

# New Analysis on Reception of $M$ -ary FSK Signals over Rician Fading Channels

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## Abstract

In this paper, we analyze the distribution of the envelope of the received signal over frequency-nonselective slow Rician fading channels with additive white Gaussian noise(AWGN). Especially, we can obtain the error rate performance of noncoherent  $M$ -ary FSK(MFSK) over slow and flat Rician fading channels and AWGN from the new probability density function(PDF) of the envelope, not the PDF of the instantaneous signal-to-noise ratio(SNR) published before, of the received signal. When coherent MFSK signals experience the Rician fading channel, the performances are derived, using the union bound.

## I. INTRODUCTION

The statistical properties of mobile radio environments can be often specified by three propagation effects: 1) short-term fading 2) long-term fading 3) propagation path loss<sup>[1]</sup>. In short-term fading, the scattering mechanism only results in numerous reflected components<sup>[2]</sup>. The Rayleigh model is used to characterize this fading in small geographical areas and sometimes does not account for large scale effects like shadowing by building and hills<sup>[3]</sup>. In long-term fading, the change of effective height for mobile communication antenna exists due to the nature of the terrain. Its statistics follow the log-normal distribution. But in the event that there are both the information signal and the interfering signal, this channel is modeled to be a Rician fading model<sup>[4]</sup>. By modeling the channel as a Rician fading channel, a result is obtained that is valid in the limit of large direct-to-diffuse power ratios for channels with no fading and in the limit of small direct-to-diffuse power ratios for Rayleigh environments as

well as for the general case when neither the direct nor the diffuse components of the signal are negligible<sup>[5]</sup>. When Rician factor  $K$  is 0, the error performances lead to those of Rayleigh fading model. A more general fading distribution which has been shown to be appropriate to model the multipath fading in urban as well as indoor situations is the Nakagami  $m$ -distribution, of which Rayleigh fading is a special case of fading index  $m=1$ <sup>[6]</sup>. Nakagami distribution can model fading conditions more or less severe than those in the Rayleigh case. The Rice and lognormal distribution can also be approximated by the Nakagami distribution<sup>[7]</sup>.

We used the PDF of the instantaneous SNR<sup>[8],[9]</sup> or that of the amplitude of the received signal<sup>[7]</sup>, to analyze the general performance of digital modulation over a fading channel. Robert<sup>[10]</sup> derives an accurate expression for the bit error probability of noncoherent FSK and DPSK for Rician fading, but without an extension to the  $M$ -ary case. Crepeau<sup>[11]</sup> evaluated the error rate performance of noncoherent MFSK and DPSK in Nakagami fading channels. Sun

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<sup>[12]</sup> presents the closed-form solution for noncoherent MFSK performance over a slow and flat Rician fading. But, until now, we could not use the distribution of the envelope, to present the performance results of the received signals under the effect of the Rician fading environments. So, we derive these PDFs of the received signal over the Rician fading channels. Especially, in Appendix, by using the new PDF of the envelope we analyze the performance of noncoherent MFSK on Rician fading channels. It can be shown that these results are perfectly equal to the results found by Sun<sup>[12]</sup>. Also, we can find the error performance of coherent MFSK over slow and flat Rician fading channels when AWGN is present, using the union bound, an upper bound on the bit error probability. Then we compare coherent MFSK performance result with that of noncoherent MFSK signals under the Rician fading channels. Error probabilities are graphically displayed for various values of  $M$  and Rician factor  $K$ . The remains of this paper are as follows. In Section 2, the distributions of the envelope, not those of the instantaneous SNR, under the Rician fading channels are analyzed. In Section 3, we can derive the performance of noncoherent MFSK signals, using the PDF of envelope on the Rician fading channels. The analytical results for the performance of coherent MFSK by using the union bound are represented in Section 4. Subsequently in Section 5, selected numerical results are presented. Finally we summarize some known results in Section 6.

## II. THE DISTRIBUTION OF THE ENVELOPE UNDER THE RICIAN FADING CHANNEL

The received signal  $r(t)$  has the form

$$r(t) = d \cos \omega_c t + n(t), \quad 0 \leq t \leq T \quad (1)$$

where the amplitude of desired signal,  $d$ , is assumed to suffer from Rician fading and  $n(t)$  is

AWGN with one-sided power spectral density  $N_0$ .

When thermal noise is passed through the narrowband filter of the receiver, the characteristics of the envelope and phase of the received signal including AWGN are given by

$$\begin{aligned} r(t) &= d \cos \omega_c t + n_c \cos \omega_c t - n_s \sin \omega_c t \\ &= [(d + n_c)^2 + n_s^2]^{1/2} \cos \left( \omega_c t + \tan^{-1} \frac{n_s}{d + n_c} \right) \\ &= R \cos(\omega_c t + \theta) \end{aligned} \quad (2)$$

where  $R$  is the envelope,  $\theta$  is the phase angle of the received signal, and two independent bandlimited noise,  $n_c$  and  $n_s$  are Gaussian-distributed, both having zero mean and the same variance  $\sigma_n^2$ .

The PDFs of  $n_c$  and  $n_s$  can be expressed as

$$f(n_c) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-n_c^2/2\sigma_n^2} \quad (3)$$

and

$$f(n_s) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-n_s^2/2\sigma_n^2}. \quad (4)$$

In a Rician faded channel, the amplitude for the desired signal with a two-parameter  $(S, \sigma_d^2)$  is characterized by the Rician PDF<sup>[4]</sup>

$$f_D(d) = \frac{d}{\sigma_d^2} e^{-\frac{S^2+d^2}{2\sigma_d^2}} I_0\left(\frac{Sd}{\sigma_d^2}\right) \quad (5)$$

where  $I_0(\cdot)$  is zeroth-order modified Bessel function of the first kind,  $\sigma_d^2$  is the mean power of the diffused components and  $\frac{S^2}{2}$  is the power in the direct components. Hence the total local mean power is  $\sigma_d^2 + \frac{S^2}{2}$ <sup>[11]</sup>.

The Rician distribution is characterized by the Rician factor  $K$ , which is defined as the ratio of the mean direct power to the mean diffused power, i.e.,

$$K = \frac{S^2}{2\sigma_d^2}.$$

There is a close fit between the  $m$ -distribution and the Rician distribution when the following parameter relationship holds<sup>[7]</sup>:

$$K = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}, \quad m \geq 1 \quad (6)$$

where the parameter  $m$  in the Nakagami fading channels is defined as the ratio of moments, called fading index,

$$m = \frac{\Omega^2}{E[(d^2 - \Omega)^2]}, \quad m \geq \frac{1}{2} \quad (7)$$

and

$$\Omega = E[d^2]. \quad (8)$$

The fit is exact in the extremes

$$m = 1, \quad K = 0 \quad (\text{Rayleigh fading})$$

$$m \rightarrow \infty, \quad K \rightarrow \infty \quad (\text{nonfading}).$$

New random variables are defined as follows:

$$x = d + n_c = R \cos \theta \quad (9)$$

$$y = n_s = R \sin \theta. \quad (10)$$

If  $d$  and  $n_c$  are independent, then the density of  $x$  equals the convolution of their respective density [13]~[15], i.e.,

$$f(x) = f_D(d)f_{N_c}(n_c). \quad (11)$$

Given two independent PDF of two rv  $x$  and  $y$ , this joint function equals the multiplication for the function of  $x$  and  $y$  [13], i.e.,

$$f(x, y) = f(x)f(y). \quad (12)$$

We express the joint function  $f(x, y)$  of the rv  $x$  and  $y$  in terms of  $r$  and  $\theta$ . Here  $f_{R, \theta}(r, \theta)$  is given by

$$f_{R, \theta}(r, \theta) = f_{X, Y}(x = r \cos \theta, y = r \sin \theta) |J| \quad (13)$$

where  $J$  is the Jacobian of the transformation.

In (11),

$$\begin{aligned} f(x) &= \int_0^\infty f_D(d)f_{N_c}(x-d)dd \\ &= \frac{1}{\sigma_d^2 \sqrt{2\pi} \sigma_n^2} e^{-\left(\frac{S^2}{2\sigma_d^2} + \frac{x^2}{2\sigma_n^2}\right)} \end{aligned}$$

$$\cdot \int_0^\infty de^{-\left(\frac{1}{2\sigma_d^2} + \frac{1}{2\sigma_n^2}\right)d^2} e^{-\frac{x}{\sigma_n^2}d} I_0\left(\frac{Sd}{\sigma_d^2}\right) dd. \quad (14)$$

Then, from (12)

$$\begin{aligned} f(x, y) &= \frac{1}{2\pi \sigma_d^2 \sigma_n^2} e^{-\frac{S^2}{2\sigma_d^2} - \frac{x^2 + y^2}{2\sigma_n^2}} \\ &\cdot \int_0^\infty de^{-\left(\frac{1}{2\sigma_d^2} + \frac{1}{2\sigma_n^2}\right)d^2 + \frac{x}{\sigma_n^2}d} \\ &\cdot I_0\left(\frac{Sd}{\sigma_d^2}\right) dd. \quad (15) \end{aligned}$$

Using the transformation of rv, the joint PDF of  $R$  and  $\theta$  becomes

$$\begin{aligned} f_{R, \theta}(r, \theta) &= \frac{r}{2\pi \sigma_d^2 \sigma_n^2} e^{-\frac{S^2}{2\sigma_d^2} - \frac{r^2}{2\sigma_n^2}} \\ &\cdot \int_0^\infty de^{-\left(\frac{1}{2\sigma_d^2} + \frac{1}{2\sigma_n^2}\right)d^2 + \frac{r \cos \theta}{\sigma_n^2}d} \\ &\cdot I_0\left(\frac{Sd}{\sigma_d^2}\right) dd. \quad (16) \end{aligned}$$

To obtain the marginal PDF of envelope  $R$ , we integrate (16) over  $\theta$  to get [16], [17]

$$\begin{aligned} f_{R, Rician}(r) &= \frac{r}{2\pi \sigma_d^2 \sigma_n^2} e^{-\frac{S^2}{2\sigma_d^2} - \frac{r^2}{2\sigma_n^2}} \\ &\cdot \int_0^\infty de^{-\left(\frac{1}{2\sigma_d^2} + \frac{1}{2\sigma_n^2}\right)d^2} I_0\left(\frac{Sd}{\sigma_d^2}\right) \\ &\cdot \int_{-\pi}^\pi e^{\frac{r \cos \theta}{\sigma_n^2}d} d\theta dd \\ &= \frac{r}{\sigma_d^2} e^{-\frac{S^2 + r^2}{2\sigma_d^2}} I_0\left(\frac{Sr}{\sigma_d^2}\right), \quad r \geq 0 \quad (17) \end{aligned}$$

where  $\sigma_t^2 = \sigma_d^2 + \sigma_n^2$ .

By normalizing  $2\sigma_n^2$ , (17) can be expressed as follows:

$$\begin{aligned} f_{R, Rician}(r) &= \frac{2(1+K)}{1+K+\gamma_0} r e^{-\frac{K}{1+K+\gamma_0} r^2} \\ &\cdot e^{-\frac{1+K}{1+K+\gamma_0} r^2} I_0\left(2\sqrt{K(K+1)\gamma_0} \frac{r}{1+K+\gamma_0}\right) \quad (18) \end{aligned}$$

where  $\gamma_0 = (1+K) \frac{\sigma_d^2}{\sigma_n^2}$ .

The distribution in a Rayleigh fading channel can be obtained by setting  $K=0$  in (18). Thus, (18) becomes

$$f_{R, \text{Rayleigh}}(\gamma) = \frac{2\gamma}{1+\gamma_0} e^{-\frac{\gamma^2}{1+\gamma_0}}. \quad (19)$$

We can find that (19) corresponds to the result of [18, Eq. (20)] for  $m=1$ .

It is well known that for a given number of Rician factor  $K$ , for smaller SNR, the distribution of the envelope is more shifted to the left like that of the envelope under the effect of the Nakagami fading environment. Given average SNR of the received signal, for larger Rician factor  $K$ , that of the envelope is more shifted to the right.

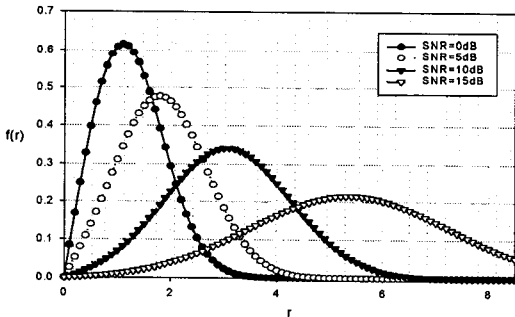


Fig. 1. The PDF of the envelope in a Rician fading for SNR=0, 5, 10, 15 dB and  $K=6$  dB.

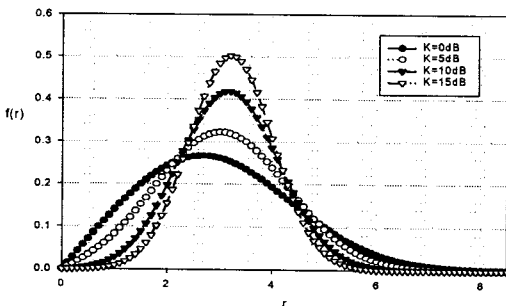


Fig. 2. The PDF of the envelope in a Rician fading for  $K=0, 5, 10, 15$  dB and SNR = 10 dB.

### III. NONCOHERENT MFSK PERFORMANCE ON RICIAN FADING CHANNELS

$M-1$  outputs of  $M$  filters are assumed to include only AWGN, which has zero mean and variance  $\sigma_n^2$ . Given that information signal was transmitted, the probability that the envelope of the received signal exceeds the threshold value  $r$  is given by [19]

$$g(r) = \sum_{n=1}^{M-1} (-1)^{n-1} \binom{M-1}{n} e^{-\frac{nr^2}{2\sigma_n^2}} \quad (20)$$

where each  $M$ -ary signal is uncorrelated.

To obtain the probability of symbol error for noncoherent MFSK over a Rician fading channel with the Rician parameter  $K$ , we multiply (17) and (20) and perform the integration with respect to  $r$ . So, this yields (See Appendix.)

$$\begin{aligned} P_{s, \text{Rician}} &= \int_0^{\infty} f_{R, \text{Rician}}(\gamma) g(\gamma) d\gamma \\ &= \sum_{n=1}^{M-1} (-1)^{n-1} \binom{M-1}{n} \frac{K+1}{n\gamma_0 + (n+1)(K+1)} \\ &\quad \cdot \exp\left[-\frac{n\gamma_0 K}{n\gamma_0 + (n+1)(K+1)}\right]. \end{aligned} \quad (21)$$

For orthogonal signaling, the average probability of bit error,  $P_b$ , is related to the average of symbol error,  $P_s$ , by [11]

$$P_b = \frac{M}{2(M-1)} P_s \quad (22)$$

where the per bit average SNR  $\gamma_b$  is related to per symbol average SNR by

$$\gamma_b = \frac{1}{\log_2 M} \gamma_0. \quad (23)$$

Evaluating (21) and converting from symbols to bits, using (22) and (23), we observe that the bit error probability in the Rician fading environment is

$$\begin{aligned} P_{b, \text{Rician}} &= \frac{M}{2(M-1)} (K+1) \sum_{n=1}^{M-1} (-1)^{n-1} \\ &\quad \cdot \binom{M-1}{n} \frac{1}{n\gamma_b \log_2 M + (n+1)(K+1)} \end{aligned}$$

$$\cdot \exp \left[ -K + \frac{(n+1)K(K+1)}{n\gamma_b \log_2 M + (n+1)(K+1)} \right]. \quad (24)$$

The bit error probability in a Rayleigh fading,  $P_{b, Rayleigh}$ , can be obtained by setting  $K=0$  in (24) as follows:

$$P_{b, Rayleigh} = \frac{M}{2(M-1)} \sum_{n=1}^{M-1} (-1)^{n-1} \cdot \left( \frac{M-1}{n} \right) \frac{1}{n\gamma_b \log_2 M + n + 1}. \quad (25)$$

We can find that the result of [18, Eq. (34)] for  $m=1$  corresponds to (25). It is interesting to note that (24) is perfectly equal to Sun's result for noncoherent MFSK in a Rician channel [12, Eq. (8)].

#### IV. COHERENT MFSK PERFORMANCE ON RICIAN FADING CHANNELS

The events that the observation vector  $z$  is nearer to the signal vector  $s_k$  than the signal vector  $s_i$  may be expressed as  $A_{ik}, i, k=1, 2, \dots, M$ , where we have assumed that the signal  $s_i$  was transmitted. On the assumption that  $B_i$  is the decision region which includes  $s_i$ , the conditional error probability of symbol,  $P(z \notin B_i | s_i)$  is equivalent to the probability of the union of mutually exclusive events  $A_{i1}, A_{i2}, \dots, A_{ii-1}, A_{ii+1}, \dots, A_{iM}$ . It can be shown that [4], [13]

$$P(z \notin B_i | s_i) \leq \sum_{k=1, k \neq i}^M P(A_{ik}), \quad i=1, 2, \dots, M \quad (26)$$

$$P(z \notin B_i | s_i) \sum_{k=1, k \neq i}^M P(s_i, s_k), \quad i=1, 2, \dots, M \quad (27)$$

where  $P(s_i, s_k)$  represents the probability that  $z$  is nearer to  $s_k$  than  $s_i$ , assuming that the signal  $s_i$  is transmitted.

Then let us assume that  $s_i$  was transmitted, the error probability of a signal is then given by

$$P(s_i, s_k) = \int_{d_{ik}/2}^{\infty} \frac{1}{\sqrt{\pi N_0}} \exp \left( -\frac{u^2}{N_0} \right) du \quad (28)$$

where  $d_{ik}$  is the Euclidean distance between a pair of signals, defined as

$$d_{ik} = \| s_i - s_k \|. \quad (29)$$

(28) can be also expressed as

$$P(s_i, s_k) = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{\gamma_{ik}}{2}} \right) \quad (30)$$

as a function of the instantaneous SNR for  $s_i$ , defined as

$$\gamma_{ik} = \frac{d_{ik}^2}{2N_0}. \quad (31)$$

Substituting (30) into (27), we have

$$P(z \notin B_i | s_i) \leq \frac{1}{2} \sum_{k=1, k \neq i}^M \operatorname{erfc} \left( \sqrt{\frac{\gamma_{ik}}{2}} \right), \quad i=1, 2, \dots, M. \quad (32)$$

Given that the transmitted signal is  $s_i$  of the binary signals,  $s_i$  and  $s_k$ , the average symbol rate, or simply the average of the probability of error in detecting the desired signal in a Rician faded channel is given by

$$P_{s, Rician} = \int_0^{\infty} f(\gamma_{ik}) P(s_i, s_k) d\gamma_{ik} \quad (33)$$

in which, for convenience, we have defined the PDF of the instantaneous SNR for  $s_i$  as

$$f(\gamma_{ik}) = f(\gamma) |_{\gamma=\gamma_{ik}}, \quad (34)$$

From (30) and (34), (33) is represented as

$$P_{s, Rician} = \frac{1}{2} \int_0^{\infty} \frac{K+1}{\gamma_0} e^{-K - \frac{(K+1)\gamma_0}{\gamma}} \cdot I_0 \left[ 2\sqrt{\frac{K(K+1)\gamma_{ik}}{\gamma_0}} \right] \operatorname{erfc} \left( \sqrt{\frac{\gamma_{ik}}{2}} \right) d\gamma_{ik}. \quad (35)$$

We substitute the infinite series expansion of the Bessel function  $I_0(z)$  as

$$I_0(z) = \sum_{l=0}^{\infty} \frac{(z/2)^{2l}}{(l!)^2} \quad (36)$$

into (35), make the change of variable and use [17].

$$\int_0^{\infty} \operatorname{erfc}(\beta x) e^{\mu^2 x^2} x^{\nu-1} dx = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\beta^{\nu}} \cdot {}_2F_1\left(\frac{\nu}{2}, \frac{\nu+1}{2}; \frac{\nu}{2} + 1; \frac{\mu^2}{\beta^2}\right),$$

$Re \beta^2 > Re \mu^2, Re \nu > 0.$  (37)

Using identity [17], [20]

$${}_2F_1(\alpha, \beta; \gamma; Z) = (1-Z)^{-\beta} \cdot {}_2F_1\left(\beta, \gamma - \alpha; \gamma; \frac{Z}{Z-1}\right) \quad (38)$$

and

$${}_2F_1\left(1, p + \frac{1}{2}; p+1; x\right) = \frac{1}{\binom{2p}{p} \left(\frac{x}{4}\right)^p} \left[ \frac{1}{\sqrt{1-x}} - \sum_{s=0}^{p-1} \binom{2s}{s} \left(\frac{x}{4}\right)^s \right]. \quad (39)$$

(35) thus becomes

$$P_{s, Rician} = \frac{1}{2} e^{-K} \sum_{l=0}^{\infty} \frac{1}{\Gamma(l+1)} \cdot K^l \beta(K) \left\{ \frac{1}{\beta(K)} - \sum_{t=0}^l \binom{2t}{t} \left[ \frac{(1+K)/4}{\mu^2 + 1 + K} \right]^t \right\} \quad (40)$$

where

$$\beta(K) = \frac{\mu}{\sqrt{\mu^2 + 1 + K}} \quad (41)$$

$$\mu = \sqrt{\frac{\gamma_0}{2}}. \quad (42)$$

Finally, for equally probable  $M$  signals, we can present that from (22) and (23), the bit error probability under the Rician fading model,  $P_{b, Rician}$  is

$$P_{b, Rician} \leq \frac{M}{4} e^{-K} \sum_{l=0}^{\infty} \frac{1}{\Gamma(l+1)} \cdot K^l \beta(K) \left\{ \frac{1}{\beta(K)} - \sum_{t=0}^l \binom{2t}{t} \left[ \frac{(1+K)/4}{\mu^2 + 1 + K} \right]^t \right\}. \quad (43)$$

For the special case of Rician factor  $K=0$ , we can find that the result of (43) is perfectly equivalent

to the result of [18, Eq (51)] for  $m=1$ .

## V. NUMERICAL RESULTS

The selected numerical results to show the performance of noncoherent and coherent MFSK in Nakagami and Rician fading channels with arbitrary fading parameters were presented<sup>[11], [12]</sup>. Also, the performance of coherent MFSK in Nakagami fading channels was represented<sup>[18]</sup>. The performances of coherent MFSK signals under the presence of Nakagami and Rician fading channels are graphically suggested for the signal alphabet sizes  $M$ ,  $m$ , and  $K$  in Fig. 3~7. It is assumed that the performance of coherent MFSK in fading channels has an upper bound. Coherent MFSK results in Nakagami and Rician fading channels are plotted with  $M=2, 4, 8$ , and 16 in Fig. 3~6, respectively. Under the assumption that coherent MFSK signals experience an independent Nakagami fading with  $m=1, 5$ , and 10, these fading severity indexes correspond to a Rician fading channel with  $K=0, 8.472$ , and 18.487. It is an expected result as Rician factor  $K$  increases, fading depth decreases. Fig. 7 shows coherent MFSK performance comparison in Nakagami and Rician fading channels for  $M=4, 8$ , and 16,  $m=10$ , and  $K=18.487$ . Note that for a given number of Rician factor  $K$ , a substantial gain in performance is achieved by increasing the signal alphabet size  $M$ . Hence, this figure illustrates that, by increasing the number of  $M$ , one can reduce the SNR per bit required to achieve a certain bit error probability for  $K$ . Also, we can find that the bit error probability fits closely for  $M=8$  and  $M=16$  in a Rician fading environment. The noncoherent and coherent MFSK results are plotted with alphabet sizes  $M=2, 4, 8$ , and 16 in Fig. 8, respectively. In each of these figures, we have a simple value of  $K$ . It is clear from these results that for a given number of Rician factor  $K$ , the improvement increases with increasing alphabet size  $M$ . Also, coherent MFSK performance can be

better than noncoherent MFSK performance in a practical SNR range.

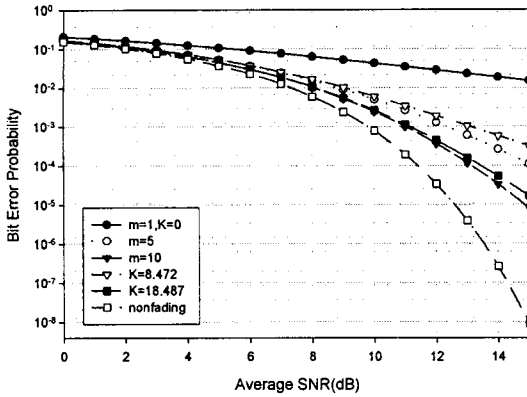


Fig. 3. Coherent MFSK performance comparison in Nakagami and Rician fading channels for  $M = 2$ .

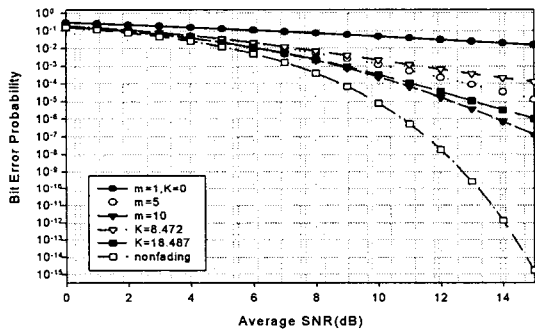


Fig. 4. Coherent MFSK performance comparison in Nakagami and Rician fading channels for  $M = 4$ .

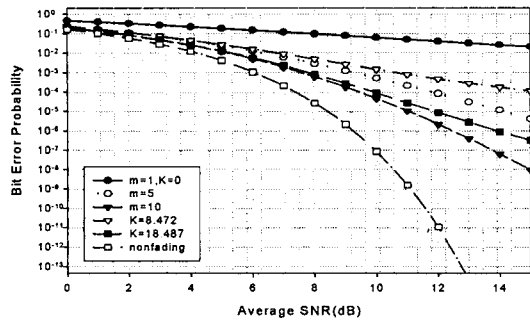


Fig. 5. Coherent MFSK performance comparison in Nakagami and Rician fading channels for  $M = 8$ .

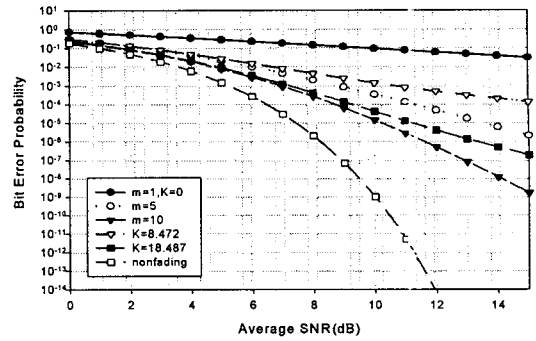


Fig. 6. Coherent MFSK performance comparison in Nakagami and Rician fading channels for  $M = 16$ .

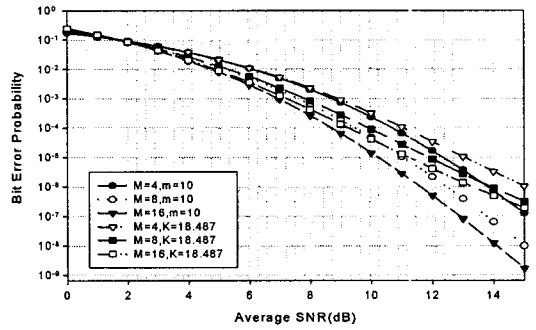


Fig. 7. Coherent MFSK performance comparison in Nakagami and Rician fading channels. This parameters for this figure are:  $M = 4, 8, 16$ ,  $m = 10$ , and  $K = 18.487$ .

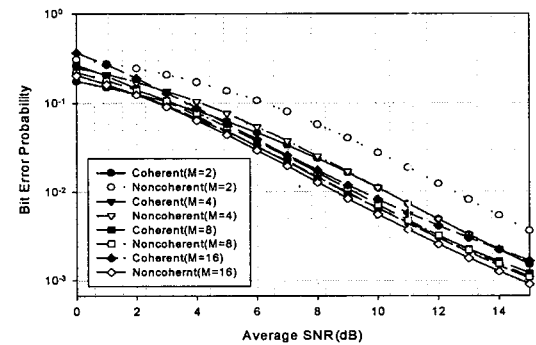


Fig. 8. Coherent/noncoherent MFSK performance comparison in a Rician fading channel for  $M = 2, 4, 8, 16$  and  $K = 5$ .

## VI. CONCLUDING REMARKS

In this paper, we have suggested the distribution of the envelope under the fading environments. By using these distributions we can analyze the error rate performance of noncoherent MFSK systems that operate on Rician fading channels. To perform the evaluation of coherent MPSK systems, it is valuable to investigate the distribution of the phase angle under wireless fading channels. Also, we have presented new expressions for the upper bound of the bit error probability for coherent MFSK in Rician fading channels. These results are sufficiently general to offer a convenient method to evaluate the performance of several current coherent and noncoherent MFSK systems that operate on channels with a wide variety of fading or scintillation conditions in wireless personal communications.

## APPENDIX

Noncoherent MFSK Performance Using the PDF of Envelope on Rician Fading Channels

We obtain the probability of symbol error for noncoherent MFSK over a Rician fading channel with the Rician parameter  $K$  from (17) and (20) as following[17]:

$$\begin{aligned}
 P_{s, Rician} &= \int_0^{\infty} f_{R, Rician}(\gamma) g(\gamma) d\gamma \\
 &= \sum_{n=1}^{M-1} (-1)^{n-1} \binom{M-1}{n} \\
 &\quad \cdot \int_0^{\infty} \frac{\gamma}{\sigma_i^2} e^{-\frac{S^2 + \gamma^2}{2\sigma_i^2}} I_0\left(\frac{S\gamma}{\sigma_i^2}\right) e^{-\frac{n\gamma^2}{2\sigma_n^2}} d\gamma \\
 &= \sum_{n=1}^{M-1} (-1)^{n-1} \binom{M-1}{n} \\
 &\quad \cdot \frac{1}{\sigma_i^2} e^{-\frac{S^2}{2\sigma_i^2}} \frac{1}{2 \left( \frac{1}{2\sigma_i^2} + \frac{n}{2\sigma_n^2} \right)}
 \end{aligned}$$

$$\begin{aligned}
 &\cdot {}_1F_1\left(1, 1; \frac{\left(\frac{S}{\sigma_i}\right)^2}{4\left(\frac{1}{2\sigma_i^2} + \frac{n}{2\sigma_n^2}\right)}\right) \\
 &= \sum_{n=1}^{M-1} (-1)^{n-1} \binom{M-1}{n} \frac{\sigma_n^2}{\sigma_n^2 + n(\sigma_d^2 + \sigma_n^2)} \\
 &\quad \cdot \exp\left[-\frac{S^2}{2} \frac{n}{\sigma_n^2 + n\sigma_d^2 + n\sigma_n^2}\right]. \quad (A.1)
 \end{aligned}$$

We can express the average symbol error probability in (A.1) in terms of the average SNR per symbol,  $\gamma_0$ . To accomplish this, (A.1) may be expressed as

$$\begin{aligned}
 P_{s, Rician} &= \sum_{n=1}^{M-1} (-1)^{n-1} \binom{M-1}{n} \\
 &\quad \cdot \frac{K+1}{n\gamma_0 + (n+1)(K+1)} \exp\left[-\frac{n\gamma_0 K}{n\gamma_0 + (n+1)(K+1)}\right] \quad (A.2)
 \end{aligned}$$

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