Journal of Statistics*, 32, 11-20.