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A Design Method of Gear Trains Using a Genetic Algorithm

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ABSTRACT

The design of gear train is a kind of mixed problems which have to determine various types of design variables; i,e., continuous, discrete, and integer variables. Therefore, the most common practice of optimum design using the derivative of objective function has difficulty in solving those kinds of problems and the optimum solution also depends on initial guess because there are many sophisticated constraints. In this study, the Genetic Algorithm is introduced for the optimum design of gear trains to solve such problems and we propose a genetic algorithm based gear design system. This system is applied for the geometrical volume(size) minimization problem of the two-stage gear train and the simple planetary gear train to show that genetic algorithm is better than the conventional algorithms for solving the problems that have continuous, discrete, and integer variables. In this system, each design factor such as strength, durability, interference, contact ratio, etc. is considered on the basis of AGMA standards to satisfy the required design specification and the performance with minimizing the geometrical volume(size) of gear trains

Key Words : Gear, design method, genetic algorithm(GA), optimization, planetary gear train, AGMA(American Gear Manufacturers Association) standards

Nomenclature

a	Operating center distance	[mm]
b	face width	[mm]
b_{s1}	face width of 1st stage	[mm]
b_{s2}	face width of 2nd stage	[mm]
m_n	normal module	[mm]
m_{ns1}	normal module of 1st stage	[mm]
m_{ns2}	normal module of 2nd stage	[mm]
r_{a1}	tip radius of external gear	[mm]
r_{a2}	tip radius of internal gear	[mm]
u	Reduction gear ratio	
v_t	pitch line velocity at operating pitch diameter	[m/s]
z	number of teeth	
z_{1s1}	number of pinion teeth in 1st stage	
z_{2s1}	number of gear teeth in 1st stage	
z_{1s2}	number of pinion teeth in 2nd stage	
z_{2s2}	number of gear teeth in 2nd stage	
z_p	number of teeth in planet pinion	
z_r	number of teeth in ring gear	
z_s	number of teeth in sun gear	
N	number of planet pinion	
T_f	flash temperature	[°C]
α_{a1}	pressure angle at external gear tip	[°]
α_{a2}	pressure angle at internal gear tip	[°]
β	helix angle	[°]
ε_α	transverse contact ratio	
ε_β	overlap contact ratio	
σ_F	bending stress	[MPa]
$\sigma_{F\lim}$	allowable bending stress	[MPa]
σ_H	contact stress	[MPa]
$\sigma_{H\lim}$	allowable contact stress	[MPa]

1. Introduction

When designing a kind of mechanical elements, there are many important factors considered in design such as lightweights, small sizes, and high strength and durability. Until now, researchers tend to use either some experimental data or empirical know-how of experts in design process in order to satisfy these factors. But, newer approaches like computer aided optimization methods are used more and more widely in fields especially where numerical analysis is possible.

However, many optimization problems in mechanical design have some design variables that are either integers or discrete values commercially available, that is, discontinuous ones. Typical examples of integer variables are number of teeth of gear, number of spring coil, number of bolts in structures, and so on. The examples of discrete ones are module of gear, diameter of bolt and cross sectional area of truss members.

Therefore, in many cases it is general to perform optimization on condition that all design variables are continuous and then, it is converted to nearest integers or discrete values[1]. But, this method has the following disadvantages[2]: First, original optimization problem is to be expanded into a lot of sub optimizing problems. Second, solutions can deviate from optimums. Finally, for problems having many design variables, infeasible solutions can be obtained. These gradient-based methods, which require the derivative of both objective functions and constraints, have the difficulty in getting solutions irrelevant to the initial design especially for multimodal design spaces. Besides, it is difficult to find global optimums using these methods. Singularity points also exist due to discrete regions in design spaces so that solution cannot be obtained easily.

The simplest and the most robust way to resolve those problems of gradient-based methods is either enumerative search or random search. These search methods are not restricted in the types of design space and design variables. On the other hand, these need a large number of function evaluations even for simple problems. So, algorithms that can solve the problems of existing gradient-based methods and inefficiency of enumerative or random search have been proposed, and one of those methods is genetic algorithm.

In this study, we apply genetic algorithm[3] to the optimization problem of gear trains which is one of the typical engineering design problems where continuous, discrete, and integer variables are mixed together, and validate its advantages. The objectives of the design are geometrical volume(size) minimization of 2-stage gear train and that of simple planetary gear train that has very valuable aspects in lightweight, small size and high strength. With regard to these design objects, we determine the number of teeth, module, face width, and helix angle, considering constraints such as strength(durability), interference, contact ratio and other factors based on AGMA standards[4] so that these gear trains can perform the tasks required in design specification.

2. Design-Specific Genetic Algorithm for Gear Train

The procedure of Genetic Algorithm in order to solve specific application problem is divided into three processes, that is, generation of populations, evaluation of fitness, and reproduction[3]. The process of generation of population is to convert the code type of needed solutions for given problem into binary codes. In this stage, length of binary code is determined according to not only the upper and the lower values of variables but also the accuracy of finding solutions and then populations are generated at random. The process of evaluation is to find the fitness of each solution by substituting populations into objective function. This fitness value of each solution is used as a measure of selecting populations of next generation. Final process of reproduction is to search optimum solutions by choosing solutions with fitness and remixing these with others.

Fig.1 shows the search procedure of genetic algorithm in this study.

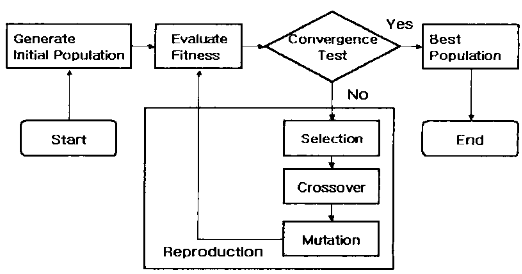


Fig. 1 Process of GA for gear train optimization

2.1 Generation of population

In order to determine the values of design variables, such as module, number of teeth, face width, and helix angle through genetic algorithm, the upper and the lower values of design variables, that is, upper bound and lower bound are to be established first. Then, length of each chromosome is determined according to the attributes of each variable, which is whether it is continuous or not. The method how to determine the length of chromosome of each variables follows equation (1), (2), (3)[5].

In case of being continuous variables,

$$2^{\lambda} \geq \frac{x_u - x_l}{\varepsilon} + 1, \qquad (1)$$

for integer variables,

$$2^{\lambda} = \text{number of integer}, \qquad (2)$$

for discrete variables,

$$2^{\lambda} = \text{number of discrete values}. \qquad (3)$$

where, λ is the length of binary code, x_u and x_l are upper and lower bound of each design variables and ε is the accuracy of continuous variables. In coding process, the accuracy of continuous variables is set up according to the attributes of them so that useless values cannot be created. Besides, when the number of integers and discrete variables are not coincident in equation (2) and (3), either penalty function method[6] or excessive distribution method[2] are used. Penalty function method is one how to dismiss solutions outside the region with posing the penalty coefficients to them when they are created. Excessive distribution method is one how to allocate the array of binary codes into the variables in region uniformly during the decoding process. The former is affected due to the selection of penalty coefficients. So excessive distribution method is used in this research.

Whole length of chromosome is determined using equation (4) after lengths of each chromosome corresponding to each design variables are obtained.

$$\Lambda = \sum_{i=1}^{n} \lambda_i \qquad (4)$$

where, Λ is the length of chromosome in binary code and n is the number of independent variables. Binary digit-chromosomes whose lengths are calculated with above procedures are randomly created, and then through the operation of genetic algorithm, each chromosome is mapped into decimal code. Finally the fitness of each chromosome is evaluated. The mapping method of continuous and integer design variables into binary codes is the same as equation (5) or equation (6

For continuous variables,

$$x_i = x_l + D_b \frac{x_u - x_l}{2^{\lambda} - 1}, \qquad (5)$$

for integer variables,

$$x_i = x_l + D_b. \qquad (6)$$

where, x_i is the physical value of each variable, and D_b is the binary code mapped into decimal code.

For discrete design variables, the physical value of x_i is determined by a mapping relationship that is illustrated by a three-bit string as shown in Table 1. The three-bit string can be decoded to an unsigned decimal integer between 0 and 7, and then, these values are mapped into discrete ones according to their one-to-one relationships.

Table 1. Mapping relationship between strings and discrete values

Binary digit strings	*Unsigned decimal integers*	*Discrete values*
000	0	D_1
001	1	D_2
010	2	D_3
011	3	D_4
100	4	D_5
101	5	D_6
110	6	D_7
111	7	D_8

2.2 Evaluation of fitness

The fitness of each chromosome is evaluated by substituting the values of decoded variables into fitness function. Exterior penalty function method[6] is used in order to convert constrained problem into unconstrained one. The fitness function for that is constructed as equation (7).

$$F_{fitness} = C_{\max} - (F_{objective} + \sum_{j=1}^{p} \gamma_j \Gamma) \qquad (7)$$

where, $F_{objective}$ is objective function, $C_{\max}$ is a positive number which has to be large enough to exclude negative fitness value, p is the number of constraints, γ_j is the penalty coefficient, and Γ is the square of the amount of constraint violation.

2.3 Reproduction

As a selection method of prominent individuals for a shift in generation, this research adopts tournament selection[7] to prevent solutions from converging in an early stage due to a large deviation of the fitness value of each solution in a population. Parental solution comprises offspring through the reproduction process of genes like crossover, mutation, and reproduction.

For crossover, this research uses random function so as to determine arbitrary point of parental chromosomes, and then, perform one point crossover that chromosomes located in a site on the determined point are swapped one another.

It is necessary to introduce a mutation operator not only to keep genetic algorithm from operating simple search but also to produce more various solutions. The application probability of mutation has to be set to an extent by which crossover is not disturbed. In this research, the application probability of mutation are set to 0.03% ~ 0.06% according to problems.

By the way, as a termination criterion of genetic algorithm, this research uses the method that iteration is performed until the number of generation reaches the predetermined maximum number. In this case, users have to set the number of a shift in generation enough to search optimums.

3. Design Application for Gear Trains and Investigation of the Results

3.1 Volume minimization of 2-stage gear train

In designing 2-stage gear train, we must determine the number of teeth that satisfies required gear ratio at first. This research performs optimization whose objectives are reduction gear ratio and geometrical volume(size) minimization of gear train. The characteristics of multi-objective optimization are not one solution but hundreds of solutions provided as well as the trade-off feature between the objectives. Therefore genetic algorithm is one of the most appropriate method resolving multi-objective optimization problems.

In order to convert this design problem having complicated constraints into unconstrained problem, the exterior penalty function method is used. The multi objective functions are mapped into a new single objective function using weighted sum method[7] where each objective function is multiplied by weight factor and united into a single objective function.

Design variables are number of teeth in each gear, module, face width, and helix angle of each stage of gear pairs.

Design constraints are bending strength, contact durability, contact ratio, aspect ratio of pinion, and minimum number of teeth that prevents from interference, pitch line velocity, and reduction gear ratio of both the first stage and the second stage of gear pairs[4,8,9].

The objective function and constraints can be written as

$$F_{objective} = w_1 F_{obj1} + w_2 F_{obj2} + \sum_{j=1}^{p} \gamma_j (MAX(G_j, 0))^2 \quad (8)$$

$$F_{obj1} = \left| u - \frac{z_{2s1} z_{2s2}}{z_{1s1} z_{1s2}} \right|$$

$$F_{obj1} = b_{s1} \left(\frac{m_{ns1}}{\cos\beta_{s1}}\right)^2 (z_{1s1}{}^2 + z_{2s1}{}^2) + b_{s2} \left(\frac{m_{ns2}}{\cos\beta_{s2}}\right)^2 (z_{1s2}{}^2 + z_{2s2}{}^2)$$

$$G_1 = 1.2\sigma_{Hs1} - \sigma_{H\lim} \qquad G_{13} = 0.8 - \varepsilon_{\beta s2}$$
$$G_2 = 1.2\sigma_{Hs2} - \sigma_{H\lim} \qquad G_{14} = \varepsilon_{\beta s2} - 6.0$$
$$G_3 = 1.15\sigma_{Fs1} - \sigma_{F\lim} \qquad G_{15} = b_{s1}\cos\beta_{s1} - 2z_{1s1}m_{ns1}$$
$$G_4 = 1.15\sigma_{Fs2} - \sigma_{F\lim} \qquad G_{16} = 0.5z_{1s1}m_{ns1} - b_{s1}\cos\beta_{s1}$$
$$G_5 = v_{ts1} - v_{t\max} \qquad G_{17} = b_{s2}\cos\beta_{s2} - 2z_{1s2}m_{ns2}$$
$$G_6 = v_{ts2} - v_{t\max} \qquad G_{18} = 0.5z_{1s2}m_{ns2} - b_{s2}\cos\beta_{s2}$$
$$G_7 = 1.0 - \varepsilon_{\alpha s1} \qquad G_{19} = z_{1s1\lim} - z_{1s1}$$
$$G_8 = \varepsilon_{\alpha s1} - 2.5 \qquad G_{20} = z_{1s2\lim} - z_{1s2}$$
$$G_9 = 1.0 - \varepsilon_{\alpha s2} \qquad G_{21} = \frac{z_{2s2}}{z_{1s2}} - \frac{z_{2s1}}{z_{1s1}}$$
$$G_{10} = \varepsilon_{\alpha s2} - 2.5$$
$$G_{11} = 0.8 - \varepsilon_{\beta s1} \qquad G_{22} = \frac{z_{2s1}m_{ns1}}{\cos\beta_{s1}} - \frac{z_{2s2}m_{ns2}}{\cos\beta_{s2}}$$
$$G_{12} = \varepsilon_{\beta s1} - 6.0$$

where, F_{obj1} is the objective function for volume minimization, F_{obj2} is the objective function for reduction gear ratio, w_1 and w_2 are weight factors for F_{obj1} and F_{obj1} respectively, γ_p is the penalty coefficient, and G_j is the violent value of constraints. $G_1 \sim G_4$ represent the constraints for the bending strength and pitting resistance on the 1st and 2nd stage gears considering factor of safety. $G_5 \sim G_6$ represent the constraints for pitch line velocity. $G_7 \sim G_{14}$ represent the constraints for contact ratios. $G_{15} \sim G_{18}$ represent the constraints for aspect ratios of pinion of 1st and 2nd stage. $G_{19} \sim G_{20}$ represent the constraints for undercut. $G_{21} \sim G_{22}$ represent constraints for reduction gear ratio and pitch diameter of gear respectively.

Fitness function is constructed as equation (9) by subtracting constructed single objective from C_{max}, what is a adequate value which prevent the fitness value from being negative

$$F_{fitness} = C_{\max} - F_{objective} \quad (9)$$

Design purpose is to reduce the volume of gears that are currently used speed reducer of escalator satisfying design specification. Upper and lower bounds of each design variable and the variable sets are shown in Table 2. Design specification and dimensions of currently used gear train for escalator is shown in Table 3.

As for parameters used in genetic algorithm, number of individuals is 30, the probability of crossover is 0.8, the probability of mutation is 0.3, and the algorithm is set to terminate when the number of a shift in a generation reaches 10000.

Table 2. Bounds of design variables for 2-stage gear train for escalator

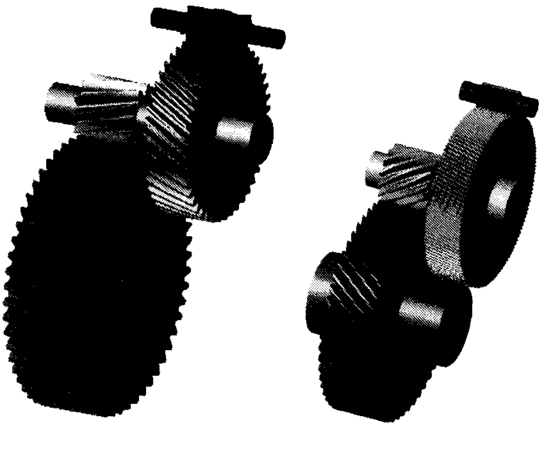
Design variable		Bounds
z_{1s1}		14 ~ 77
z_{2s1}		14 ~ 141
z_{1s2}		14 ~ 141
z_{2s2}		14 ~ 141
Normal module	[mm]	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
b_{s1}	[mm]	1 ~ 500
b_{s2}	[mm]	1 ~ 500
β_{s1}	[°]	0 ~ 45
β_{s2}	[°]	0 ~ 45

Table 3. Design specification of 2-stage gear train for escalator

Item		1-stage		2-stage	
		pinion	gear	pinion	gear
Normal module	[mm]	2.75		3.75	
Number of teeth		7	43	11	53
Total gear ratio		29.5974026			
Helix angle	[°]	28		15	
Addendum modification coefficient		0.52	-0.5060	0.4330	-0.4830
Effective face width	[mm]	37		50	
Pressure angle	[°]	20			
Rotational speed of input driver	[rpm]	1750			
Transmitted power	[kW]	7.5			
Grade(AGMA)		9			
Material		SCM415			
Heat treatment		Carburized & case hardened			
Surface hardness		HRC 60			
Volume (pitch circle)	[mm³]	2269249.613			

Table 4. Design result of optimization

Item		1-stage		2-stage	
		pinion	gear	pinion	gear
Normal module	[mm]	1		2.5	
Number of teeth		18	134	14	54
Total gear ratio		28.714286			
Helix angle	[°]	12.9		35.5	
Addendum modification coefficient		0	0	0	0
Effective face width	[mm]	30		40.1	
Pressure angle	[°]	20			
Rotational speed of input driver	[rpm]	1750			
Contact stress	[MPa]	1309		1330	
Allowable contact stress	[MPa]	1531		1531	
Bending Stress	[MPa]	351		338	
Allowable bending stress	[MPa]	423		423	
Volume (pitch circle)	[mm³]	1377539.057			



(a) Old model (b) New model

Fig.2 Model of 2-stage gear trains of escalator

Table 4 shows design results that are obtained at 8664th generation, and from this, we can easily notice optimum values of integer and discrete variables are found adequately satisfying constraints. In this design results, the correction of solution according to the types of variables after design is not needed. Therefore genetic algorithm is the effective method in designing gear trains.

Fig.2 shows the comparison between the model of existing product and the model having dimensions found by optimization in this research.

In this result, the volume of pitch diameter and face width is reduced about 40%, and the error of reduction gear ratio between objective and result are about 3%.

3.2 Volume minimization of simple planetary gear train

Planetary gear train[10] is very useful at the view of lightweight, small size, and high strength. Comparing this gear train with parallel gear train, planetary gear train is superior to parallel gear trains because each gear comprising this gear train is not only configured efficiently in the volumetric aspect but also distributes overall load to make it possible to transmit higher load per its size. Thus, planetary gear train is widely used in many industry fields and its utility value is more and more raised up. Therefore, the environment where planetary gear train is used becomes various, so that the requirement of design for compact is increased.

This research does optimum design for volume minimization of planetary gear train using genetic algorithm and compares these results with traditional optimum design method. The basic constitution of simple planetary gear train is shown in Fig.3.

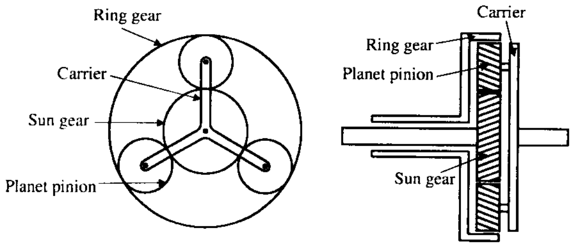


Fig.3 Simple planetary gear train

Exterior penalty function method[6] is used so as to map this complicatedly constrained problem into unconstrained problem. The objective of design is minimizing the volume of the pitch diameter of ring gear. Each gear is standard involute gear with pressure angle 20 ° and design variables are number of teeth in sun gear, normal module, helix angle, and face width. As for the variable for face width, face width, aspect ratio, and face width factor k $(=b/m_n)$ are used respectively. Design constraints are bending strength, contact durability, scoring, contact ratio, pitch line velocity, involute interference, trochoid interference, and trimming interference[8,9,10]. The objective function made up is shown as equation (10).

$$F_{objective} = (\frac{z_r m_n}{2\cos\beta})^2 b + \sum_{j=1}^{p} \gamma_j (MAX(G_j, 0))^2 \qquad (10)$$

$$G_1 = \sigma_{Hs} - \sigma_{H\lim} \qquad G_9 = b\cos\beta - 1.15 z_s m_n$$

$$G_2 = \sigma_{Fs} - \sigma_{F\lim} \qquad G_{10} = 0.25 z_s m_n - b\cos\beta$$

$$G_3 = v_{ts} - v_{t\max} \qquad G_{11} = b\cos\beta - 1.15 z_p m_n$$

$$G_4 = 1.0 - \varepsilon_{\alpha s} \qquad G_{12} = 0.25 z_p m_n - b\cos\beta$$

$$G_5 = \varepsilon_{\alpha s} - 2.5 \qquad G_{13} = b\cos\beta - 1.15 z_r m_n$$

$$G_6 = 1.0 - \varepsilon_{\beta s} \qquad G_{14} = 0.17 z_r m_n - b\cos\beta$$

$$G_7 = \varepsilon_{\beta s} - 6.0 \qquad G_{15} = z_s m_n - 500\cos\beta$$

$$G_8 = T_f - T_{f\lim} \qquad G_{16} = (z_p + 2) - (z_p + z_s)\sin(\frac{\pi}{N})$$

$$G_{17} = \cos^{-1}\left(\frac{a^2 + r_{a2}^2 - r_{a1}^2 -}{2ar_{a2}}\right) - \left(\cos^{-1}\left(\frac{r_{a2}^2 - r_{a1}^2 - a^2}{2ar_{a1}}\right)\frac{z_p}{z_r} + inv\alpha_{wt} - inv\alpha_{a1}\right)$$

$$G_{18} = \frac{z_r}{z_p}\left(\sin^{-1}\sqrt{\frac{\left(\frac{\cos\alpha_{al}}{\cos\alpha_{a2}}\right)^2 - 1}{\left(\frac{z_1}{z_2}\right)^2 - 1}} + inv\alpha_{a2} - inv\alpha_w \right) - \left(\sin^{-1}\sqrt{\frac{1-\left(\frac{\cos\alpha_{al}}{\cos\alpha_{a2}}\right)^2}{1-\left(\frac{z_1}{z_2}\right)^2}} + inv\alpha_{al} - inv\alpha_w \right)$$

$$G_{19} : \frac{z_r + z_s}{N} = \text{integer}$$

$$G_{20} : \frac{z_s}{N} \neq \text{integer}$$

where, $G_1 \sim G_2$ represent the constraints for the bending strength and pitting resistance of planet pinion and sun gear. G_3 represents the constraint for pitch line velocity. $G_4 \sim G_7$ represent the constraints for contact ratios. G_8 represents the constraint for scoring. $G_9 \sim G_{14}$ represent the constraint for aspect ratios of sun gear, planet pinion, and ring gear, respectively. G_{15} represents the constraint for pitch diameter of sun gear. G_{16} represents the constraint for interference between planet pinions. $G_{17} \sim G_{18}$ represent the constraints for trochoid interference and trimming between planet pinion and ring gear respectively. G_{19} is a geometrical constraint to compose planet gear train. G_{20} is a constraint that prevents same meshing phase between each planet pinion and sun gear.

By subtracting this newly generated objective function $F_{objective}$ from C_{max}, what is the adequate value preventing the fitness value from being negative, the fitness function $F_{fitness}$ is same as equation (9).

Using this design method, optimum design is carried out for planetary helical gear train. Configuration is that sun gear transfers input power into output carrier with fixed ring gear. Required reduction gear ratio is 4.2 and considering suggested constraints, we have found design solution to minimize volume of planetary gear train. Upper bounds and lower bounds of each design variable and variable set are determined as shown in Table 5. Table 6 shows design specification of planetary gear train used in optimum design. As for parameters in genetic algorithm, number of generation is 30, the probability of crossover is 0.7, the probability of mutation is 0.05, and termination number of a shift in a generation is set to 3000.

Table 5. Design variables for planetary gear train

Design variables			Bounds
	Number of teeth in ring gear		14 ~ 45
	Normal module	[mm]	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
	Helix angle	[°]	0 ~ 45
Face width	b	[mm]	1 ~ 500
	Aspect ratio (b/d_s)		0.2 ~ 1.2
	k		10 ~ 20

Table 6. Design specification of planetary gear train

Specification		
Rotational speed of input driver	[rpm]	3400
Transmitted power	[kW]	75
Reduction gear ratio		4.2
Pressure angle	[°]	20
Load cycle for life factor		1.0E7
Surface hardness	[BHN]	540
Gear operating state		Commercial enclosed units
Lubricant		SAE 50 motor oil with mild EP
Surface condition		Fine finish
AGMA quality		11

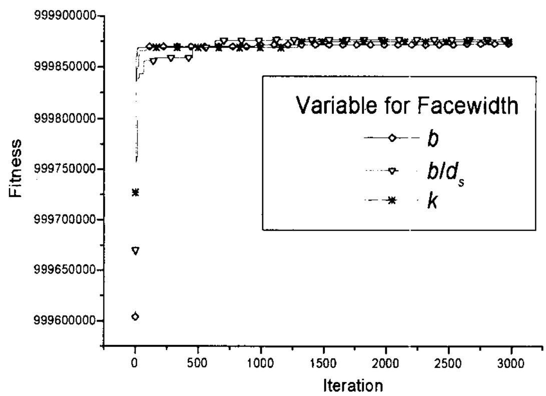


Fig.4 Iteration history of best fitness for planetary gear train

Table 7. Design result of planetary gear train

	b	b/d_s	k
Number of iteration	2610	1082	2585
Number of teeth in sun gear	38	43	29
Number of teeth in planet pinion	42	47	32
Number of teeth in ring gear	122	137	93
Normal module [mm]	1.0	1.0	1.5
Face width [mm]	25.4	20.138	15.0
Helix angle [°]	30.4	28.9	40.1
Reduction gear ratio	4.21	4.186	4.207
Contact stress [MPa]	933.77	934.20	934.10
Allowable contact stress [MPa]	934.22	934.22	934.22
Bending stress [MPa]	273.62	299.04	262.28
Allowable Bending stress [MPa]	309.36	309.36	309.36
Volume [mm^3]	399127	387318	391828
Pitch radius of ring gear [mm]	141.45	156.49	182.37

Fig.4 shows evolution history that genetic algorithm searches optimums in the case of face width b, aspect ratio b/d_s, and face width factor $k(=b/m_n)$ as a variable for face width respectively[8]. Table 7 shows each design result for them. It is easily shown that the fitness rises suddenly in an early stage of evolution history but after that stage, the degree of adaptation slopes gently. As for design results, each result differs in the coefficient of face width slightly, while each design variable approaches to adequate design value satisfying constraints.

Table 8. Comparison of optimum solution using GA and ADS

method			Volume	m_n	z_s	b	β
GA			387318	1.0	43	20.138	28.9
ADS	Initial value : GA	1st	371776	0.99586	43	19.7569	28.9
		Modify	388976	1.0	43	20.5	28.9
	Initial value : Arbitrary	1st	1246920	2.063	25.5173	40.81	29.4541
		2nd	738940	1.3383	39.4258	24.4941	29.2562
		3rd	304741	0.82	44.4938	21.7624	29.8389

Table 8 shows the design results of genetic algorithm where design specifications are the same and the aspect ratio is the variable of face width, in comparison with those of optimum design based on gradient-based methods. As an optimum design software for gradient-based methods, ADS[11] is used. Since ADS is severely affected by initial design value, two conditions are investigated and compared. At one condition, initial design value for ADS is set up from solution, which is found by genetic algorithm, optimization is performed to obtain solution, and correction is made for it to be available design value. At another condition, initial design value is arbitrarily selected and optimization is done. As for the former condition, the objective value is lower but optimum module is not available for commercial purpose, so that the optimum value of module is replaced by the nearest value commercially available and then whether or not the replaced value violates constraints is investigated. If the replaced value violates, other values of design variables are adjusted towards constraints being satisfied, and resulting values are shown in field named *modify*. Here, only module and face width are changed, because in many cases redesign process is needed if considering helix angle with these variables. As for the latter condition, initial design value is arbitrarily selected as far as being considered to be adequate by designers and optimum is found. In this condition, most cases are design results violating constraints, so that three cases of design values that satisfy design constraints are shown in the table. The results of the first and second case are very inferior to the results from initial design value by genetic algorithm. The result of the third case seems to be considerably good, but in this case the optimum module is not commercially available and number of teeth is also not integer but real value. Therefore, the optimum module and optimum number of teeth has to be converted into available values and then design values satisfying constraints are to be found while face width and helix angle are changed. But, this makes reduction gear ratio away from target value of design specification and constraints to be violated. And designers have to determine the amount of changes of the face width and helix angle. It results in making the optimum design depend primarily on designers' decision and efforts. When considering these features, the third case design result is not superior to the optimum result of genetic algorithm and likely to be inadequate value from a point of view of module and number of teeth.

4. Conclusions

By applying genetic algorithm to optimization problems of gear train, we obtain a good design result in comparison with the existing design and the design result of conventional design method. Conclusions are as follows.

1. This design approach solves the problem of conventional gradient-based optimization methods that design values primarily depend on initial guess.

2. Integer, discrete, and continuous variables are directly treated, so that not only random adjustment is excluded but also reliability of design value is raised. As a result, this research is able to get more excellent solutions than existing method especially for optimization problem of gear train that has mixed type design variables.

3. Genetic algorithm is known to be applicable to many various fields, but it has difficulty in being applied to complicated constrained problem. However, genetic algorithm is applied to this type of problem, that is, optimization problem of gear train so that the obtained result is better than those by conventional gradient-based method.

4. Genetic algorithm has the disadvantage of inefficiency rather than gradient-based methods, but its design result is more excellent and reliable than the results of other methods.

5. As for selection of several parameters of genetic algorithm such as penalty coefficient, probability of mutation, probability of crossover, and population size, either empirical or trial and error method are used according to the features of problems.

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