Hybrid Fuzzy Learning Controller for an Unstable Nonlinear System

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ABSTRACT

Although it is well known that fuzzy learning controller is powerful for nonlinear systems, it is very difficult to apply a learning method if they are unstable. An unstable system diverges for impulse input. This divergence makes it difficult to learn the rules unless we can find the initial rules to make the system stable prior to learning. Therefore, we introduced LQR(Linear Quadratic Regulator) technique to stabilize the system. It is a state feedback control to move unstable poles of a linear system to stable ones. But, if the system is nonlinear or complicated to get a linear model, we cannot expect good results with only LQR. In this paper, we propose that the LQR law is derived from a roughly approximated linear model, and next the fuzzy controller is tuned by the adaptive on-line learning with the real nonlinear plant. This hybrid controller of LQR and fuzzy learning was superior to the LQR of a linearized model in unstable nonlinear systems.

Key Words: Hybrid control, Fuzzy learning, LQR(Linear Quadratic Regulator), Unstable system, Nonlinear system.

1. Introduction

Most fuzzy logic controllers have been built just to emulate human decision- making behavior. However, it is necessary to construct the rule bases by using a learning method with self-improvement when it is difficult or impossible to get them only by expert's experience. The fuzzy controller based on learning accomplishes its learning for a given control task through repetition or sequence of operations. The control input sequence for the next operation is determined by utilizing the knowledge acquired from observations of the past operations so that the outputs can follow the given target trajectories as closely as possible. The rule modification mechanism monitors the control performance at every trial and makes the rule base change if the performance is not satisfactory. While the process is adequately controlled according to the pre-specified performance, the rule base is iteratively improved until no further rule changes are necessary[1-4].

Thus, such adaptive on-line learning control necessitates the premise that the system is stable. In a stable system, during a meaningful task, the learning mechanism plays a role to improve the performance of controller in spite of the change of environment. However, in an unstable system, the output is not definitely dependent on the input, because a small input can produce a large output. It is nearly impossible to tune or update the control rules without the aid of additional controllers that make it stable. In the case of an inverted pendulum system, it is relatively easy to find the initial control rules to make it stable because of its simplicity. We had obtained the good fuzzy control rules by using the learning method even though the system was unstable[5]. But the seesaw's balancing is another problem. In this system, the control input is the position of a moving cart on the seesaw, and the output is the slope of the seesaw. The control of the cart's position that makes the seesaw remain horizontal is very difficult because the system is too sensitive. Unlike the inverted pendulum problem, of the cart's position that makes the seesaw remain horizontal is very difficult because the system is too sensitive. Unlike the inverted pendulum problem, it is also not easy to find the initial control rules for the stability by intuition. To make the seesaw system stable prior to on-line learning, it is necessary to use an additional control technique. Most linear control techniques need a linear modeling equation. In real systems, it is very difficult to find the mathematical models. They are mainly nonlinear models even though the modeling equations are found. In the case of an unstable nonlinear model, it is possible to linearize a nonlinear model and to make the linear model stable by using linear control techniques. This paper will explain how to control an unstable nonlinear system. First, we get a roughly approximated model and linearize it. Next, we can obtain the state feedback controller by designing LQR with the linearized model. At least, this controller makes the real system stable. Finally, we tune the fuzzy controller by the adaptive on-line learning with the real nonlinear plant. Fig.1 shows a block diagram to control an unstable nonlinear plant. The hybrid fuzzy learning controller is consisted of the LQR controller and the fuzzy learning controller(FLC).

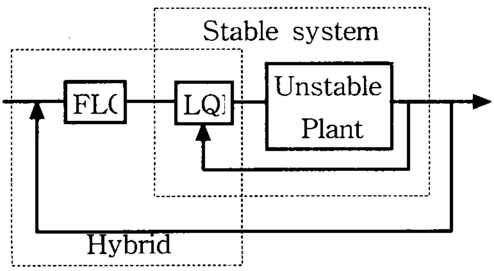

Fig. 1 Block diagram of hybrid controller

2. Problem Description & Stable Control

The seesaw module consists of two long arms hinged onto a support fulcrum. The axis is coupled to a potentiometer, which is used to measure the angular deflection of the seesaw. The cart is placed on top of the seesaw and moves through a rack and pinion gear. In this experimental setup, the position of the cart is also measured by another potentiometer. The objective of the experiment is to design a feedback control system that makes the seesaw remain horizontal.

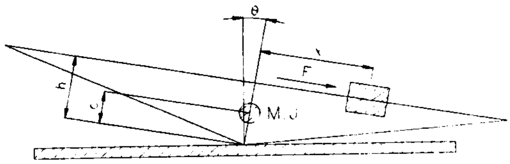

Fig. 2 Simplified model for seesaw experiment

Fig. 2 shows the simplified model for the seesaw experiment. The kinetic and potential energies of each element in the system are individually derived, and the non-linear differential equations are obtained as:

$$m\ddot{x} + mh\ddot{\theta} - mx\dot{\theta}^2 - mg\sin(\theta) = F \qquad (1)$$

$$(J + mh^2 + 2mx\dot{x}\dot{\theta} + mx^2)\ddot{\theta} + mh\ddot{x} - mgh\sin(\theta) - mgx\cos(\theta) - Mgc\sin(\theta) = 0 \qquad (2)$$

where,

F = input force to the cart (N)
m = mass of the cart (Kg)
M = Mass of [seesaw+track] (Kg)
c = distance of center of gravity of [seesaw + track] from pivot point (m)
h = height of track from pivot point (m)

The linear model is derived as :

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mhg}{J} & -g\frac{Mhc-J}{J} & 0 & 0 \\ \frac{mg}{J} & \frac{Mgc}{J} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{mh^2+J}{Jm} \\ -\frac{h}{J} \end{bmatrix} F \qquad (3)$$

The states of the system are :

x = *position of the cart relative to the center of the track (m)*

$\dot{x}$ = *velocity of the cart (m/sec)*

θ = *angle of the seesaw relative to the vertical (rad)*

$\dot{\theta}$ = *angular velocity of the seesaw (rad/sec)*

Using MATLAB to design the LQR controller, we obtain the optimal feedback gain *K*, which minimizes the quadratic performance index;

$$J=\int(x'Qx+rv^2)dt \tag{4}$$

If this linearized model is the exact model of the real system, LQR could supply an optimal controller for the real system. In an unstable nonlinear system, we cannot expect a good solution because LQR is obtained from a roughly approximated model. However, it is sufficient that LQR makes the real system stable, and we can supplement a fuzzy learning method as a controller for the stable nonlinear system.

3. Fuzzy Learning for Optimal Control

In the generalized fuzzy rules, the *k*-th control rule R_k is presented by the following expression.

$$R_k\ :\ A_{1,k}(x_1)\times A_{2,k}(x_2)\times\ldots\times A_{M,k}(x_M)\ \Rightarrow\ v_k \qquad k=1,\ \ldots\ ,\ N. \tag{5}$$

where, M and N represents the number of input variables and the total rules respectively and v_k typically represents the centroid of a membership function or a singleton defining an output variable. If the premise of the rule is inferred by the product-sum method, the fitness of the *k*-th rule, Φ_k is defined by

$$\Phi_k=\frac{\prod_{i=1}^{M}\mu_{A_{i,k}}(x_i)}{\sum_{k=1}^{N}\prod_{i=1}^{M}\mu_{A_{i,k}}(x_i)} \tag{6}$$

The control input which is attributed to the *k*-th rule is obtained by multiplying the fitness by the output value of the *k*-th rule. The total control input of a plant is the summation of all the rules.

$$u=\sum_{k=1}^{N}\Phi_k\times v_k \tag{7}$$

The control objective is to minimize an error quantity so that an actual output can exactly follow the desired output of the reference model[3,4]. If a cost function J is defined as the summation of the output error,

$$J=\sum_{h=0}^{\infty}J(k+h) \tag{8}$$

where, $J(h)=\frac{1}{2}\ \{y_d(h)-y(h)\}^2$

To minimize the cost function J, it is necessary to change the control input in the direction of the negative gradient of J.

$$\Delta u(k)\propto-\frac{\partial J}{\partial u(k)} \tag{9}$$

In the fuzzy control, it is not easy to get the gradient of J with respect to the input because the modeling equation is not known. If we assume that the current control input $u(k)$ has a dominant effect on the n steps ahead output $y(k+n)$, we can determine the control input $u(k)$ to minimize the n steps ahead output error.

$$\begin{aligned}\Delta u(k)&\propto-\frac{\partial J(k+n)}{\partial u(k)}\\&=\varepsilon(k+n)\frac{\partial y(k+n)}{\partial u(k)}\end{aligned} \tag{10}$$

where, $\varepsilon(k+n) = y_d(k+n) - y(k+n)$

Thus, the learning algorithm is expressed by

$$u(k)_{new} = u(k)_{old} + \Delta u(k) \tag{11}$$

$$\Delta u(k) = \eta\varepsilon(k+n) \tag{12}$$

where, η : learning rate with a positive constant if at least $\frac{\partial y(k+n)}{u(k)}$ is positive.

This future step n is then determined to make the derivative of the output with respect to the input positive[3].

4. Experiment

In the case of seesaw system, LQR has a fairly good solution for the linearized model even though it is a nonlinear system. If the system is too complex to get the mathematical model, it is no problem to use a model with a little error because the role of LQR is to make the system stable. To observe more definitely the learning effect, we found the LQR controller from the linearized model with additional 30% error. The dotted line in Fig. 3 is the result of the LQR controller with a modeling error. We can see that the LQR control system is stable even though the output had high oscillation because of modeling error. When the controller was improved by the fuzzy learning, the output had little oscillation as shown in the solid line of Fig. 3.

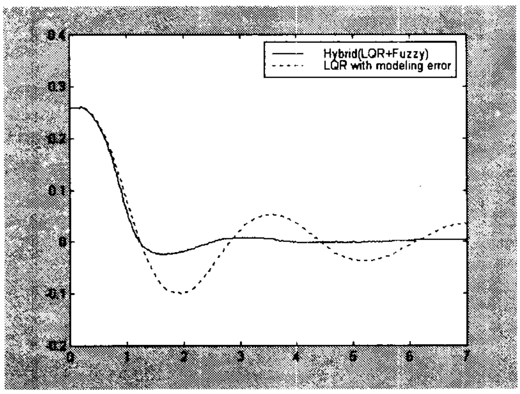

Fig. 3 Simulations of Seesaw control

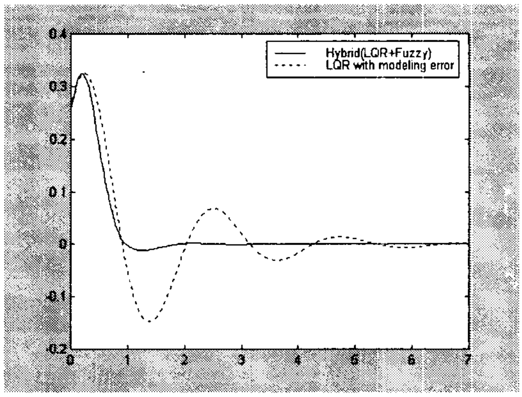

Fig. 4 Experiments of Seesaw control

Fig. 4 shows the experimental results. When the initial angle of seesaw was inclined by 15° (0.26 rad), the seesaw maintained horizontal after a little overshoot, and it took about 2 seconds. They were very similar to the simulation results. In the initial control response, the output did not change in the experiment whereas it was increasing in the simulation. Because of the seesaw's slope, the mass center of the seesaw was initially deviated from the vertical line of the fulcrum. It is a moment that makes the seesaw incline more and more. In the experiment, however, the phenomenon did not occur because of the friction effect. The moment did not overcome friction. Although friction exists in all the real system, it is very difficult to consider the friction effect in the simulation. The response of the experiment was also slower than the simulation's because of the friction.

5. Conclusion

It was proved that the adaptive on-line fuzzy learning method was very powerful in the control of a nonlinear system. In the seesaw system, although there were some restrictions hard to consider in the simulation like the friction effect, the fuzzy controller simply solved the nonlinear problem.

We proposed a hybrid controller in an unstable system because it was difficult to acquire the fuzzy rules by on-line learning. As an assistant controller, the LQR controller easily made the linear system stable. Next, the fuzzy controller was updated through on-line learning to control the stable nonlinear system. The proposed hybrid fuzzy learning controller was consisted of the LQR controller and the fuzzy learning controller. In this approach, it was not necessary to try to find more exact model because the modeling equation was only used to make the system stable. The hybrid method gave excellent control results without the exact modeling equation.

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