Issues When Estimating Fatigue Life of Structures

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ABSTRACT

When estimating fatigue crack growth (FCG) life of structures, the use of crack growth models and knowledge of the values of their corresponding parameters are of vital importance. Inconsistency in using models with appropriate parameters can lead to enormous errors in FCG life prediction. In this paper examples are analyzed and compared with test results to show the possible problems. Consistency checks are necessary for avoiding some pitfalls, and also necessary for verifying the correct performance and accuracy of the used computer program.

Keywords : Fatigue crack growth, fracture mechanics, fatigue life, crack growth model, consistency, computer program performance

1. Introduction

Estimation of crack growth life in a cracked body is composed of two steps.

The first step is the material property representation. It usually involves testing the material according to some standard to obtain the material property. For crack analysis using fracture mechanics, the direct test results will be a~N curve for a cracked specimen as shown in Fig. 1, where a is the crack length, N the number of fatigue cycles. Then the data are processed to obtain a crack growth rate curve as shown in Fig. 2.

The second step will be the crack growth analysis based on $da/dN \sim \Delta K$ relation.

Bridging these two steps, at the center is the crack growth rate (CGR) model. Consistency issues in these two steps are the main concern of this paper. The problems will be discussed based on real test and analysis results.

Fig. 1 and Fig. 2 are actual test results for a Center Cracked Tension (CCT) specimen made of 2024-T3 thin sheet [1,2], which will be used extensively to illustrate our points. The width of the specimen is 60mm and its thickness is 1.016mm.

The chemical composition of the material is as follows (%wt): Si=0.11, Fe=0.23, Cu=4.46, Mn=0.58, Mg=1.44, Cr=0.04, Zn=0.03, Ti=0.02.

The related mechanical properties of the 2024-T3 are as follow [1]:

Young's modulus E=70000MPa, Poisson's ratio ν =0.3, yield stress σ_y=370MPa, fracture elongation =17%, tensile strength=500MPa, plane stress fracture toughness K_c=80MPa $\sqrt{m}$.

2. Consistency between basic models and their parameters

There are many models that can be used in crack growth analysis. To make the discussion clear, specific simple models will be used in examples.

The most simple and widely used one is the Paris-Erdogan model(or, as often simply called, Paris law)[3], i.e.:

$$da/dN = C(\Delta K)^n \quad (1)$$

Also equally well known, Forman Equation [4] can take the stress ratio R into account:

$$da/dN = C(\Delta K)^n / [(1-R)K_c - \Delta K] \quad (2)$$

Walker Equation [5] is another one to account the R effects:

$$da/dN = C(\Delta K)^n/(1-R)^m \quad (3)$$

where R : stress ratio ($= \frac{\text{min . stress}}{\text{max . stress}}$)

It is all apparent and well known that the so-called "material property constants" (C, n and m) are not the same using different models. From the same da/dN test data in Fig. 2, values of these material constants are listed in Tab. 1. They are obtained by statistic regression. As the data are only for one stress ratio, it is not possible to fit a sensible m value. Walker Eq. (3) degenerates to simple Eq. (1) by forcing m=0.

The solid line curve shown in Fig. 2 is actually the Paris model (Eq. 1) with the experimental points (symbols) shown as well. It is seen from Table 1 that the values of C differ by the order of nearly 2 decimal positions. The predicted curves using both the fitted Paris and Forman equations are compared with test points in Fig. 3. It is seen that they can represent test points almost equally well.

The most important thing is not the question which model is used, but what matters mostly is that parameters to be used must be consistent with a chosen model. The parameters are constant only relative to a specific model, but not so with different models. In a word, they are not truly material constants. As this is a well-known fact, we are not discussing it in detail, but simply showing the brief table 1 to emphasize the difference. The other issues of consistency that could be overlooked will be discussed in more details.

3. Consistency with modified models

Enormous models are derived from the above basic equations by making modification using "effective stress intensity" concept to account for the variation of stress ratio R, or the effect of "crack closure" [6]. In simple form, the effective stress intensity factor is,

$$\Delta K_{eff} = U\Delta K_{app} \quad (4)$$

($= U\Delta\sigma\sqrt{\pi a} \cdot \sec(\pi a/W)$ for CCT)

Combined with the Paris law, we have:

$$da/dN = C(\Delta K_{eff})^n \quad (5)$$

Elber [6] showed empirically for 2024-T3 Al alloy that

$$U = 0.5 + 0.4R, \quad \text{for} \quad 0 < R < 0.7 \quad (6)$$

We call this model (eqs. 4+5+6) the Paris-Elber model for clearness.

Other modifications, such threshold correction, retardation effect correction etc, can make the compound models even more complex.

With all kinds of modified models, inconsistency problems may occur in 2 manners. Here the Paris-Elber model is used as an example for its simplicity.

3.1 Inconsistency case 1

This kind of inconsistency often occurs in following situation.

The researchers may have conducted quite a lot of experiments, say, crack growth rate tests under different stress ratios When test results are processed it is found that they can be represented best using the above "effective stress intensity" concept. So the results are expressed in the form of $da/dN \sim \Delta K_{eff}$ curve(Eq.5) as shown in Fig. 4.

As the results are put into usage for analyzing structure component life, some kind of analysis program (available commercially or from other source, such as in [7]) is used. The program gives out life predictions and it seems no problem whatsoever. However, actually the analysis program may not treat the da/dN data as an "effective" one, or the program may not implement the Elber model at all, which is out of the experimenters' control If that is the case, the predicted life is seriously in error.

Here in Fig 4, the curve was reprocessed using Paris-Elber model based on the same data in Fig. 1. Firstly, we would like to point out that the values of C and n for the Paris-Elber model (C=2.2585E-10, n=3.23) are not the same as (quite different from) those for simple Paris model shown in Table 1 in the last section (Not material constants). Secondly, if these values are put into a program that did not implement Elber model, the calculated crack growth behavior will not resemble the origin Fig. 1 at all. In Fig. 5 shown below, the dotted line is the response from the inconsistent prediction. The solid line is the predicted a~N curve using consistent Paris law (values of C and n in Table 1), which agrees well with the test symbol points The difference is so great that we do not need to say any more about it This kind of inconsistent error will make the resulting structure too conservative and cost too high. In short, if the analysis program does not incorporate Elber model, the values of c and n input to the program must be values in Tab. 1 instead of values derived from Eq.5.

3.2 Inconsistency case 2

There are other chances that the analysts have developed their own program that has many complex models implemented. They get some material property data from literature or other sources, such as in reference [8], and start to analyze the crack growth behavior under variable amplitude load spectrum. To suit the varied R ratios, some kind of modified model is used in the analysis. In this situation, the predicted life could also be misleading as there might be a potential inconsistency problem.

The data from literature (or other sources) are usually processed against apparent applied stress intensity, certainly not according to their modified models. As we said, again and again, that the parameters in a specific model (including detail modifications) are not material constants. This point can also be seen clearly by comparing figures 2 and 4.

The solid line curve 1 in Fig. 6 is the predicted crack growth behavior using consistent Paris-Elber model, i.e., values of C and n derived from Eq.5 (C=2.2585E-10, n=3.23 as stated in last section). It agrees well with the test points. The dotted line 2 predicts the behavior wrongly due to the inconsistency problem values of parameters for simple Paris model from Table 1 (simulating data from literature) are used in the Paris-Elber model prediction. It can be seen this kind of inconsistency may predict a much longer life that leads not only to no conservative but also dangerous.

In actual analysis, such big errors as we illustrated in case1 and 2 will be probably easily spotted if a consistency check is made. A re-analysis is then inevitable. If the consistency issue was well kept in analysts mind, the analysis could be done rightly at the first time. Whats more important, if some errors (with other kind of modifications, such as for threshold) were not easily spotted for variable amplitude fatigue analysis of a structure and no consistency check were attempted, an inconsistency problem could present in the results without noticed. However, a consistency analysis for a constant amplitude test applied back to the same standard specimen, such as we did here, will be very useful for checking the inconsistency problem.

4. Other consistency issues

First we are going to show a minor error of inconsistency due to the test data processing which could be easily overlooked.

During rate formula fitting, discrete measurements of crack length a_i at loading cycle N_i were used to calculate $(\Delta N/\Delta a)$ as approximation of (da/dN) within the interval from N_i to N_{i+1}. This average rate may seem to be related to any cycle N from N_i to N_{i+1}, but the stress intensity range ΔK at different N are not the same. One could use the right end value of ΔK_{i+1} at cycle N_{i+1}, or use the left end value of ΔK_i at cycle N_i, together with the $(\Delta N/\Delta a)$ to make a data point (a pair), and it is usually done that way. Certainly there is some difference between these 2 end values, and different fitted parameters (C and n, for Paris' law) will be obtained. The predicted crack growth curves based on them are shown in Fig. 7. It can been seen neither curve using left end K nor that using right end K resembles test points properly, especially when crack growth longer. Instead, using the K values at the middle of the interval will make a much better prediction as also shown by Fig. 7. If life prediction is made for structure under variable loading without a consistency check, the incorporated error will not be noticed.

As the computer software technology advances, now it is easy to represent material characteristics in a numerical discrete table format without using any analytical functions.

Since the inconsistency problems discussed above all associated with some analytical rate models, one might think that there would be no such problem with numerical rate tables. However, this is not true. The inconsistency is not coherent with analytical model, but resulted from inconsistent use of data. If the data table used to represent the material is not in the corresponding form for life prediction, the same sort of error can also occur as discussed above.

Shown in Fig. 8 are similar errors as discussed above. The tabular rate data were processed using Elbers effective stress intensity concept (as in Fig. 4) and input into the analysis program. If the crack growth analysis is done correctly using Elbers model, the predicted growth curve will be the solid line that agrees with test results. If growth analysis is made based on applied stress intensity (without Elbers correction) the predicted life curve will be the dot-dashed line which gives a much shorter life. Another dotted line shows the opposite error (rate data table of applied SIF used in Elbers model) similar as the case 2 discussed.

If the rate data table is in the form of applied stress intensities, it must be a 2-way (2 dimensional) table. One-dimensional numerical table will not be able to predict crack growth under random loading since the CGR is also stress ratio dependent (apart from dependent on stress intensity). Overlooking this issue will result in some other inconsistency error.

Consistency check should also be performed for retardation (interaction) models. Any analysis for constant amplitude test using whatever retardation model must agree with the result of non-retardation model. This fact can be used to verify whether a retardation model has been implemented correctly.

5. Conclusions

As illustrated above, the consistency issue is very important in fatigue crack growth analysis. In order to perform correct crack analysis, we strongly suggest that consistency check must be made before any structural analysis. The consistency check on individual standard test specimen under constant amplitude fatigue loading serves as multi-purposes :

- To check that the analysts fully understand the problem and know exactly what model to be used.
- To check whether the analysis program has actually implemented the model needed.
- To check whether the proper data (including their presentation form) are used for analysis input.
- To verify whether the computing software performs correctly and how good its performance. This is necessary no matter the program is self-developed or commercially available.

We have developed our own computer software that includes the rate data model fitting (parameter determination) and the crack growth analysis in one package consistently. We have run consistent analyses extensively to check the program and found its performance satisfactory.

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