Quasi-LQG/H_∞/LTR Control for a Nonlinear Servo System with Coulomb Friction and Dead-zone

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ABSTRACT

In this paper we propose a controller design method, called Quasi-LQG/H_∞/LTR for nonlinear servo systems with hard nonlinearities such as Coulomb friction, dead-zone. Introducing the RIDF method to model Coulomb friction and dead-zone, the statistically linearized system is built. Then, we consider the H_∞ performance constraint for the optimization of statistically linearized systems, by replacing a covariance Lyapunov equation into a modified Riccati equation of which solution leads to an upper bound of the LQG performance. As a result, the nonlinear correction term is included in coupled Riccati equation, which is generally very difficult to have a numerical solution. To solve this problem, we use the modified loop shaping technique and show some analytic proofs on LTR condition. Finally, the Quasi-LQG/H_∞/LTR controller for a nonlinear system is synthesized by inverse random input describing function techniques (IRIDF). It is shown that the proposed design method has a better performance robustness to the hard nonlinearity than the LQG/H_∞/LTR method via simulations and experiments for the timing-belt driving servo system that contains the Coulomb friction and dead-zone.

Keywords : RIDF, LQG/ H_∞, LTR, coulomb friction, dead-zone, timing-belt driving servo system

1. Introduction

For a servomechanism, hard nonlinear elements such as Coulomb friction, dead-zone and backlash are often appeared. These nonlinear elements make the exact control of a servo system to be difficult. To avoid this problem, several different control methods need to design a nonlinear controller as to each hard nonlinear characteristic such as friction [1], piecewise saturation [2] and saturation [3], but unified and systematic approaches for a hard nonlinear multivariable servo system have not fully developed yet. Generally, the linear approximation control methods by Taylor series expansion are limited in their applicability due to the discontinuous differentiability of nonlinearities. It is known that the use of statistical linearization techniques can be effective for many problems for hard nonlinear servo systems. If the systematic multivariable control methods of the linear systems can be applied to hard nonlinear servo systems under any acceptable conditions, it is very desirable to develop the general nonlinear controller design methods.

For this problem, Beaman developed the nonlinear quadratic Gaussian control method [4], which combines the optimal estimation and control for statistically linearized systems. This method, however, has the drawbacks that selecting design parameters is complex and the nonlinear correction term is often complicated. In addition, NQG control does not fully address performance and stability robustness issues. In order to solve these and other issues, the nonlinear quadratic Gaussian control with loop transfer recovery (QLQG/LTR) [5] that has the LQG performance criterion. The QLQG/LTR method, however, often has a weak point in face of the uncertainty of the system than a H_∞ norm-based control method.

In this paper, we will develop the Quasi-LQG/H_∞/LTR method for a hard nonlinear system as a generalization of the QLQG/LTR method to consider additional H_∞ robustness. For a linear system, the $LQG/H_\infty/LTR$ method [6] have developed, which was derived from the Doyle's mixed H2/ H_∞ method [7]. But Yeh's $LQG/H_\infty/LTR$ method mainly focused on the loop shaping method only and was not interested in a hard nonlinearity. The proposed Quasi-$LQG/H_\infty/LTR$ method derives from LQG/H_∞ control method [8] and the QLQG/LTR method to get advantages of two methods.

The main design techniques that are included in our method are the random input describing function (RIDF) technique [9], IRIDF [10,11] and modified LTR. Therefore, our proposed method can guarantee stability-robustness (in sense of H_∞ - norm bound) and nominal performance (in sense of LQG cost bound) as well as robustness to hard nonlinearities. Our method is a general version of the QLQG/LTR method because if H_∞ - norm parameter γ approaches infinity, then Quasi-$LQG/H_\infty/LTR$ control becomes the QLQG/LTR. A timing-belt driving servo system containing Coulomb frictions and dead-zone is to examine the robustness of the controller to the hard nonlinear effects through simulations and experiments to show performance of the proposed control method.

2. Modeling the Nonlinear Servo System

A schematic diagram of the nonlinear servo system with Coulomb friction and dead-zone is shown in Fig. 1.

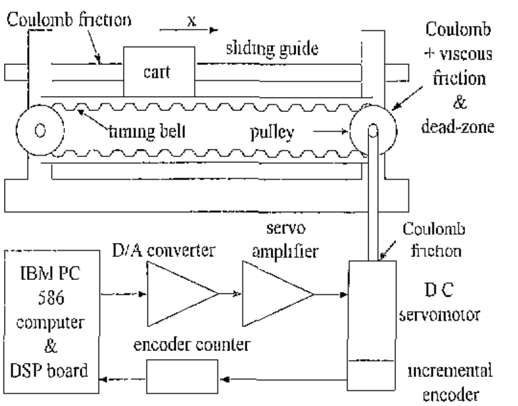

Fig. 1 A schematic diagram of the timing-belt driving servo system

In servo mechanism, in order to transmit a power between axes or to carry a cart for the translation motion, a timing-belt often is used. For a light load, a timing-belt system need no lubrication and no sliding between belt and pulley teeth have fast transmission of the power under lower noise than a gear system. Specially, when the distances between two axes are large, the timing-belt is very appropriate. Because of these properties, the timing-belt is often chosen for a robot system and many automatic mechanisms. However, if the distances between the axes are larger and moving loads that are connected to belt exist, a guide for moving loads must be considered to prevent the deflection of the belt. In this case, the Coulomb friction between the contacting surfaces of guides and moving loads can appear. Furthermore, a dead-zone may be accompanied due to the loose engagements between pulley and belt. Therefore, these nonlinearties must be considered where the timing-belt is chosen as a power transmission device.

In this paper, the cart is connected with timing-belt in its lower part, and moves through two sliding guide axes. Two pulleys are connected to a timing-belt, and one side of pulley is combined with a servo DC motor. A FDD-102PD motor driver drives the servo DC motor of FMD-E10EA of LG incorporation attaching the incremental rotary encoder with 1000 pulse/rev resolution. The PC containing DSP system composed of TMS320C processor of dSPACE incorporation reads the pulse from the rotary encoder and sends the control command through D/A converter to the motor driver. Fig 2 gives the photograph of the timing-belt driving servo system.

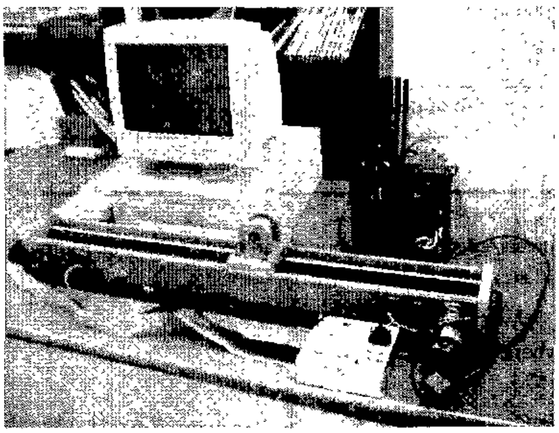

Fig. 2 Photograph of the timing-belt driving servo system

In this cart, a Coulomb friction exists between the sliding guide and cart contacting part and a dead-zone phenomenon is observed in combing part of the pulley and timing-belt. Since the cart system have the motions of translation and rotation exist, in order to simplify the problem, two motions can be transformed into a equivalent translation motion as the following dynamic equation:

$$M_{eq}\ddot{x}(t)+C_{eq}\dot{x}(t)+F_{feq}sgn(\dot{x})=Dz\cdot F_{eq}(t) \qquad (1)$$

where $x(t)$ is the position of the cart and M_{eq}, C_{eq}, F_{feq}, Dz, $F_{eq}(t)$ represent the equivalent values of mass, friction, Coulomb friction force and dead-zone of the entire system. Table 1 shows the value of system parameters. If the nonlinear plant is linearized via statistical linearization techniques, then the statistically

Table 1 Values of the system parameter

Name	Values of parameter
guide length	100 cm
distance between pulley	92 cm
guide diameter	2 cm
r_p (radius of pulley)	2.83 cm
pitch of pulley	17.78 cm/rev
M_{eq}	0.003767 $Kgf\cdot sec^2/cm$
C_{eq}	0.24853 $Kgf\cdot sec/cm$
F_{feq}	0.42714 Kgf
δ (width of dead-zone)	0.25117
K_{amp}	1.5714 $Kgf\cdot cm/V$

linearized state space model can be described as follows:

$$\begin{aligned}\dot{x}(t)&=N(\sigma_x)x(t)+Bu(t)\\ y(t)&=Cx(t)\end{aligned} \qquad (2)$$

where

$$N(\sigma_x)=\begin{bmatrix}0 & 1\\ 0 & -\dfrac{C_{eq}+N_f}{M_{eq}}\end{bmatrix},\quad B=\begin{bmatrix}0\\ \dfrac{K_{amp}}{M_{eq}}N_{dz}\end{bmatrix},$$

$$C=[1\quad 0],$$

$$N_f=\frac{C_{feq}\sqrt{2/\pi}}{\sigma_{x2}},\quad N_{dz}=1-erf\left(\frac{\delta}{\sigma_u}\right).$$

N_f and N_{dz} are the DF gains for Coulomb friction and dead-zone, δ is the width of dead-zone and σ_{x2} and σ_u are the standard deviations of the state variable x_2 and control input $u(t)$, respectively.

3. Quasi-LQG/H$_\infty$/LTR Control

We will develop the Quasi-LQG/H$_\infty$/LTR control in order to design the controller for the previously given statistically linearized model. The nth-order stabilizable and detectable plant and weighted errors can be given by

$$\begin{aligned}\dot{x}(t)&=N(\sigma_x)+Bu(t)+D_1w_1(t)\\ z_1(t)&=E_1(t)x(t),\ z_2(t)=E_2(t)u(t)\\ z_{1\infty}(t)&=E_{1\infty}(t)x(t),\ z_{2\infty}(t)=E_{2\infty}(t)u(t)\\ y(t)&=Cx(t)+D_2w_2(t)\end{aligned} \qquad (3)$$

where $N(\sigma_x)$ is the $(n\times n)$ statistically linearized plant and σ_x is the standard deviation of the plant states. Then the n_c -order nonlinear dynamic controller can be obtained by

$$\begin{aligned}\dot{z}(t)&=N_z(\sigma_z)z(t)+Hy(t)\\ u(t)&=Gz(t)\end{aligned} \qquad (4)$$

where $N(\sigma_z)$ is the $(n_c\times n_c)$ statistically linearized controller matrix and σ_z is the standard deviation of the controller states, and which meets the following design criteria :

1) the closed-loop system equation (2) is asymptotically stable;

2) the $q_\infty\times p$ closed-loop transfer function matrix

$$T_{NC}(s)=\tilde{E}_\infty(s\tilde{I}_{\tilde{n}}-\tilde{N}(\sigma))^{-1}\tilde{D} \qquad (5)$$

from $\tilde{w}(t)$ to $\tilde{z}_\infty$ satisfies the constraint

$$\|T_{NC}\|_\infty\le\gamma \qquad (6)$$

where γ is a given positive constant and

$$\tilde{w}(t)=[w_1(t),w_2(t)]^T,\quad \tilde{z}_\infty(t)=[z_{1\infty}(t),z_{2\infty}(t)]^T.$$

$$\tilde{N}=\begin{bmatrix}N(\sigma_x) & BG\\ HC & N_z(\sigma_z)\end{bmatrix},\ \tilde{D}=\begin{bmatrix}D_1 & 0\\ 0 & HD_2\end{bmatrix},\ \tilde{E}_\infty=\begin{bmatrix}E_{1\infty} & 0\\ 0 & E_{2\infty}\end{bmatrix}.$$

3) the performance functional

$$J(N_z(\sigma_z),H,G)=\lim_{t\to\infty}E[x^T(t)R_1x(t)+u^T(t)R_2u(t)]$$
$$=tr(\tilde{Q}\tilde{R})$$

where $\tilde{Q}(=\lim_{t\to\infty}E[\tilde{x}(t)\tilde{x}^T(t)])$ is the steady-state closed-loop state covariance, which satisfies the $(\tilde{n}\times\tilde{n})$

algebraic Lyapunov equation

$$\tilde{N}\tilde{Q}+\tilde{Q}\tilde{N}^T+\tilde{V}=0 \quad (7)$$

where $\tilde{V}$ is the power spectral density of the Gaussian white noise input, $\tilde{R}$ is the control weighting matrix. and

$$\tilde{R}=\begin{bmatrix} R_1 & 0 \\ 0 & G^T R_2 G \end{bmatrix},\ R_1=E_1^T E_1, R_2=E_2^T E_2,$$

$$\tilde{V}=\begin{bmatrix} V_1 & 0 \\ 0 & HV_2H^T \end{bmatrix}, V_1=D_1D_1^T, V_2=D_2D_2^T.$$

The state equation of the closed-loop system may be written as

$$\dot{\tilde{x}}(t)=\tilde{N}\tilde{x}(t)+\tilde{D}\tilde{w}(t) \quad (8)$$

where $\tilde{x}(t)=[x(t)z(t)]^T$.

3.1 Quasi-LQG with the Constraint of H_∞ Disturbance Attenuation

The key step of the minimization issue in 3) is to satisfy the following result [8]. Let $(N_z(\sigma_z),H,G)$ be given and nonnegative matrix $\tilde{Q}_\infty \in R^{\tilde{n}\times\tilde{n}}$ satisfying the algebraic Riccati equation

$$\tilde{N}\tilde{Q}_\infty+\tilde{Q}_\infty\tilde{N}+\gamma^{-2}\tilde{Q}_\infty\tilde{R}_\infty\tilde{Q}_\infty+\tilde{V}=0 \quad (9)$$

where

$$\tilde{R}_\infty=\begin{bmatrix} R_{1\infty} & 0 \\ 0 & G^T R_{2\infty} G \end{bmatrix},\ R_{1\infty}=E_{1\infty}^T E_{1\infty},$$

$R_{1\infty}=E_{1\infty}^T E_{1\infty}=\beta^2 R_2$, β is a nonnegative constant.

Now, similar to the linear case [8], the following results can be obtained except the statistic functions and nonlinear correction terms.

Riccati equations:

$$N\overline{Q}+\overline{Q}N^T+\gamma^{-2}\overline{Q}R_{1\infty}\overline{Q}-\overline{Q}C^TV_2^{-1}C\overline{Q}+V_1=0 \quad (10)$$

$$(N+\gamma^{-2}[\overline{Q}+\hat{Q}]R_{1\infty})^T\overline{P}+\overline{P}(N+\gamma^{-2}[\overline{Q}+\hat{Q}]R_{1\infty})+R_1 - S^T\overline{P}BR_2^{-1}B^T\overline{P}S+\Psi(\overline{P},\overline{Q},\hat{Q},N)=0 \quad (11)$$

$$(N-BR_2^{-1}B^T\overline{P}S+\gamma^{-2}\overline{Q}R_{1\infty})\hat{Q}+\hat{Q}(N-BR_2^{-1}B^T\overline{P}S +\gamma^{-2}\overline{Q}R_{1\infty})^T+\gamma^{-2}\hat{Q}(R_{1\infty}+\beta^2S^T\overline{P}BR_2^{-1}B^T\overline{P}S)\hat{Q} +\overline{Q}C^TV^{-2}C\overline{Q}=0 \quad (12)$$

where

$$S=(I_n+\beta^2\gamma^{-2}\hat{Q}\overline{P})^{-1},$$

$$\Psi(\overline{P},\overline{Q},\hat{Q},N(\sigma_x))=2\mathrm{tr}[\overline{P}\frac{\partial N(\sigma_x)}{\partial(\overline{Q}+\hat{Q})}(\overline{Q}+\hat{Q})]$$

Controller parameters:

$$N_z(\sigma_z)=N-BR_2^{-1}B^T\overline{P}S-\overline{Q}C^TV_2^{-1}C+\gamma^{-2}\overline{Q}R_{1\infty} \quad (13)$$

$$H=\overline{Q}C^TV_2^{-1},\quad G=-R_2^{-1}B^T\overline{P}S \quad (14)$$

Note that these results are the version for a statistically linearized system as the counterpart for the linear case [8] except DF and nonlinear correction terms in equation (12). Equation (10) can be solved independently, but equation (11) and (12) are coupled and so must be solved simultaneously. Furthermore, equation (12) contains nonlinear correction terms, Ψ, which is very difficult to obtain the solution. If the nonlinear correction term can be neglected by simple techniques, the controller for hard nonlinear systems can be easily designed similar to linear systems. For this problem, thus, we will show that LTR techniques for statistically linearized systems give another advantage to eliminate nonlinear correction term in equation (12).

3.2 Design of the Target Filter Loop

To develop our loop shaping problems, without loss of generality, we suggest some assumptions with terms commonly used in the optimal control problems. A fictitious process and measurement white noise is considered for loop shaping and the modified Kalman filter frequency domain equality (KFDE) is also used. The statistically linearized design plant

$$\dot{x}(t)=Nx(t)+Bu(t)+Lw_1(t)$$
$$y(t)=Cx(t)+D_2w_2(t) \quad (15)$$

where

$E[w_1(t)]=0, E[w_2(t)]=0, E[w_1(t)w_1^T(t+\tau)]=I\delta(\tau)$,

$E[w_2(t)w_2^T(t+\tau)]=I\delta(\tau). V_1=D_1D_1^T=LL^T$,

$V_2=D_2D_2^T=\mu I$, μ = a positive constant as a design parameter.

And weighting matrices for estimated errors and control are defined as

$$R_1=C^TC, R_2=\rho I, R_{1\infty}=C^TC/\mu,\ \beta=0, R_{2\infty}=0.$$

$$S=I_n, \alpha^{-2}=1-\gamma^{-2}, 1<\gamma<\infty, 1<\alpha<\infty,$$

ρ = a positive constant as a design parameter.

Then the Riccati equations can be expressed as follows:

$$N\overline{Q}+\overline{Q}N^T+LL^T-\frac{\alpha^{-2}}{\mu}\overline{Q}C^TC\overline{Q}=0 \quad (16)$$

$$(N+\frac{\gamma^{-2}}{\mu}[\overline{Q}+\hat{Q}]C^TC)^T\overline{P}+\overline{P}(N+\frac{\gamma^{-2}}{\mu}[\overline{Q}+\hat{Q}]C^TC)$$
$$+C^TC-\frac{1}{\rho}\overline{P}BB^T\overline{P}+\Psi(\overline{P},\overline{Q},\hat{Q},N)=0 \quad (17)$$

$$(N-\frac{1}{\rho}BB^T\overline{P}+\frac{\gamma^{-2}}{\mu}\overline{Q}C^TC)\hat{Q}+(N-\frac{1}{\rho}BB^T\overline{P}+$$
$$\frac{\gamma^{-2}}{\mu}QC^TC)^T+\frac{\gamma^{-2}}{\mu}\hat{Q}C^TC\hat{Q}+\frac{1}{\mu}\overline{Q}C^TC\overline{Q}=0 \quad (18)$$

Because the principle of separation holds one direction, equation (16) corresponds to the Riccati equation to obtain a filter gain that is broken at the plant output. The target filter loop function matrix $G_{KF}(s)$ is written by

$$G_{KF}(s)=C(sI-N)^{-1}H \quad (19)$$

The filter gain H can be obtained from equation (16).

$$H=\frac{1}{\mu}\overline{Q}C^T \quad (20)$$

The remaining controller dynamics are

$$N_z(\sigma_z)=N-\alpha^{-2}HC+BG \quad (21)$$

$$G=-\frac{1}{\rho}B^TP \quad (22)$$

Fig. 3 shows the structure of the Quasi-LQG/H_∞/LTR control system for the statistically linearized system.

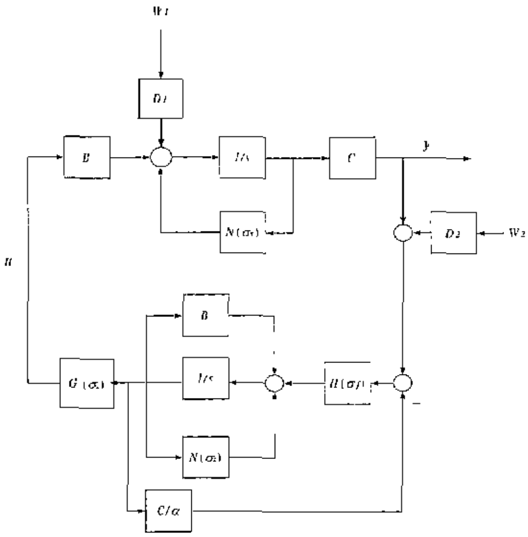

Fig. 3 The structure of the Quasi-LQG/H_∞/LTR control system

3.3 LTR using the Nonlinear Cheap Control

In order to design the control gain G, equation (17) and (18) must be solved. However, the control gain G cannot be obtained since, as stated previously, equation (18) contains the nonlinear correction term, $\Psi()$, which is very difficult to calculate directly. Therefore, let us introduce the nonlinear cheap control problem for this problem by examining the order of magnitude of the nonlinear correction term. Then let us define the LTR index q to show that the order of magnitude of the LTR index is same as that of the nonlinear correction term.

Definition 1. The LTR index q is defined as following form:

$$q=\left\|(N+\gamma^{-2}/\mu[\overline{Q}+\hat{Q}])C^TC\right\|\cdot(\rho\cdot(\left\|BB^T\right\|\cdot\left\|C^TC\right\|)^{-1})^{1/2}$$
$$<<1 \quad (23)$$

For scalar case ($B=C=L=1$),

$$q=\left\|N+\gamma^{-2}/\mu[\overline{Q}+\hat{Q}]\right\|\cdot\rho^{1/2}<<1 \quad (24)$$

This definition comes from equation (18) as following statements:

When $\rho\to 0$, then $\left\|\overline{P}(N+\gamma^{-2}/\mu[\overline{Q}+\hat{Q}])C^TC\right\|$ and $\left\|C^TC\right\|$ and $\left\|BB^T\right\|$ are finite, and

$$\left\|C^TC\right\|\cong\left\|\overline{P}BB^T\overline{P}\right\|/\rho\le\left\|BB^T\right\|\cdot\left\|\overline{P}\right\|^2/\rho \quad (25)$$

or

$$\left\|\overline{P}\right\|\ge(\rho\left\|C^TC\right\|(\left\|BB^T\right\|)^{-1})^{1/2} \quad (26)$$

Rearranging the above equation, then

$$\left\|\overline{P}(N+\gamma^{-2}/\mu[\overline{Q}+\hat{Q}])C^TC\right\|\le$$
$$\left\|\overline{P}\right\|\cdot\left\|(N+\gamma^{-2}/\mu[\overline{Q}+\hat{Q}])C^TC\right\|<<\left\|C^TC\right\| \quad (27)$$

or

$$\left\|(N+\gamma^{-2}/\mu[\overline{Q}+\hat{Q}])C^TC\right\|<<\left\|C^TC\right\|\cdot\left\|\overline{P}\right\|^{-1}$$
$$=(\left\|BB^T\right\|\left\|CC\right\|/\rho)^{1/2} \quad (28)$$

Dividing the left term by the right term from equation (28), the definition for the LTR index q can be established.

From now, we will show that the order of magnitude for $\left\|\Psi\right\|$ is equivalent to that of the LTR index (q) as $\rho\to 0$. For simple scalar case and the parameters of the controller are

$$N_z=N-\alpha^{-2}H+G,\quad H=\overline{Q}/\mu,\quad G=-\overline{P}/\rho. \quad (29)$$

Let us examine the limit behavior of the LTR index related to the simplifying of this problem. Next we can check the order of magnitude of H and G, respectively from equation (16), (17), (18) and (29).

1) $|N| >> 1/\sqrt{\mu}$ and $|N| >> 1/\sqrt{\rho}$

$$o(H)=o(2\alpha^2 N),\quad o(G)=o(2N+4\gamma^{-2}N) \quad (30)$$

2) $|N| << 1/\sqrt{\mu}$ and $|N| << 1/\sqrt{\rho}$ $(1/\sqrt{\mu} << 1/\sqrt{\rho})$

$$o(H)=o(\alpha^2/\sqrt{\mu}),\quad o(G)=o(1/\sqrt{\rho}) \quad (31)$$

and from equation (9), the weighted scalar variance can be calculated as

$$\gamma^{-2}(\overline{Q}+\hat{Q})/\mu = -(N(2N+2G-\alpha^{-2}H)-GH)/(2N+2G-\alpha^{-2}H) + \{(N(2N+2G-\alpha^{-2}H)-GH)/(2N+2G-\alpha^{-2}H))^2 - \gamma^{-2}V_1/\mu\}^{1/2} \quad (32)$$

Finally, checking the different orders of magnitude for the LTR index, it will be shown that the nonlinear correction term in Riccati equation can be neglected and the modified coupled Riccati equation is the same as the linear one.

Theorem 1. If the good LTR is guaranteed, the order of magnitude of the LTR (q) approaches 0 as $\rho \to 0$.

Proof : From equation (30), (31) and (32), let us divide the procedure of proof into the following four cases.

Case 1. : $|N| >> 1/\sqrt{\mu}$ and $|N| >> 1/\sqrt{\rho}$ $(q >> 1)$

$$\begin{cases} o(H)=o(2\alpha^2 N) \\ o(G)=o(2N+4\gamma^{-2}N) \\ o(\gamma^{-2}/\mu(\overline{Q}+\hat{Q}))=o(0) \end{cases}$$

Since $o(q(=\|N\|)) << 1/\sqrt{\rho}$, the given condition $|N| >> 1/\sqrt{\rho}$ is not satisfied if $\rho \to 0$. Therefore, in case $\rho \to 0$, the possibility of $q >> 1$ does not exist.

Case 2. : $|N| \cong 1/\sqrt{\rho}$ $(q \cong 1)$

$$\begin{cases} o(H)=o(2\alpha^2 N) \\ o(G)=o(2N+4\gamma^{-2}N) \\ o(\gamma^{-2}/\mu(\overline{Q}+\hat{Q}))=o(0) \end{cases}$$

Since $o(q(=\|N\|)) \cong 1$, the given condition $|N| \cong 1/\sqrt{\rho}$ is not satisfied if $\rho \to 0$ Therefore, in case $\rho \to 0$, the possibility of $q \cong 1$ does not exist.

Case 3. : $1/\sqrt{\mu} << |N| << 1/\sqrt{\rho}$ $(q << 1)$

$$\begin{cases} o(H)=o(2\alpha^2 N) \\ o(G)=o(1/\sqrt{\rho}) \\ o(\gamma^{-2}/\mu(\overline{Q}+\hat{Q}))=o(0) \end{cases}$$

If $\rho \to 0$, the given condition $|N| << 1/\sqrt{\rho}$ is satisfied. Therefore, in case $\rho \to 0$, the condition of $q << 1$ is satisfied.

Case 4. : $|N| << 1/\sqrt{\mu} << 1/\sqrt{\rho}$ $(q << 1)$

$$\begin{cases} o(H)=o(\alpha^2/\sqrt{\mu}) \\ o(G)=o(1/\sqrt{\rho}) \\ o(\gamma^{-2}/\mu(\overline{Q}+\hat{Q}))=o(1/\sqrt{\mu}[\alpha^2/2+\sqrt{\alpha^4/4-\gamma^{-2}V_1}]) \end{cases}$$

If α, V_1 and μ are finite, then $\gamma^{-2}/\mu(\overline{Q}+\hat{Q})$ is finite in the stable system and $o(q(=\|N+\gamma^{-2}/\mu(\overline{Q}+\hat{Q})\|)) << 1/\sqrt{\rho}$.

If $\rho \to 0$, the given condition $|N| << 1/\sqrt{\rho}$ is satisfied and $q << 1$ is also satisfied. Therefore, q approaches 0 as $\rho \to 0$ because $\|N\|$ is finite in the stable system that has finite inputs.

Note that from Theorem 1, the LTR conditions for the statistically linearized system are the same as the linear one since the order of magnitude of the nonlinear correction term $\|\Psi\|$ is that of q. When the LTR conditions are satisfied, the correction term can be neglected and the modified coupled Riccati equations are the same form as the linear case. When the LTR conditions are satisfied, the correction term can be neglected and the modified coupled Riccati equations are the same form as the linear case.

Next, we will derive the limiting behavior of the loop transfer function matrix at the plant output. From equation (17), if the LTR condition is satisfied, limiting behavior as $\rho \to 0$ is

$$\boldsymbol{C}^T\boldsymbol{C}-(\boldsymbol{PB}/\sqrt{\rho})\cdot(\boldsymbol{B}^T\boldsymbol{P}/\sqrt{\rho}) \to \boldsymbol{0} \quad (33)$$

Substituting control gain $\boldsymbol{G}=-\boldsymbol{B}^T\boldsymbol{P}/\rho$ into equation (33)

$$(\sqrt{\rho}\boldsymbol{G})^T(\sqrt{\rho}\boldsymbol{G}) \to \boldsymbol{UC} \quad (34)$$

This implies that

$$\lim_{\rho \to 0} \sqrt{\rho}\boldsymbol{G} \to \boldsymbol{UC} \quad (35)$$

where $\boldsymbol{U}$ is the $m \times m$ unitary matrix, i.e., $\boldsymbol{U}^T\boldsymbol{U}=\boldsymbol{I}_m$. We consider the controller TFM, $\boldsymbol{K}(s)$ as

$$\boldsymbol{K}(s) = -\boldsymbol{G}(s\boldsymbol{I}-\boldsymbol{N}-\boldsymbol{BG}+\alpha^{-2}\boldsymbol{HC})^{-1}\boldsymbol{H} \quad (36)$$

Theorem 2. If $\mathrm{Re}\,\lambda_i(\boldsymbol{N}+\boldsymbol{BG})<0$, $\mathrm{Re}\,\lambda_i(\boldsymbol{N}+\alpha^{-2}\boldsymbol{HC})<0$, and $\lim_{\rho \to 0}\sqrt{\rho}\boldsymbol{G} \to \boldsymbol{UC}$, then the limiting behavior of the ***K(s)*** as $\rho \to 0$ is as follows:

$$\lim_{\rho \to 0} K(s) \to [C(sI-N)B]^{-1}[C(sI-N)^{-1}H]$$
$$= G^{-1}(s) \cdot G_{KF}(s) \qquad (37)$$

Proof : Proof is simple and is omitted.

3.4 Quasi-LQG/H$_\infty$/LTR Controller Synthesis

In order to calculate the statistic values of the controller, DF gains for nonlinearties should be assumed before they are calculated since the statistic values of nonlinearity cannot be known previously. Their exact values can be obtained by solving the modified Riccati equation (9) of the closed-loop for the compensated system. But the DF with respect to hard nonlinearity is originally derived in the sense of LQG optimization. Thus instead of equation (9), the following Lyapunov equation of the closed-loop for compensated system must be introduced in order to calculate the exact values of DF gains and other statistic properties.

$$\tilde{N}\tilde{Q}_\infty + \tilde{Q}_\infty \tilde{N} + \tilde{V} = 0 \qquad (38)$$

where

$$\tilde{N} = \begin{bmatrix} N(\sigma_x) & BG \\ HC & N_z(\sigma_z) \end{bmatrix}, \tilde{V} = \begin{bmatrix} V_1 & 0 \\ 0 & HV_2H^T \end{bmatrix}$$

If we consider the nonlinear correction term in equation (17) in the cheap control problem, then we must solve equation (17) and equation (38) simultaneously with respect to the guessed unknown variables $[\tilde{Q}_\infty : n(2n+1), \overline{P}; n(n+1)/2]$ where n is the number of plant states. Since it requires a great deal of computation time for a higher-order plant, it is very difficult to find the solution. Fortunately, as a result of cheap control problem shown in theorem 1, when the good LTR condition is satisfied, the nonlinear correction term, $\Psi(\cdot)$, in equation (17) can be neglected. Hence the control gains and the stationary statistics of the compensated system can be separately calculated from the LTR procedure and modified Riccati equation.

Next, the set of DF gains whose parameters depend on the stationary statistics of several input ranges can be converted into the nonlinear function that can be executed via the IRIDF techniques. Atherton [10] presented the theoretical explanations of the IRIDF techniques. For practical applications, however, the approximated method suggested by Suzuki [11] is more convenient to convert DF into the nonlinear function..

The design procedure of the Quasi-LQG/H$_\infty$/LTR control system is similar to QLQG/LTR case [5], and thus further explanation is omitted.

4. Control System Design and Discussion

4.1 Problem Formulation

The state space model of the nonlinear servo system with Coulomb friction and dead-zone is given in eqation (2). The design specifications considered are as follows:

1) Steady state tracking errors should be zero for arbitrary constant inputs.
2) Gain crossover frequency should be about 12 rad/s.
3) The maximum singular value of the sensitivity transfer function matrix should be less than -20 dB for all $\omega < 1$ rad/s for the good command following and disturbance rejection.

4.2 LQG/H$_\infty$/LTR Controller Design

A linear plant is required to apply the LQG/H$_\infty$/LTR method. For linear case, the Coulomb friction and dead-zone are no considered. Then the design plant model dynamics are represented as follows:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \qquad (39)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{C_{eq}}{M_{eq}} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \dfrac{K_{amp}}{M_{eq}} \end{bmatrix}.$$

and output matrix C is the same as the matrices in equation (2). The design plant model is found to be completely controllable from the input $u(t)$ and completely observable through the output $y(t)$, and is also a minimum phase plant. Therefore, the LQG/H$_\infty$/LTR controller can be designed with a guarantee of the good LTR. The target filter loop is designed by matching the high frequency singular values. Then the design parameter L is chosen as follows:

$$L = \alpha^{-1}[1 \quad 0]^T \qquad (40)$$

To determine the filter gain matrix H, the values of *1.02* and *0.00125* for μ and γ, respectively are chosen to provide a crossover frequency of about *12 rad/s* for the target filter loop. After choosing L, μ and γ to satisfy the desired target filter loop shaping, the filter gain matrix $\boldsymbol{H}$ is calculated from equations (16) and (20). The resulting filter gain matrix $\boldsymbol{H}$ is.

$$\boldsymbol{H} = \begin{bmatrix} 28.2843 & 0 \end{bmatrix}^T \tag{41}$$

The LTR is attempted with the nonlinear cheap control problem. The target filter loop is usually recovered up to a decade beyond the crossover frequency. This level of recovery is obtained with a value of 10^{-5} for ρ. Then the control gain matrix $\boldsymbol{G}$ is calculated from equations (17), (18) and (22) without the correction term (17) as follows·

$$\boldsymbol{G} = \begin{bmatrix} -227.07 & -0.7641 \end{bmatrix} \tag{42}$$

Let us evaluate the performance and stability robustness for the nonlinear plant with the LQG/H_∞/LTR controller. For this purpose, the frequency responses are checked for 3 different command inputs, which are assumed as zero mean white noises for the statistical linearization of nonlinear plant. The white noise intensities of the selected inputs (V_2) are 5×10^5, 10^6 and 10^{10} which represent the small, medium and large input cases, respectively. Singular values of the loop transfer function matrix and target filter of the nonlinear plant with the LQG/H_∞/LTR controller are shown in Fig. 4, and the experimental normalized step responses of the nonlinear plant with the LQG/H_∞/LTR controller is shown in Fig. 5 and Fig. 6

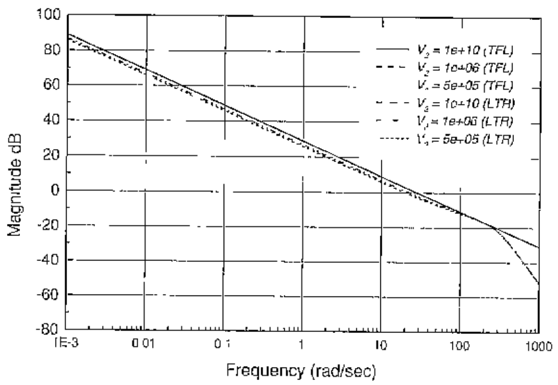

Fig. 4 Singular values of the loop transfer function matrix and target filter loop of the nonlinear plant with the LQG/H_∞/LTR controller

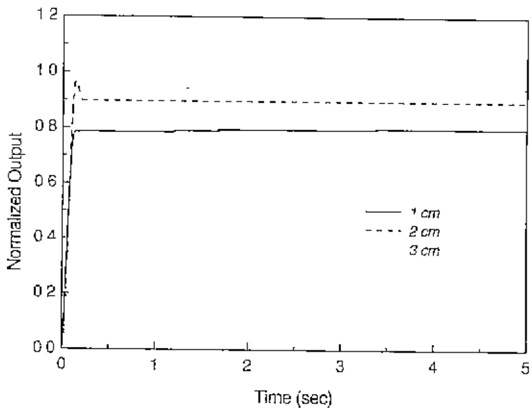

Fig. 5 Step responses of the LQG/H_∞/LTR control system; a case of the small reference inputs (experiment)

The LQG/H_∞/LTR control system maintains the stability robustness for any input magnitude and direction, but it does not meet the performance requirements for small inputs. In the time response, there are some steady state errors for the most of step inputs. This is due to the effect of the Coulomb friction and dead-zone. However, as the size of the step inputs is larger, it is seen that the steady state errors are decreased. This is the fact that the control forces of the case in a lager step input are greater comparing the case of a small step input. Thus, the nonlinear effects on the performance of the control system vary according to the magnitude of the control force. In addition, some overshoot exists for large inputs due to the elastic of the timing-belt. In this paper, let this problem not to be treated.

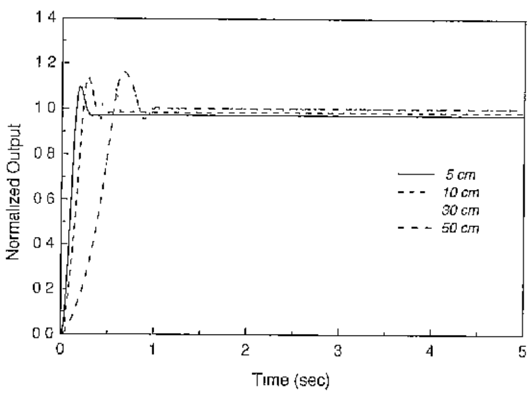

Fig. 6 Step responses of the LQG/H_∞/LTR control system : a case of the large reference inputs (experiment)

Therefore, the LQG/H$_\infty$/LTR controller cannot be used for a large operating range. In order to improve the performance, a nonlinear controller is required which can capture the effect of the Coulomb friction and dead-zone and in addition, adapt to changes in the magnitude and direction of the command input.

4.3 Quasi-LQG/H$_\infty$/LTR Controller Design

The statistically linearized plant of equation (2) and the selection of several operating points to cover an operating range of interest are required to apply the Quasi-LQG/H$_\infty$/LTR method. The zero mean white noise intensities of the noise inputs (V_2) are selected between $5\times10^5 \sim 10^{10}$. Results of singular values for 3 input cases ($V_2 = 5\times10^5$, 10^6 and 10^{10}) are presented. The gains (filter, control and DF) and the stationary statistics (controller states and filter innovations) are stored for all linear designs. The filter gains are the same as the LQG/H$_\infty$/LTR case. Since G are almost constant for any noise intensity V_2, then G can be chosen as follows:

$$G = [-227.063 \quad 0.76385] \tag{43}$$

The nonlinearities of $F_{feq}\,\mathrm{sgn}(\dot{x})$ and Dz are functions of inputs $z_2(t)$ and $u(t)$ respectively. The DF gains (N_{fz} and N_{dz}) that are implemneted in the controller and the standard deviations of the controller states are given in Table 2. The desired nonlinear functions $f_{cz}(t)$ and $f_{dz}(t)$ for Coulomb friction and dead-zone, respectively are obtained via IRIDF techniques, which are shown in Fig. 7. The structure of the Quasi-LQG/H$_\infty$/LTR of the timing-belt driving servo system is presented in Fig. 8.

Table 2 DF gains and standard deviations of the compensator states at all operating points

V_2	N_{fz}	N_{dz}	σ_{z2}	σ_u
5×10^5	0.08587	0.99945	3.96897	523.422
10^6	0.05716	0.99954	5.96184	622.927
5×10^6	0.02362	0.99969	14.4285	932.099
10^7	0.01639	0.99974	47.6380	1108.58
10^8	0.00503	0.99985	152.618	1971.66
10^9	0.00158	0.99991	216.221	3506.63
10^{10}	0.00049	0.99995	685.784	6235.28

Singular values of the loop transfer function matrix and target filter of the nonlinear plant with the Quasi-LQG/H$_\infty$/LTR controller are shown in Fig. 9.

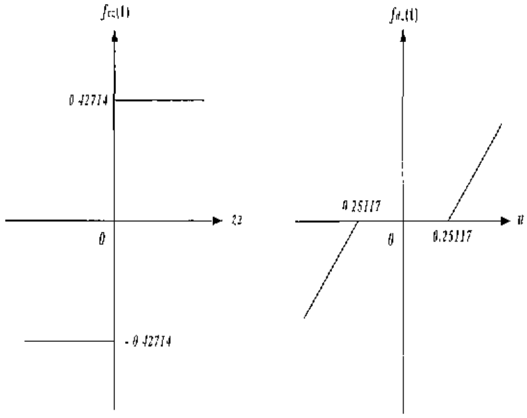

Fig. 7 Desired nonlinear functions via the IRIDF techniques

The simulation results of the normalized step responses of the nonlinear plant with the Quasi-LQG/H$_\infty$/LTR controller are shown in Fig. 10 and Fig. 11, and the experimental results are shown in Fig. 12 and Fig. 13. From the simulation and experimental results, it is found that the Quasi-LQG/H$_\infty$/LTR control system is insensitive to the magnitude and direction of the input. In addition, no steady state error exists for constant inputs so that the precise position control for the timing-belt driving servo system can be done. But unlike to simulation results, in experimental results the small overshoot and delay of the

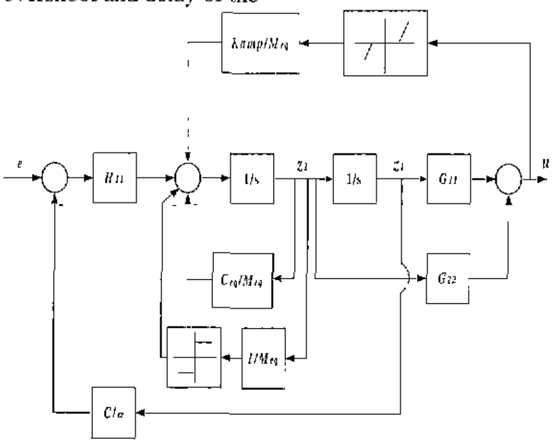

Fig. 8 The structures of the Quasi-LQG/H$_\infty$/LTR controller the timing-belt driving servo system

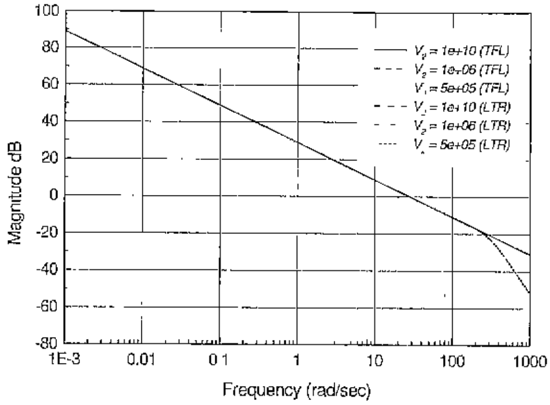

Fig. 9 Singular values of the loop transfer function matrix and target filter loop of the Quasi-LQG/H$_\infty$/LTR control system

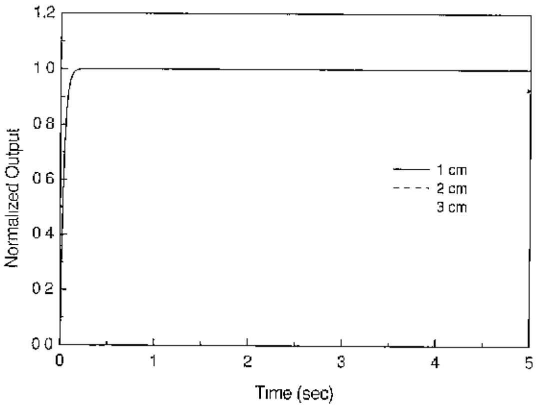

Fig. 10 Step responses of the Quasi-LQG/H$_\infty$/LTR control system; a case of the small reference inputs (simulation)

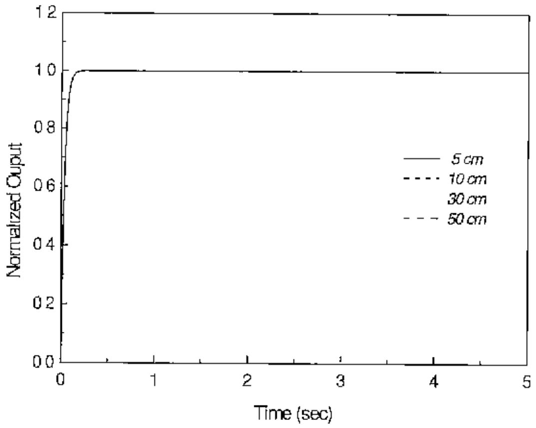

Fig. 11 Step responses of the Quasi-LQG/H$_\infty$/LTR control system; a case of the large reference inputs (simulation)

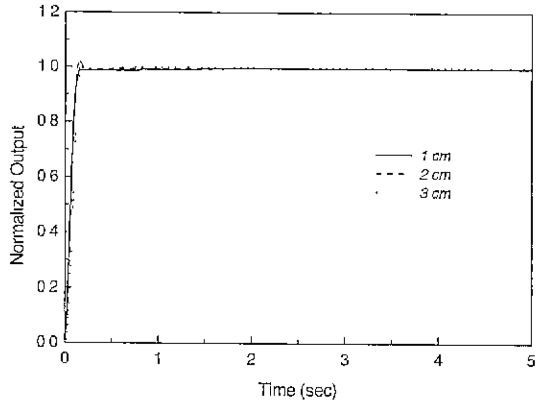

Fig. 12 Step responses of the Quasi-LQG/H$_\infty$/LTR control system; a case of the small reference inputs (experiment)

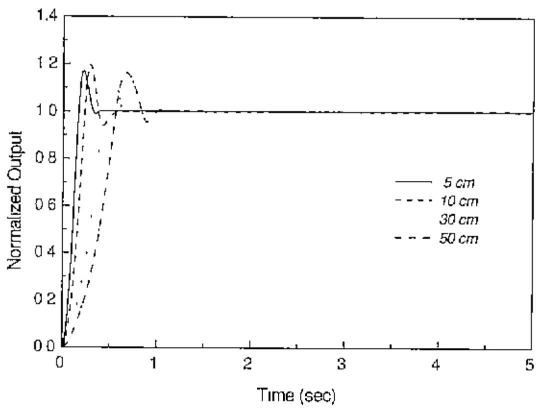

Fig. 13 Step responses of the Quasi-LQG/H$_\infty$/LTR control system; a case of the large reference inputs (experiment)

rising time can be seen for the large magnitude of the inputs like the LQG/H$_\infty$/LTR control system due to elastic effect of the timing-belt.

4.4 Discussion

In the frequency response, the LQG/H$_\infty$/LTR control system meets the design specifications for a small operating range, but the Quasi-LQG/H$_\infty$/LTR control system meets them for the entire operating range. And in the time response, the LQG/H$_\infty$/LTR control system as steady state errors for constant inputs even if the system has free integrators, because the LQG/H$_\infty$/LTR controller cannot adapt to the effect of the Coulomb friction and dead-zone which cannot be captured in the linear model. In addition, the system responses are sensitive to the input direction. However, the Quasi-LQG/H_∞/LTR control system meets the design specifications for all the operating range. The system responses are insensitive to the magnitude and direction of the command input. The response has no steady state error and has fast settling time for all the operating range. This is so because the Quasi-LQG/H_∞/LTR controller reflects in its model and, therefore, can adapt to the effect of the Coulomb friction and dead-zone.

5. Conclusion

A Quasi-LQG/H_∞/LTR design method for a nonlinear servo system with Coulomb friction and dead-zone has been presented. The method is essentially an integration of statistical linearization, loop shaping and loop transfer recovery techniques. By using statistical linearization techniques, nonlinear effects are considered in the design of nonlinear controllers that adapt to changes in input magnitude for nonlinear systems with hard nonlinearities such as Coulomb friction, backlash and dead-zone. In addition, using loop-shaping techniques the performance requirements and stability can address robustness simultaneously.

The LTR conditions for the statistically linearized system were discussed. It is found that the LTR conditions for the statistically linearized system are basically the same as in the linear case. The only difference is that the nonlinear cheap control problem includes the correction term in the modified Riccati equation. Fortunately, the correction term is not dominant in the good LTR. Therefore, it can be neglected in this situation. Then, the modified Riccati equation has the same form as the standard Riccati equation. Therefore, the required computation becomes much simpler by neglecting the correction term.

Finally, the Quasi-LQG/H_∞/LTR method is applied to a timing-belt driving servo system with Coulomb friction and dead-zone. It is verified that the Quasi-LQG/H_∞/LTR controller is insensitive to the magnitude and direction of the command input. Thus, the Quasi-LQG/H_∞/LTR method can be suggested to design controllers for the nonlinear multivariable servo systems with hard nonlinearities to meet the performance requirements and to maintain the stability robustness for a large operating range. Therefore, we can suggest that the Quasi-LQG/H_∞/LTR control method is suitable for the precise position control of a hard nonlinear servomechanism that often appears in automatic plants.

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