International Journal of the Korean Society of Precision Engineering, Vol. 1, No. 1, June 2000.

Interference and Efficiency Analysis of 2K-H I Type Differential Gear Unit

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ABSTRACT

In the design of epicyclic gearing, the analysis of interference and mechanical efficiency is an important index. As an applied way, epicyclic gearing can be used for planetary gear drive and differential gear unit. In case that one of its components is fixed with intend, it is called planetary gear drive. On the contrary, in case that no component is fixed, it is called differential gear unit. In this paper, various design constraints and interferences are defined for 2K-H I type epicyclic gearing which is a basic arrangement of diverse epicyclic gearings. And various interferences are analyzed, and mechanical efficiency is calculated in case that 2K-H I epicyclic gearing is used for a differential gear unit as the change of gear ratio, cutter pressure angle, addendum modification coefficient. As that results, trend of mechanical efficiency is investigated in the ranges of addendum modification coefficients which would not lead to interferences, and the optimal range of addendum modification coefficient which can generate the maximum mechanical efficiency are presented. In order to prove results of theoretical efficiency analysis, experimental studies are performed.

Key Words: Epicyclic gearing, differential gear unit, efficiency, interference

Nomenclature

z = number of teeth
x = addendum modification coefficients
d = diameter of pitch circle
d_{kp} = diameter of tip circle of planet gear
α_c = cutter pressure angle
α_{kp} = pressure angle at the tip of planet gear
α_{brp} = working pressure angle between ring and planet gears
a_{rp} = center distance between ring and planet gears
a_{sp} = center distance between sun and planet gears
N_p = number of planet gears
i_0 = gear ratio(z_r/z_s)
η_0 = basic efficiency
ω = angular velocity
Subscripts
r:ring gear, s:sun gear, p:planet gear, c:carrier

1. Introduction

In general, the epicyclic gearing is used widely in a field of power transmission including automatic transmission and continuously variable transmission of automobiles, transmission of light rail vehicle and so on because of its compact size, light weight, the capability of a high speed ratio, and the ability to provide a differential action. However, special attention should be paid not only efficiency in high speed ratio but also interferences among its components. That is, since epicyclic gearing is composed of internal gear called ring gear, sun gear, planet gear and carrier, it should be designed to avoid many interferences and to make the transmission run smoothly. Therefore, it has many constraints in design and manufacturing process.

Many previous contributions related to mechanical efficiency in differential mechanism had been performed. However, most of them had only executed power flows and mechanical losses, kinematic relationships of its mechanism, not carried out interference analysis(1-6). Only a part of them had investigated on the efficiency of systems composed of differential gear unit and speed variable unit. For interference conditions and limitations of addendum modification coefficient in epicyclic gearing, some contributions had performed(7-8). However, they had only achieved for meshing between internal gear and pinion, which have about the same number of teeth, not considered assembly conditions inherent to epicyclic gearing.

Based on previous contributions, interferences of 2K-H I type, which is a basic arrangement of various epicyclic gearing, are investigated using kinematic characteristics, interference conditions of meshing, assembly conditions of it. The occurrence of interference is analyzed as the change of gear ratio, cutter pressure angle, addendum modification coefficient, and mechanical efficiencies are calculated for 6 operating cases. The trend of mechanical efficiencies is investigated in ranges of addendum modification coefficients which would not lead to interferences, and the optimal ranges of addendum modification coefficient which generate the maximum mechanical efficiency are presented. In order to prove results of theoretical efficiency analysis, experimental studies are performed.

2. 2K-H I type epicyclic gearing

2K-H I type epicyclic gearing is shown in Fig. 1, which is composed of ring gear(internal gear), sun gear, planet gear and carrier. The shaft of ring gear, of sun gear and of carrier are called basic shafts, which are concentric. As an applied way, epicyclic gearing can be used for planetary gear drive and differential gear unit. In case that one of its 3 basic shatfs is fixed with intend, it is called planetary gear drive where one basic shaft not fixed is used for input and another not fixed is used for output. On the contrary, in case that no basic shaft is fixed, it is called differential gear unit which is able to provide a differential action.

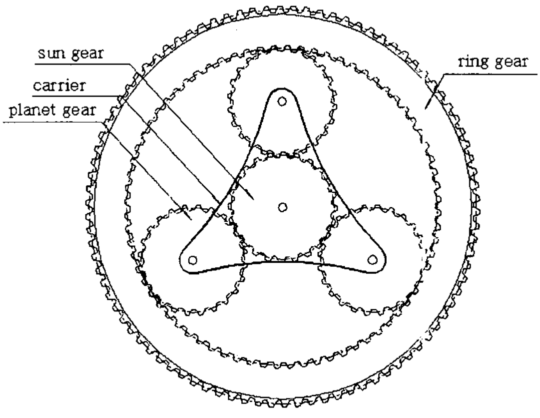

Fig. 1 2K-H I type epicyclic gearing

3. Interference and assembly conditions

According to M. Muneharu(8), there are various patterns of meshing interference between internal gear and pinion. Based on those patterns, meshing interferences of 2K-H I type epicyclic gearing are followed.

3.1 Interference conditions

1) tip circle and base circle of ring gear

The tip circle diameter of ring gear should be larger than the base circle diameter so that tooth profile of ring gear is a complete involute curve, namely, relation (1) should be satisfied.

$$z_r \geq \frac{2(1-x_r)}{1-\cos\alpha_c} \tag{1}$$

2) tooth thickness at tip circle

The tooth thickness of ring gear, planet gear and sun gear at the tip circle should not be pointed for the purpose of durability and smooth action. Therefore, addendum modification coefficients should be selected in those ranges.

3) undercut of planet and sun gear

Addendum modification coefficients of planet and sun gear should be selected for the sake of no undercut, namely, relation (2) should be satisfied.

$$x \geq 1-\frac{1}{2}z\sin^2\alpha_c \tag{2}$$

4) working pressure angle of ring and planet gear

If the working pressure angle between ring and planet gear is less than zero, tooth profiles of them are intersected each other. Therefore, addendum modification coefficients of them should be selected so that working pressure angle is more than zero.

$$z_r \geq z_p + \frac{2\tan\alpha_c(x_p - x_r)}{\mathrm{inv}\alpha_c} \tag{3}$$

5) contact ratio

Contact ratio in epcyclic gearing, which is one between sun and planet gear, or ring and planet gear, should be more than 1.

6) involute interference

Involute interference is shown in Fig. 2, which is possible to occur at the tip of ring gear or the root of planet gear. In order to avoid it, relation (4) should be satisfied.

$$\frac{z_p}{z_r} \geq 1 - \frac{\tan\alpha_{kr}}{\tan\alpha_{brp}} \tag{4}$$

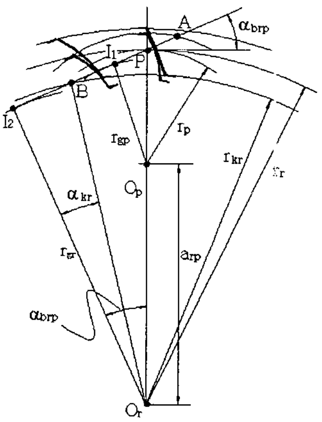

Fig. 2 Involute interference of differential gear unit

7) trochoid interference

Trochoid interference is shown in Fig. 3, which is possible to occur between the tip of planet gear and of ring gear after meshing. It is easy to occur when the teeth difference between planet and ring gear is of small number. Therefore, the tip of ring gear should be over a point B in Fig. 3 when the tip of planet gear is rotated to a point B.

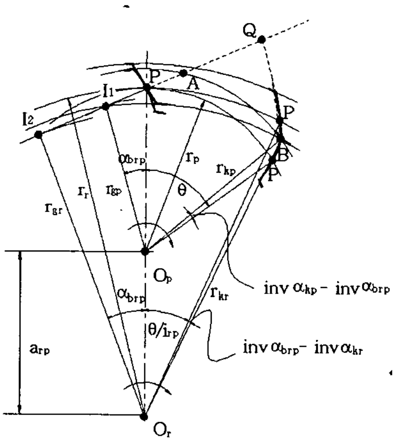

Fig. 3 Trochoid interference of differential gear unit

8) trimming interference

Though planet gear is able to assemble to the axial direction of ring gear, planet gear is unable to assemble to the radial direction of ring gear as the case may be. This phenonenon is called trimming interference, which is shown in Fig. 4. It is also easy to occur when the teeth difference between planet and ring gear is of small number. To avoid it, distance EF should be larger than distance CD in Fig. 4.

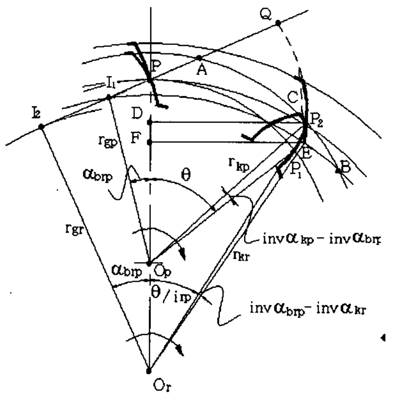

Fig. 4 Trimming interference of differential gear unit

3.2 Assembly conditions

Since 2K-H I epicyclic gearing is composed of ring gear, sun gear, 3 planet gears and carrier, it should be satisfied with various assembly conditions to assemble smoothly(7-9).

1) concentric condition

In 2K-H I type epicyclic gearing, three basic shafts should be concentric. That is to say, center distance between ring and planet gear should be equal to center distance between sun and planet gear. Therefore, relation (5) should be satisfied.

$$a_{rp} = \frac{1}{2}(d_r - d_p) = \frac{1}{2}(d_s + d_p) \quad (5)$$

2) adjacent condition

If numbers of planet gear are too many, each planet gears would occur a tip interference, so that assembly is impossible. Therefore, relation (6) should be satisfied.

$$d_{kp} < 2a_{sp}\sin\left(\frac{\pi}{N_p}\right) \quad (6)$$

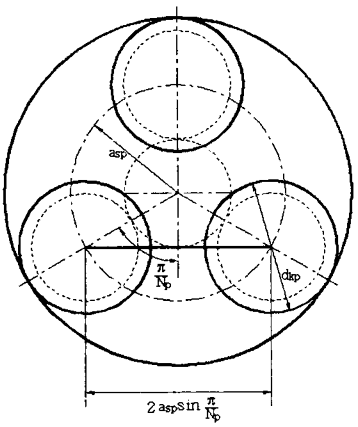

Fig. 5 Adjacent condition of differential gear unit

3) assembling condition

Epicyclic gearing with multiple planet gears must follow definite rule concerning numbers of teeth and numbers of planet gear to allow assembly. Therefore, relation (7) should be followed.

$$\frac{z_r + z_s}{N_p} = \text{integer} \quad (7)$$

4. Simulation and Experiment

4.1 Theoretical efficiency

1) basic efficiency

When a carrier is fixed, the efficiency of epicyclic gearing is defined as basic efficiency. That is, it is a meshing efficiency between ring and planet gear times meshing efficiency between sun and planet gear. Since the overall efficiency of epicyclic gearing is obtained by gear ratio, angular velocity of components, it is easy to be obtained through the introducing of basic efficiency. Since contact ratio is an important factor, basic efficiency is calculated by considering contact ratio ranges(1～2 or 2～3).

2) overall efficiency of differential gear unit

Many previous contributions concerning mechanical efficiency in differential mechanism had been performed. In this paper, the method presented by M. Muneharu(8) is applied. According to his contribution, 2K-H I type differential gear unit is capable of 6 operating cases as an applied way. Each theoretical efficiency relations are shown in Table 1.

3) assumptions

① Though the friction coefficient of tooth flank is varied complicatedly during meshing, it is assumed to 0.1 which is constant.

② The power losses of epicyclic gearing are distinct constituent losses which are mesh losses, windage and churning losses, bearing losses, oil pump losses, seal loss. However, windage and churning losses, bearing losses, oil pump losses is often omitted because those are very small. Therefore mesh loss is assumed to the only loss of epicyclic gearing.

③ The distributions of transmitted power between planet gears are uniform.

④ The normal load between tooth flanks is uniform during the rotation of gears.

⑤ Epicyclic gearing is designed for a standard gear.

⑥ Since backlash hardly affect to efficiency, it is ignored.

4.2 Calculation

For 2K-H I type differential gear unit, theoretical efficiency analysis of each 6 cases is performed after interference analysis. Flow chart of the analyzing interference and efficiency is shown in Fig. 6, which is to analyze the trend of interference and efficiency. Using it, interferences and each efficiencies are calculated as changing addendum modification coefficients, gear ratios, cutter pressure angles. Those specifications of 2K-H I type differential gear unit are shown in Table 2.

Table 1 Efficiency of 2K-H I type differential gear unit

Cases	Driving	Driven	Direction of Rotation	Efficiency(η)
1	Sun Ring	Carrier	$0<\omega_r<\omega_c<\omega_s$	$\dfrac{(1+\eta_0 i_0)(\omega_s+i_0\omega_r)}{(1+i_0)(\omega_s+\eta_0 i_0\omega_r)}$
			$0<\omega_s<\omega_c<\omega_r$	$\dfrac{(\eta_0+i_0)(\omega_s+i_0\omega_r)}{(1+i_0)(\eta_0\omega_s+i_0\omega_r)}$
2	Sun	Ring Carrier	$\omega_r<\omega_c<\omega_s$ $\omega_r<0$	$\dfrac{(1+\eta_0 i_0)\omega_c-i_0\eta_0\omega_r}{(1+i_0)\omega_c-i_0\omega_r}$
3	Carrier	Sun Ring	$0<\omega_r<\omega_c<\omega_s$	$\dfrac{(1+i_0)(\eta_0\omega_s+i_0\omega_r)}{(\eta_0+i_0)(\omega_s+i_0\omega_r)}$
			$0<\omega_s<\omega_c<\omega_r$	$\dfrac{(1+i_0)(\omega_s+i_0\eta_0\omega_r)}{(1+\eta_0 i_0)(\omega_s+i_0\omega_r)}$
4	Ring Carrier	Sun	$\omega_r<\omega_c<\omega_s$ $\omega_r<0$	$\dfrac{\eta_0\{(1+i_0)\omega_c-i_0\omega_r\}}{(\eta_0+i_0)\omega_c-i_0\omega_r}$
5	Sun Carrier	Ring	$\omega_s<\omega_c<\omega_r$ $\omega_s<0$	$\dfrac{\eta_0\{(1+i_0)\omega_c-\omega_s\}}{(1+\eta_0 i_0)\omega_c-\omega_s}$
6	Ring	Sun Carrier	$\omega_s<\omega_c<\omega_r$ $\omega_s<0$	$\dfrac{(\eta_0+i_0)\omega_c-\eta_0\omega_s}{(1+i_0)\omega_c-\omega_s}$

Table 2 Specifications of 2K-H I type differential gear unit

Number of teeth			Cutter pressure angle (Degree)	Module	Gear type
Sun gear	Planet gear	Ring gear			
24	18	60	14.5, 20.0, 26.0	2.5	Spur gear
24	24	72			
24	30	84			
24	36	96			
24	42	108			

4.3 Experiment

1) manufacture of differential gear unit

Differential gear unit manufactured is a standard gear, namely, addendum modification coefficient is zero and addendum is equal to module. And it is designed for a spur gear with z_s=24, z_p=24, z_r=72 which has 20° cutter pressure angle.

2) experimental bench

The experimental bench for efficiency is shown in Fig. 7, and its schematic diagram is shown in Fig. 8. Differential gear unit is driven by using an induction motor(11kW) as the direction of rotation in Table 1, and torque is generated by using an electro-magnetic particle brake(100Nm). Torque and angular velocity of input and output shaft are measured by using torque meters(120Nm), speed meters installed at each shafts. Therefore overall efficiency of differential gear unit can be written by relation (8).

$$\eta = \frac{T_{out}\omega_{out}}{T_{in}\omega_{in}} \tag{8}$$

where T_{in}, T_{out} are measured torques of input and output shaft, ω_{in}, ω_{out} are measured angular velocities of input and output shaft.

Experimental bench is able to realize 6 operating cases of differential gear unit, and to change speed ratio of it. If an applied load at output shaft is increased, the angular velocity of electric motor will

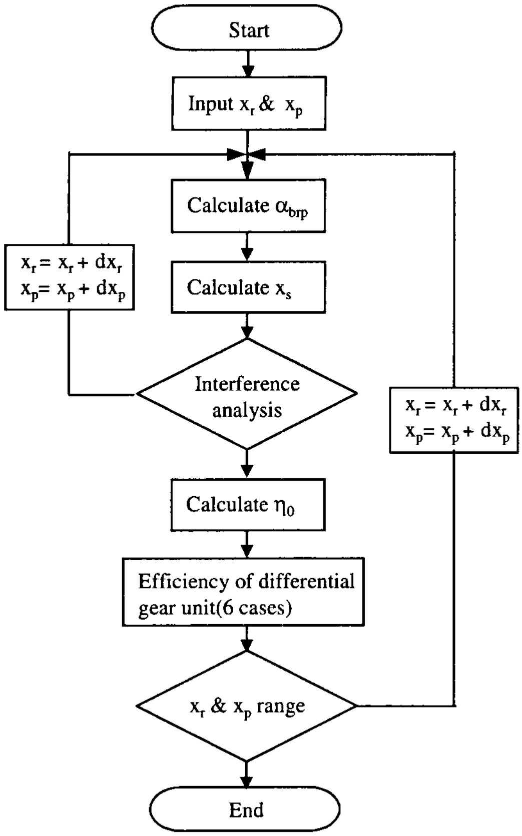

Fig. 6 Computation flow chart of analyzing interference and efficiency

be decreased irregularly. Therefore, the angular velocity of electric motor is controlled to preserve an ordered value regardless of an applied load. Before experiment, warming up(10) is performed during 30 minutes.

Fig. 7 Experimental bench for efficiency analysis

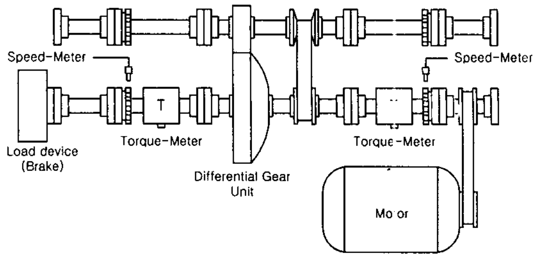

Fig. 8 Schematic diagram of experimental bench

5. Results and discussions

5.1 Simulation results

Ranges of addendum modification coefficients which would not lead to interferences are extended as the increase of cutter pressure angle and gear ratio in differential gear unit. In differential gear unit with z_s=24, z_p=24, z_r=72, ranges of addendum modification coefficients without any interference are shown in Fig. 9(a)-(c) as the increase of cutter pressure angle, and are shown in Fig. 10(a)-(e) as the increase of gear ratio(z_r/z_s), which show basic efficiencies of differential gear unit.

Efficiency analysis for 6 operating cases in Table 1 is performed, whose specifications are shown in Table 2. Results of case 1 and case 3 are much the same because input and output are just converted. Similarly, case 2 and case 4, case 5 and case 6 are shown very similar results each other.

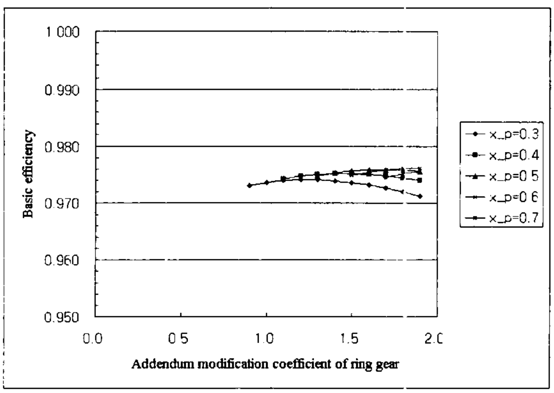

Fig. 9(a) Basic efficiency of differential gear unit (z_s=24, z_p=24, z_r=72) at α_c=14.5°

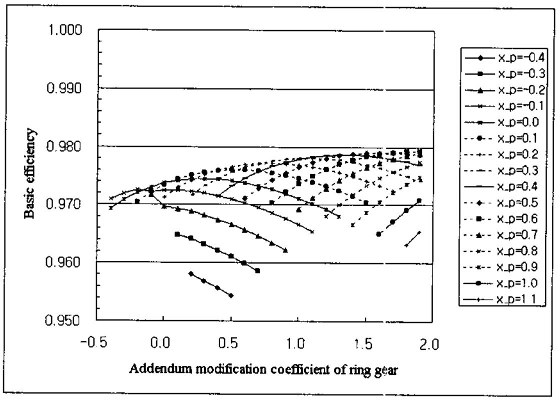

Fig. 9(b) Basic efficiency of differential gear unit (z_s=24, z_p=24, z_r=72) at α_c=20.0°

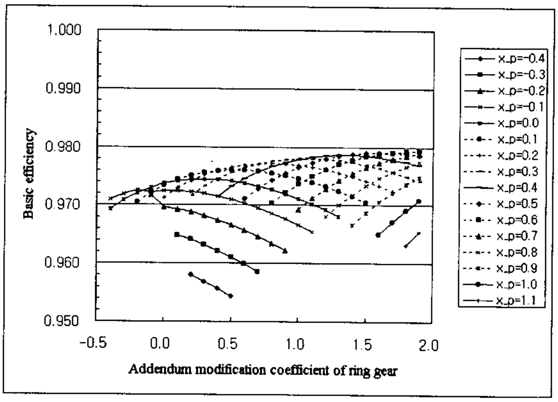

Fig. 10(b) Basic efficiency of differential gear unit (α_c=20°) with z_s=24, z_p=24, z_r=72

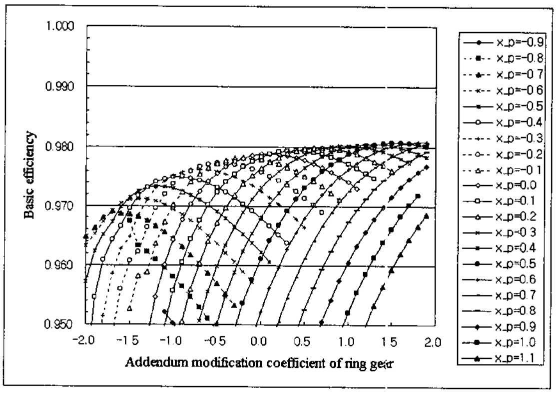

Fig. 9(c) Basic efficiency of differential gear unit (z_s=24, z_p=24, z_r=72) at α_c=26.0°

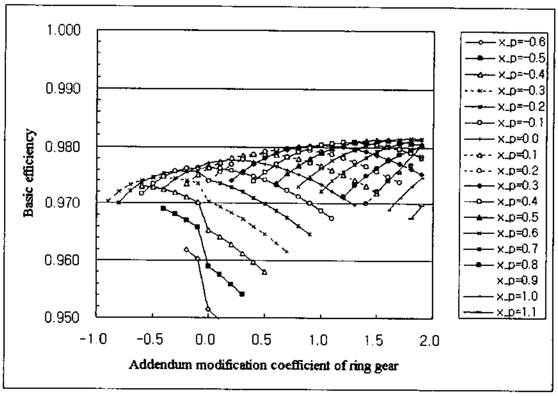

Fig. 10(c) Basic efficiency of differential gear unit (α_c=20°) with z_s=24, z_p=30, z_r=84

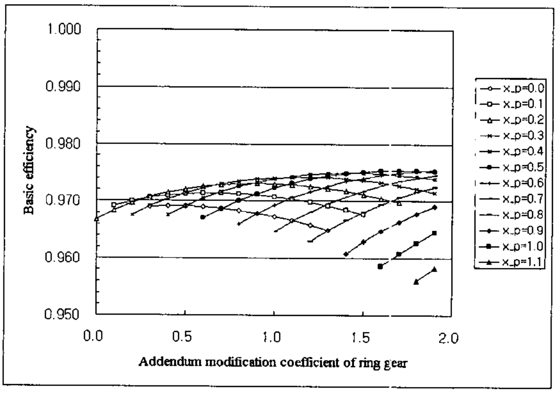

Fig. 10(a) Basic efficiency of differential gear unit (α_c=20°) with z_s=24, z_p=18, z_r=60

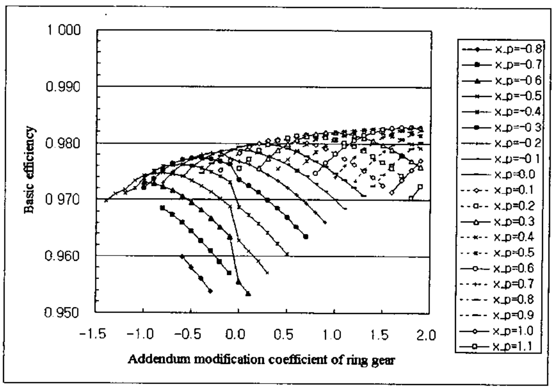

Fig. 10(d) Basic efficiency of differential gear unit (α_c=20°) with z_s=24, z_p=36, z_r=96

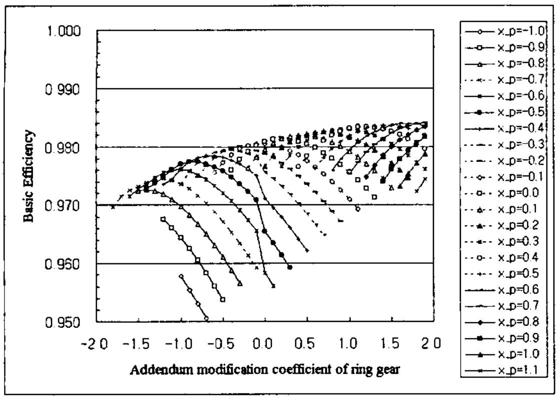

Fig. 10(e) Basic efficiency of differential gear unit (α_c=20°) with z_s=24, z_p=42, z_r=108

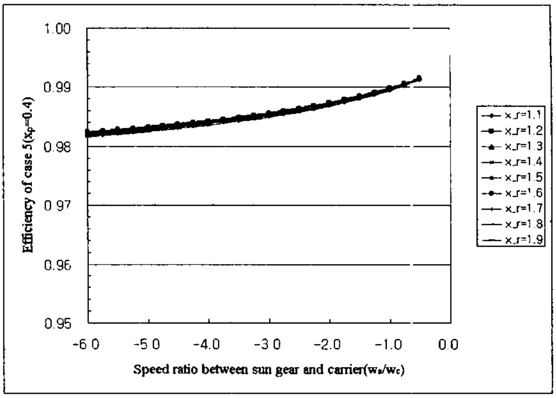

Fig. 11(a) Efficiency of case 5 (z_s=24, z_p=24, z_r=72) at α_c=14.5°

Efficiencies of case 1, 3 and case 5, 6 are higher than those of case 2 and 4, which are mainly influenced on the change of cutter pressure angle. That is to say, those differences are 1.0~1.1% at the cutter pressure angle 14.5° , 0.8~1.0% at 20° , 0.7~0.9% at 26° . However, those differences are of no relevance to the change of gear ratio(z_r/z_s).

As the increase of cutter pressure angle, efficiencies of each cases are only grown to the maximum 0.3~0.5% at the same gear ratio, which can't show an eminent variation. For differential gear unit with z_s=24, z_p=24(x_p=0.4), z_r=72, efficiencies of case 5 are shown in Fig. 11(a)-(c) as cutter pressure angle changes. They show the results stated before. The more speed ratio between sun gear and carrier(ω_s/ω_c) is close to -1, the more efficiency is somewhat increased at the same addendum modification coefficient. Similarly, as speed ratio between ring and sun gear(ω_r/ω_s) is closed to 1 in case of case 1 and 3, and as speed ratio between ring gear and carrier(ω_r/ω_c) is closed to -1 in case of case 2 and 4, efficiency is somewhat increased.

As the increase of gear ratio(z_r/z_s), efficiencies of each cases are only grown to the maximum 0.5% at the same cutter pressure angle, which can't also show an eminent variation. For differential gear unit with α_c=20.0° , x_p=0.4, efficiencies of case 5 are shown in Fig. 12(a)-(e) as gear ratio changes. As the increase of gear ratio(z_r/z_s), efficiency trends of case 1 and 3, 2 and 4 are similar to case 5 and 6 at the same cutter pressure angle.

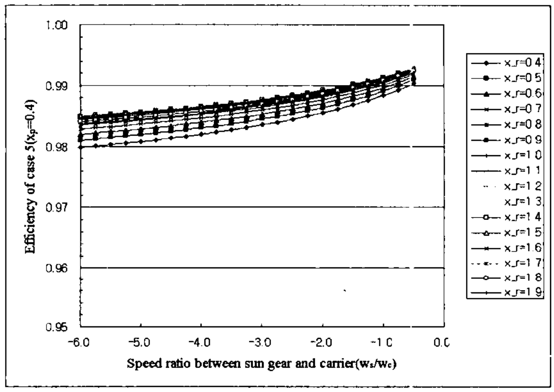

Fig. 11(b) Efficiency of case 5 (z_s=24, z_p=24, z_r=72) at α_c=20.0°

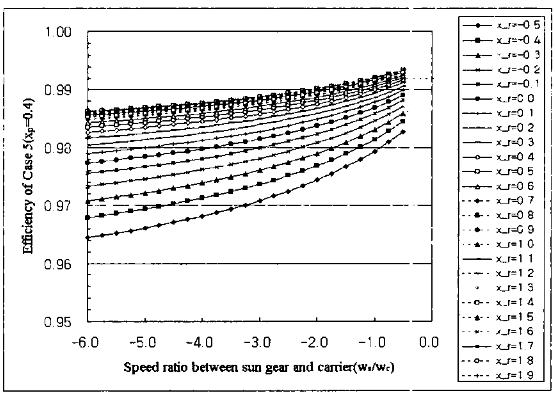

Fig. 11(c) Efficiency of case 5 (z_s=24, z_p=24, z_r=72) at α_c=26.0°

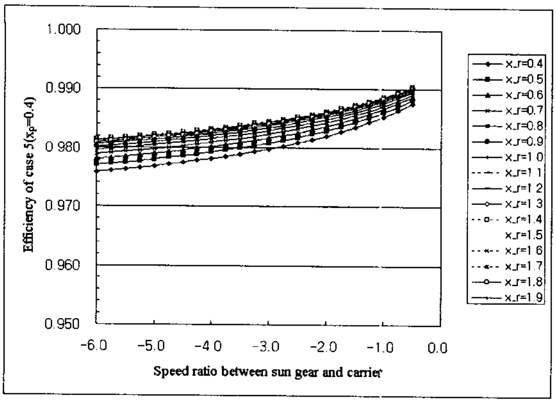

Fig. 12(a) Efficiency of case 5 (α_c=20°) with z_s=24, z_p=18, z_r=60

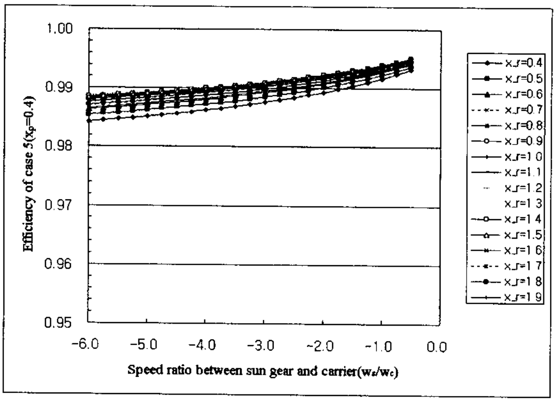

Fig. 12(b) Efficiency of case 5 (α_c=20°) with z_s=24, z_p=24, z_r=72

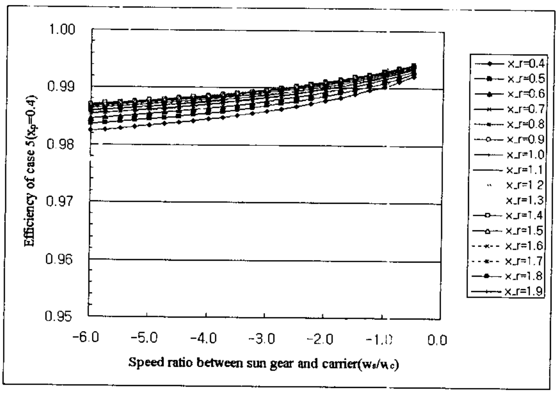

Fig. 12(c) Efficiency of case 5 (α_c=20°) with z_s=24, z_p=30, z_r=84

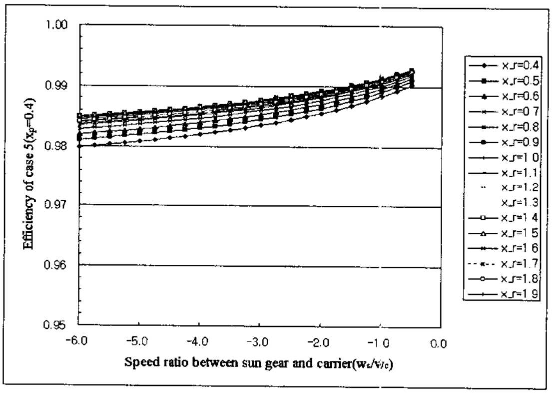

Fig. 12(d) Efficiency of case 5 (α_c=20°) with z_s=24, z_p=36, z_r=96

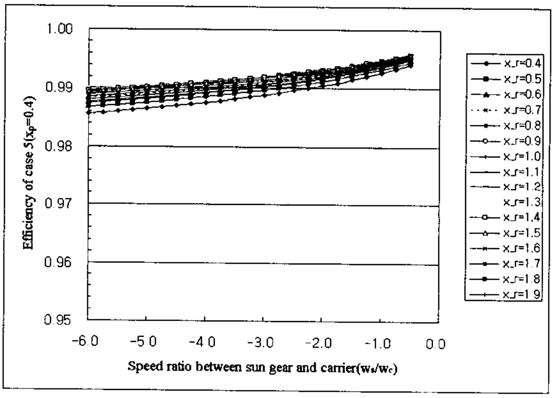

Fig. 12(e) Efficiency of case 5 (α_c=20°) with z_s=24, z_p=42, z_r=108

However the range of addendum modification coefficients which would not lead to interferences is different each other at each conditions, the range of addendum modification coefficients which would generate the maximum efficiency is x_p=0.2~0.6 at x_r=0.6 or more.

5.2 Experimental results

Fig. 13, 14 show experimental results and theoretical results of case 1 and 5 for differential gear unit with z_s=24, z_p=24, z_r=72 manufactured as a standard gear. They show that experimental results are on the whole analogous with theoretical results. The difference between experimental results and theoretical results is caused by the inertia, clearances of experimental components(gears, shafts, bearings, etc.), disparity between actual conditions and ideal conditions used in simulation.

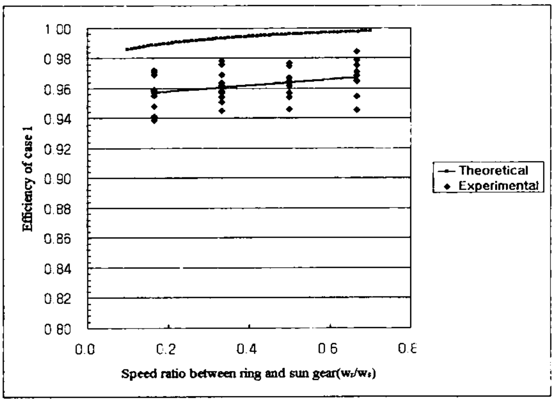

Fig. 13 Experimental results and theoretical result of Case 1

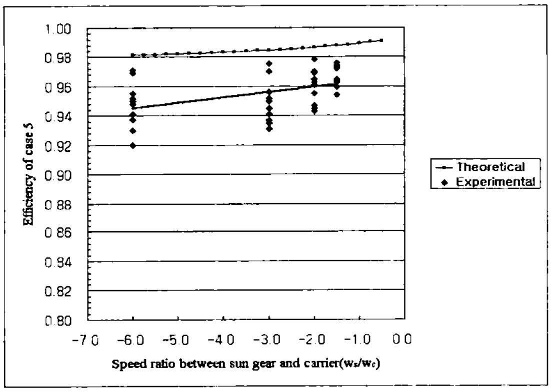

Fig. 14 Experimental results and theoretical result of Case 5

6. Conclusions

Interferences of 2K-H I type differential gear unit are analyzed as the change of gear ratio, cutter pressure angle, addendum modification coefficient, and mechanical efficiencies are calculated for 6 operating cases with addendum modification coefficient not lead to any interference. The summations are followed

1) Ranges of addendum modification coefficients which would not lead to interferences are extended as the increase of cutter pressure angle and gear ratio. Based on these ranges, the range of addendum modification coefficients which would generate the maximum efficiency is x_p=0.2~0.6 at x_r=0.6 or more.

2) Efficiencies of case 1, 3 and case 5, 6 are on the whole higher than those of case 2 and 4, and the difference is about 0.7~1.2%. However, this difference is mainly influence on the change of cutter pressure angle, and is of no relevance to the change of gear ratio(z_r/z_s).

3) The validity of efficiency analysis is proven by an experiment.

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