SAMPLING ERROR ANALYSIS FOR SOIL MOISTURE ESTIMATION

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Abstract: A spectral formalism was applied to quantify the sampling errors due to spatial and/or temporal gaps in soil moisture measurements. The lack of temporal measurements of the two-dimensional soil moisture field makes it difficult to compute the spectra directly from observed records. Therefore, the space-time soil moisture spectra derived by stochastic models of rainfall and soil moisture was used in their record. Parameters for both models were tuned with Southern Great Plains Hydrology Experiment (SGP '97) data and the Oklahoma Mesonet data. The structure of soil moisture data is discrete in space and time. A design filter was developed to compute the sampling errors for discrete measurements in space and time. This filter has the advantage in its general form applicable for all kinds of sampling designs. Sampling errors of the soil moisture estimation during the SGP '97 Hydrology Experiment period were estimated. The sampling errors for various sampling designs such as satellite over pass and point measurement ground probe were estimated under the climate condition between June and August 1997 and soil properties of the SGP '97 experimental area. The ground truth design was evaluated to 25km and 50 km spatial gap and the temporal gap from zero to 5 days.

Key words: soil moisture, rainfall, spectral formalism, sampling design, sampling error

1. INTRODUCTION

The amount of soil moisture is insignificant in the global water budget, but it plays an important role in the linkage between the hydrologic and the energy cycles through evapotranspiration, in the surface runoff processes and in the subsurface hydrologic system (Lin *et al.*, 1994). This is particularly true in the mid-latitudes and Rind (1982) concluded that, "*knowledge of the ground moisture at the beginning of the summer might allow for improved summer temperature forecasts*". Also, Shukla and Mintz (1982) affirmed that "*in the extratropics, with its large seasonal changes, the soil plays a role analogous to that of the ocean by storing of the precipitation it receives in winter and using it to the humidify the atmosphere in summer*". A number of modeling approaches have been conducted to investigate the interaction between soil moisture and precipitation (Rind, 1982; Shukla and Mintz, 1982; Rodriguez-Iturbe *et al.*, 1991a,b; Eltahir, 1998; Zheng and Eltahir, 1998). Due to the lack of a global data set with temporal measurement of soil moisture conditions, the conclusions of these modeling studies have not been confirmed with observations. Estimates of the probability density function of soil moisture have been developed (Rodriguez-Iturbe *et al.*, 1991a, b; Valdés *et al.*, 1990). Soil moisture has been estimated by extrapolating point measurements (Yu *et al.*, 1997). Recent advances in low frequency microwave remote sensing can provide direct measurement of surface soil moisture under the various topographic and land cover conditions within reasonable error bounds (Engman, 1990; Wood *et al.*, 1993; Jackson and Le Vine, 1996).

A very important issue is to decide the spatial and temporal sampling frequency in order to resolve the essential statistical features of the soil moisture field. Therefore, appropriate sampling strategies are necessary to observe and determine the soil moisture field distributions. Soil moisture data are generally estimated from samples taken discretely in space and time. In this work, procedures to develop a design filter, which is applicable to all kinds of sampling design including discrete design in space and time, and to evaluate the sampling strategy of soil moisture measurements, were presented.

North and Nakamoto (1989) derived an analytical formalism to obtain the mean-square sampling errors incurred in estimating the mean rain rates due to the intermittent observations in space and/or in time. In this formalism, the mean square error ε^2 consists of two factors: a design dependent filter $|H(\nu_1,\nu_2,f)|^2$ and the space–time spectral density of the precipitation field $S(\nu_1,\nu_2,f)$. These factors are integrated over all frequencies and wave numbers as follows

$$\varepsilon^2 = \iiint \left|H(\nu_1,\nu_2,f)\right|^2 S(\nu_1,\nu_2,f)\, d\nu_1\, d\nu_2 df \qquad (1)$$

The components of Eq. (1) allow the user to consider the sampling design and properties of the soil moisture field separately. The lack of extensive temporal measurements for the two-dimensional soil moisture field makes it difficult to estimate the spectra directly from observed records. Therefore, the space-time soil moisture spectra derived by a stochastic model were used.

Entekhabi and Rodriguez-Iturbe (1994) derived an analytical model for soil moisture dynamics by adopting a linear reservoir concept which can include rainfall forcing, the effect of diffusion on the soil moisture propagation, and soil moisture content decay. They also analyzed the model based on the Fourier analysis and derived the space-time spectral density of the soil moisture field by introducing a gain function. The relation between the rainfall spectrum and the soil moisture spectrum will be compactly written as

$$S_s(\nu_1, \nu_2, f) = G_n(\nu_1,\nu_2,f)\, S_r(\nu_1,\nu_2,f) \qquad (2)$$

where $S_S(\nu_1,\nu_2,f)$ is the soil moisture spectrum, $S_r(\nu_1,\nu_2,f)$ is the rainfall spectrum, and $G_n(\nu_1,\nu_2,f)$ is the gain function, which has space and time characteristics through its dependence on the wavenumber and frequency.

The mean square error ε^2 can be written from Eqs. (1) and (2)

$$\varepsilon^2 = \iiint \left|H(\nu_1,\nu_2,f)\right|^2 G_n(\nu_1,\nu_2,f)\, S_r(\nu_1,\nu_2,f)\, d\nu_1 d\nu_2 df \qquad (3)$$

Eq. (1) shows that if we know the second-moment statistics of the rainfall field and the properties of the soil moisture field, we can estimate the mean square error of the soil moisture field with a certain sampling design.

There are several sampling schemes for soil moisture field measurement which observe a random field discretely in space and/or time, but none can observe a field continuously in both space and time. Error related to estimating the space and time averages of a continuous field are caused by discrete measurements.

The objectives of this study are 1) to extend the spectral formalism derived by North and Nakamoto (1989) to evaluate the sampling strategy for soil moisture measurements; 2) to develop a sampling filter which is applicable to all kinds of sampling design such as satellite or air plane over pass, soil moisture gage network and 3) to evaluate the sampling errors due to spatial and/or temporal gaps in measurements from space and ground sensor networks for estimation of area averaged soil moisture field. Stochastic models for the rainfall and soil moisture fields were used to evaluate the sampling error with tuned parameters by using the SGP '97 experimental data and the Oklahoma Mesonet data.

2. FORMALISM FOR THE SOIL MOISTURE ESTIMATION DESIGNS

2. 1 Spectral Formalism

North and Nakamoto (1989) considered a random field $\psi(l_1,l_2,t)$, whose ensemble average is zero and whose variance at a point in space-time is σ^2. The space-time average of $\psi(l_1,l_2,t)$ is

$$\Psi = \frac{1}{L_1 L_2 T}\int_{-\frac{L_1}{2}}^{\frac{L_1}{2}}\int_{-\frac{L_2}{2}}^{\frac{L_2}{2}}\int_{-\frac{T}{2}}^{\frac{T}{2}} \psi(l_1,l_2,t)\, dl_1 dl_2 dt \qquad (4)$$

They assumed the field $\psi(l_1,l_2,t)$, is statistically homogeneous, isotropic in space, and stationary in time, and systematically analyzed the errors incurred in estimating Ψ by various statistical measurement designs. Fig. 1 shows a schematic diagram of the space-time volume.

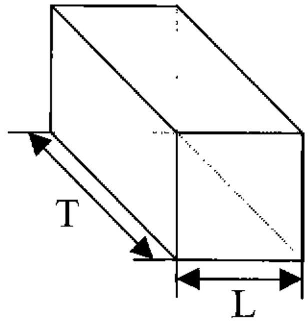

Fig. 1. A schematic Diagram of the Space-Time Volume Used in the Averaging Process

The discrete estimator in space and time can be written as

$$\Psi_D = \frac{1}{N_1 N_2 N_3}\sum_{n_1=-\frac{N_1}{2}}^{\frac{N_1}{2}-1}\sum_{n_2=-\frac{N_2}{2}}^{\frac{N_2}{2}-1}\sum_{n_3=-\frac{N_3}{2}}^{\frac{N_3}{2}-1} \psi\left(\left(n_1+\frac{1}{2}\right)\Delta l_1, \left(n_2+\frac{1}{2}\right)\Delta l_2, \left(n_3+\frac{1}{2}\right)\Delta t\right)$$

where N_1 is the total number of estimates made through x-axis in the width, L_1 ($L_1=N_1\Delta l_1$) and N_2 is the total number of estimates made through y-axis in the width, L_2 ($L_2=N_2\Delta l_2$) and N_3 is the total number of visits made by the satellite in the period ($T=N_3\Delta t$) and Eq (4) can be written as

$$\Psi_D = \frac{1}{L_1 L_2 T}\iiint K_D(l_1,l_2,t)\psi(l_1,l_2,t)\, dl_1 dl_2 dt \qquad (5)$$

where

$$K_D(l_1,l_2,t)=\Delta l_1\Delta l_2\Delta t\sum_{n_1=-\frac{N_1}{2}}^{\frac{N_1}{2}-1}\delta\left(l_1-\left(n_1+\frac{1}{2}\right)\Delta l_1\right)\sum_{n_2=-\frac{N_2}{2}}^{\frac{N_2}{2}-1}\delta\left(l_2-\left(n_2+\frac{1}{2}\right)\Delta l_2\right)\sum_{n_3=-\frac{N_3}{2}}^{\frac{N_3}{2}-1}\delta\left(t-\left(n_3+\frac{1}{2}\right)\Delta t\right) \quad (6)$$

is a kernel representative of the discrete sampling design in time and space.

Figs. 2, 3 and 4 represent the schematic diagrams and data structures of the sampling designs.

The mean square error incurred in estimating ψ, using each design, is used to indicate the error size.

$$\varepsilon^2=<(\Psi-\Psi_k)^2> \quad (7)$$

where k=S represents the estimator using the satellite design, and k=G represents the estimator using the ground soil moisture gage design, and k=D represents the estimator using discrete space and time sampling design.

Eq. (7) can be written as

$$\varepsilon^2=\frac{1}{L_1^2L_2^2T^2}\iiiiiint\langle\psi(l_1',l_2',t')\psi(l_1,l_2,t)\rangle\left[1-K(l_1',l_2',t')\right]\left[1-K(l_1,l_2,t)\right]dl_1'dl_2'dt'dl_1dl_2dt \quad (8)$$

By using the Fourier transform, (7) can be written compactly as

$$\varepsilon^2=\iiint\left|H(\nu_1,\nu_2,f)\right|^2S(\nu_1,\nu_2,f)\,d\nu_1\,d\nu_2df \quad (9)$$

where $S(\nu_1,\nu_2,f)$ is the space-time power spectrum and $H(\nu_1,\nu_2,f)$ is a complex-valued design dependent filter.

The components of Eq. (9) are neatly separated into factors, which are individually dependent on design information and properties of the rainfall and the soil moisture field. The appealing feature of Eq. (9) is that the mean square error is dependent only upon second moment statistics of the field and the sampling design.

2.2 Design Filters for Soil Moisture Estimations

We can consider the general case of a soil moisture estimation design with discrete sampling in space and time. The design filter for this case can be derived as follows

$$\left|H_D(\nu_1,\nu_2,f)\right|=G(\pi\nu_1L_1)G(\pi\nu_2L_2)G(\pi fT)\times\left[1-\frac{1}{G(\pi\nu_1\Delta l_1)G(\pi\nu_2\Delta l_2)G(\pi f\Delta T)}\right] \quad (10)$$

where $G(\pi x)=[\sin(\pi x)/\pi x]$

A convenient form of the filter is

$$\left|H_D(\nu_1,\nu_2,f)\right|^2=G(\pi\nu_1L_1)^2G(\pi\nu_2L_2)^2G(\pi fT)^2\times\left[1-\frac{1}{G(\pi\nu_1\Delta l_1)G(\pi\nu_2\Delta l_2)G(\pi f\Delta T)}\right]^2 \quad (11)$$

which can be written as

$$\left|H_D(\nu_1,\nu_2,f)\right|^2=\frac{\sin^2(\pi\nu_1L_1)}{N_1^2\sin^2(\pi\nu_1\Delta l_1)}\frac{\sin^2(\pi\nu_2L_2)}{N_2^2\sin^2(\pi\nu_2\Delta l_2)}\frac{\sin^2(\pi fT)}{N_3^2\sin^2(\pi f\Delta t)}\times\left[1-G(\pi\nu_1\Delta l_1)G(\pi\nu_2\Delta l_2)G(\pi f\Delta t)\right]^2 \quad (12)$$

This form is easily analyzed if it is noted that

$$\lim_{N\to\infty}\frac{\sin^2(\pi fN\Delta t)}{N\sin^2(\pi f\Delta t)}=\frac{1}{\Delta t}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{\Delta t}\right) \quad (13)$$

which is a series of spikes at $f=n/\Delta t$ (Blackmam and Tuckey, 1959).

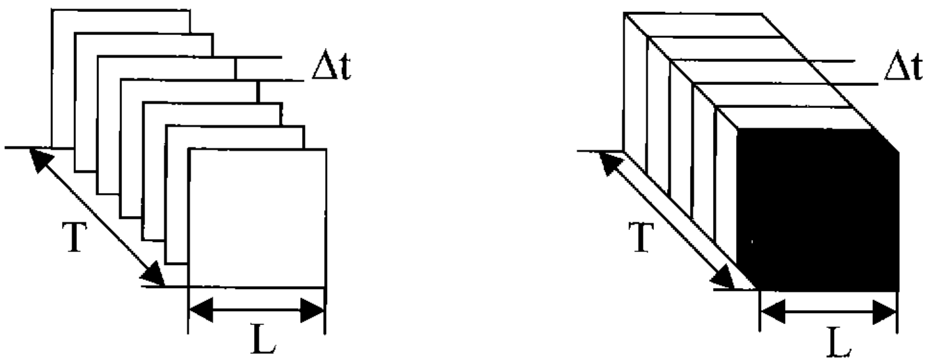

Fig. 2. A schematic Diagram and Data structure of the Satellite Design

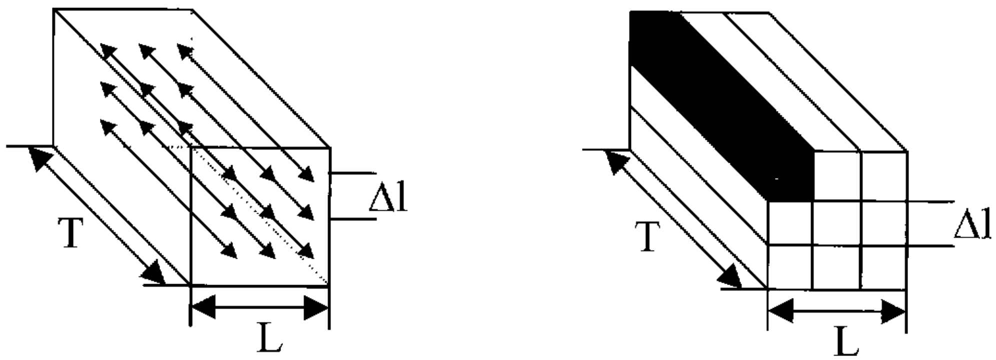

Fig. 3. A Schematic Diagram and Data Structure of the Soil Moisture Gage Design

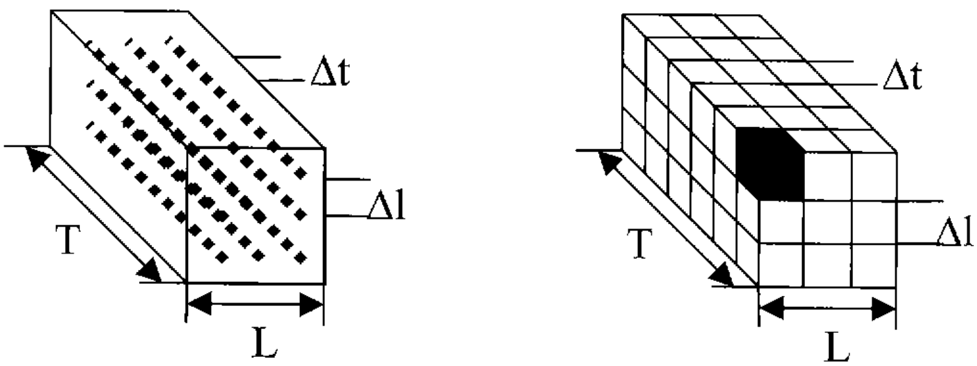

Fig. 4. A Schematic Diagram and Data Structure of Discrete Sampling in Space and Time

Eq. (12) can be written as

$$|H_D(\nu_1,\nu_2,f)|^2 \approx \frac{1}{L_1L_2T} \sum_{n_1=-\infty}^{\infty} \delta\left(\nu_1-\frac{n_1}{\Delta l_1}\right) \sum_{n_2=-\infty}^{\infty} \delta\left(\nu_2-\frac{n_2}{\Delta l_2}\right) \sum_{n_3=-\infty}^{\infty} \delta\left(f-\frac{n_3}{\Delta t}\right) \times [1-G(\pi\nu_1\Delta l_1)G(\pi\nu_2\Delta l_2)G(\pi f\Delta t)]^2 \tag{14}$$

which is a three-dimensional Dirac comb in the (ν_1,ν_2,f) space.

North and Nakamoto (1989) derived the filter for the case of temporally discrete visits of a satellite as follows

$$|H_S(\nu_1,\nu_2,f)|^2 = \frac{1}{T}G^2(\pi\nu_1L_1)G^2(\pi\nu_2L_2) \times \sum_{n_3=-\infty}^{\infty} \delta\left(f-\frac{n_3}{\Delta t}\right)[1-G(\pi f\Delta t)]^2 \tag{15}$$

which is a one-dimensional Dirac comb along the frequency.

For the periodic rectangular array of raingages in the limit of a large number of gages

$$|H_G(\nu_1,\nu_2,f)|^2 = \frac{1}{L_1L_2}G^2(\pi fT) \times \sum_{n_1=-\infty}^{\infty} \delta\left(\nu_1-\frac{n_1}{\Delta l_1}\right) \sum_{n_2=-\infty}^{\infty} \delta\left(\nu_2-\frac{n_2}{\Delta l_2}\right)[1-G(\pi\nu_1\Delta l_1)G(\pi\nu_2\Delta l_2)]^2 \tag{16}$$

which is a two-dimensional Dirac comb in the (ν_1,ν_2) plane.

3. MODELS FOR THE RAINFALL AND SOIL MOISTURE FIELDS

3.1 Model for Rainfall Forcing

The Waymire, Gupta, and Rodriguez-Iturbe (WGR) model is a multidimensional rainfall model to represent mesoscale precipitation (Waymire *et al.*, 1984). The WGR precipitation model shows a good link between atmospheric dynamics and a statistical description of mesoscale precipitation and incorporates many observed physical features of rainfall at mesoscale level. This model is characterized by the arrival mechanism of storm events through space and time. The model represents rainfall in a hierarchical approach with rain cells embedded in cluster potential centers, which, in turn, are embedded in rain bands. The Poisson process and the spatial Poisson process were introduced for the rain bands arrival scheme and to distribute the cluster potentials within a rain band, respectively. The occurrences of rain cells within the cluster potentials and the rain bands are assumed to be random, independently and identically distributed in the space-time cylinder with a common probability density function.

Valdés *et al.* (1990) derived the analytical form of the frequency-wavenumber spectrum of the WGR model. This spectrum depends on all nine parameters, and its analytical expression is as follows

$$S_p(f,\nu_1,\nu_2) = \theta_1 \frac{\alpha U(D,0)}{\alpha^2+14\pi^2\Theta^2} + \left[\theta_2 + \frac{\theta_3}{4\pi(D^2+\sigma^2)}\right]\alpha\beta U(D,\sigma) \left[\frac{1}{\alpha^2+14\pi^2\Theta} - \frac{1}{\beta^2+14\pi^2\Theta}\right] \tag{20}$$

where

$$U(D,\sigma) = 8\pi D^2 + \sigma^2 \times \exp\left[-4\pi^2(D^2+\sigma^2)(\nu_1^2+\nu_2^2)\right]$$

$$\Theta = \nu_1 u_x + \nu_2 u_y + f \qquad \theta_1 = \frac{\lambda_m E[\nu]\rho_L \pi D^2 i_0^2}{2\alpha}$$

$$\theta_2 = \frac{2\lambda_m \beta E^2[\nu]\rho_L^2 \pi^2 D^4 \iota_0^2}{\alpha(\beta^2 - \alpha^2)} \qquad \theta_3 = \frac{2\lambda_m \beta E^2[\nu]\rho_L \pi^2 D^4 \iota_0^2}{\alpha(\beta^2 - \alpha^2)}$$

3.2 A Model for the Soil Moisture Dynamics

Entekhabi and Rodriguez-Iturbe (1994) proposed an analytical model for soil moisture dynamics by adopting the linear reservoir concept and considering the diffusion impact on the soil moisture propagation. The model can be written as

$$nZ_r \frac{\partial s(l_1,l_2,t)}{\partial t} = -\eta s(l_1,l_2,t) + nZ_r \kappa \nabla^2 s(l_1,l_2,t) + P(l_1,l_2,t) \tag{21}$$

where $s(l_1,l_2,t)$ is relative soil moisture, n is the soil porosity, Z_r is the active soil depth, η is loss coefficient, κ is diffusion coefficient, and $P(l_1,l_2,t)$ is rainfall input.

They analyzed this model by using Fourier nalysis and derived the relationship between the soil moisture spectrum and the rainfall spectrum as follows

$$S_s(\nu_1,\nu_2,f) = Gn(\nu_1,\nu_2,f)\, S_p(\nu_1,\nu_2,f) \tag{22}$$

where $S_S(\nu_1,\nu_2,f)$ is the soil moisture spectrum, $S_P(\nu_1,\nu_2,f)$ is the rainfall spectrum, and $Gn(\nu_1,\nu_2,f)$ is the gain function, which has space and time characteristics through its dependence on the wavenumber and frequency and can be written as

$$Gn(\nu_1,\nu_2,f) = \frac{\left(\frac{1}{nZ_r}\right)^2}{\left((\nu_1+\nu_2)^2\kappa + \frac{\eta}{nZ_r}\right)^2 + f^2} \tag{23}$$

4. SAMPLING ERRORS FOR SOIL MOISTURE ESTIMATIONS

4. 1 Satellite and Soil Moisture Point Meas Urement Design

The mean square sampling error can be written according to the sampling designs for the soil moisture field with several rainfall processes. By applying the WGR precipitation model for the rainfall process, the space-time power spectrum of soil moisture field, $S_S(\nu_1,\nu_2,f)$, becomes

$$S_s(\nu_1,\nu_2,f) = \frac{\left(\frac{1}{nZ_r}\right)^2}{\left((\nu_1+\nu_2)^2\kappa + \frac{\eta}{nZ_r}\right)^2 + f^2} \left(\theta_1 \frac{\alpha U(D,0)}{\alpha^2 + 14\pi^2\Theta^2} + \left[\theta_2 + \frac{\theta_3}{4\pi(D^2+\sigma^2)}\right]\alpha\beta U(D,\sigma)\left[\frac{1}{\alpha^2+14\pi^2\Theta} - \frac{1}{\beta^2+14\pi^2\Theta}\right]\right) \tag{24}$$

By inserting Eqs. (14) and (22) into Eq. (3), the approximation formula of the error for discrete measurement in space and time becomes

$$\begin{aligned} \varepsilon_G^2 = \frac{\sigma^2}{L_1 L_2 T} \Bigg(& \sum_{n_1\neq 0} S\left(\frac{n_1}{\Delta l_1},0,0\right) + \sum_{n_2\neq 0} S\left(0,\frac{n_2}{\Delta l_2},0\right) \\ & + \sum_{n_3\neq 0} S\left(0,0,\frac{n_3}{\Delta t}\right) + \sum_{n_1\neq 0}\sum_{n_2\neq 0} S\left(\frac{n_1}{\Delta l_1},\frac{n_2}{\Delta l_2},0\right) \\ & + \sum_{n_1\neq 0}\sum_{n_3\neq 0} S\left(\frac{n_1}{\Delta l_1},0,\frac{n_3}{\Delta t}\right) + \sum_{n_2\neq 0}\sum_{n_3\neq 0} S\left(0,\frac{n_2}{\Delta l_2},\frac{n_3}{\Delta t}\right) \\ & + \sum_{n_1\neq 0}\sum_{n_2\neq 0}\sum_{n_3\neq 0} S\left(\frac{n_1}{\Delta l_1},\frac{n_2}{\Delta l_2},\frac{n_3}{\Delta t}\right)\Bigg) \end{aligned} \tag{25}$$

The mean square error (MSE) for discrete measurement in space and time be written as

$$
\varepsilon_{S,G}{}^{2} = \frac{\sigma^{2}\left(\frac{1}{nZ_r}\right)^{2}}{TL_1L_2} \times
$$

$$
\Bigg[\sum_{n_1 \neq 0} \frac{1}{\left(\left(\frac{n_1}{\Delta l_1}\right)^{2}\kappa + \frac{\eta}{nZ_r}\right)^{2}} \left(\frac{\theta_4}{\alpha^2 + 14\pi^2\Theta_1^2} + \theta_5 U_1\left[\frac{1}{\alpha^2 + 14\pi^2\Theta_1} - \frac{1}{\beta^2 + 14\pi^2\Theta_1}\right]\right)
$$

$$
+ \sum_{n_2 \neq 0} \frac{1}{\left(\kappa\left(\frac{n_2}{\Delta l_2}\right)^{2} + \frac{\eta}{nZ_r}\right)^{2}} \left(\frac{\theta_4}{\alpha^2 + 14\pi^2\Theta_2^2} + \theta_5 U_2\left[\frac{1}{\alpha^2 + 14\pi^2\Theta_2} - \frac{1}{\beta^2 + 14\pi^2\Theta_2}\right]\right)
$$

$$
+ \sum_{n_3 \neq 0} \frac{1}{\left(\frac{\eta}{nZ_r}\right)^{2} + \left(\frac{n_3}{\Delta t}\right)^{2}} \left(\frac{\theta_4}{\alpha^2 + 14\pi^2\Theta_3^2} + \theta_5 U_4\left[\frac{1}{\alpha^2 + 14\pi^2\Theta_3} - \frac{1}{\beta^2 + 14\pi^2\Theta_3}\right]\right)
$$

$$
+ \sum_{n_1 \neq 0}\sum_{n_2 \neq 0} \frac{1}{\left(\kappa\left(\left(\frac{n_1}{\Delta l_1}\right)^{2} + \left(\frac{n_2}{\Delta l_2}\right)^{2}\right) + \frac{\eta}{nZ_r}\right)^{2}} \left(\frac{\theta_4}{\alpha^2 + 14\pi^2\Theta_4^2} + \theta_5 U_3\left[\frac{1}{\alpha^2 + 14\pi^2\Theta_4} - \frac{1}{\beta^2 + 14\pi^2\Theta_4}\right]\right)
$$

$$
+ \sum_{n_1 \neq 0}\sum_{n_3 \neq 0} \frac{1}{\left(\kappa\left(\frac{n_1}{\Delta l_1}\right)^{2} + \frac{\eta}{nZ_r}\right)^{2} + \left(\frac{n_3}{\Delta t}\right)^{2}} \left(\frac{\theta_4}{\alpha^2 + 14\pi^2\Theta_5^2} + \theta_5 U_1\left[\frac{1}{\alpha^2 + 14\pi^2\Theta_5} - \frac{1}{\beta^2 + 14\pi^2\Theta_5}\right]\right)
$$

$$
+ \sum_{n_2 \neq 0}\sum_{n_3 \neq 0} \frac{1}{\left(\kappa\left(\frac{n_2}{\Delta l_2}\right)^{2} + \frac{\eta}{nZ_r}\right)^{2} + \left(\frac{n_3}{\Delta t}\right)^{2}} \left(\frac{\theta_4}{\alpha^2 + 14\pi^2\Theta_6^2} + \theta_5 U_2\left[\frac{1}{\alpha^2 + 14\pi^2\Theta_6} - \frac{1}{\beta^2 + 14\pi^2\Theta_6}\right]\right)
$$

$$
+ \sum_{n_1 \neq 0}\sum_{n_2 \neq 0}\sum_{n_3 \neq 0} \frac{1}{\left(\kappa\left(\left(\frac{n_1}{\Delta l_1}\right)^{2} + \left(\frac{n_2}{\Delta l_2}\right)^{2}\right) + \frac{\eta}{nZ_r}\right)^{2} + \left(\frac{n_3}{\Delta t}\right)^{2}} \left(\frac{\theta_4}{\alpha^2 + 14\pi^2\Theta_7^2} + \theta_5 U_3\left[\frac{1}{\alpha^2 + 14\pi^2\Theta_7} - \frac{1}{\beta^2 + 14\pi^2\Theta_7}\right]\right) \Bigg]
\tag{26}
$$

where

$$U_1 = 8\pi D^2 + \sigma^2 \times \exp\left[-4\pi^2(D^2+\sigma^2)\left(\frac{n_1}{\Delta l_1}\right)^2\right]$$

$$U_2 = 8\pi D^2 + \sigma^2 \times \exp\left[-4\pi^2(D^2+\sigma^2)\left(\frac{n_2}{\Delta l_2}\right)^2\right]$$

$$U_3 = 8\pi D^2 + \sigma^2 \times \exp\left[-4\pi^2(D^2+\sigma^2)\left(\left(\frac{n_1}{\Delta l_1}\right)^2+\left(\frac{n_2}{\Delta l_2}\right)^2\right)\right]$$

$$U_4 = 8\pi D^2 + \sigma^2$$

$$U_5 = 8\pi D^2$$

$$\Theta_1 = \left(\frac{n_1}{\Delta l_1}\right)u_x \qquad \Theta_2 = \left(\frac{n_2}{\Delta l_2}\right)u_y \qquad \Theta_3 = \left(\frac{n_3}{\Delta t}\right)$$

$$\Theta_4 = \left(\frac{n_1}{\Delta l_1}\right)u_x + \left(\frac{n_2}{\Delta l_2}\right)u_y \qquad \Theta_5 = \left(\frac{n_1}{\Delta l_1}\right)u_x + \left(\frac{n_3}{\Delta t}\right)$$

$$\Theta_6 = \left(\frac{n_2}{\Delta l_2}\right)u_y + \left(\frac{n_3}{\Delta t}\right) \qquad \Theta_7 = \left(\frac{n_1}{\Delta l_1}\right)u_x + \left(\frac{n_2}{\Delta l_2}\right)u_y + \left(\frac{n_3}{\Delta t}\right)$$

$$\theta_4 = \theta_1\alpha 8\pi D^2 \qquad \theta_5 = \left[\theta_2 + \frac{\theta_3}{4\pi(D^2+\sigma^2)}\right]\alpha\beta$$

By inserting Eqs. (16) and Eq. (22) into Eq. (3), the MSE for soil moisture gages design can be written as

$$\varepsilon_G^{\,2} = \frac{\sigma^2\left(\frac{1}{nZ_t}\right)^2}{TL_1L_2}\times$$
$$\Bigg[\sum_{n_1=0}\frac{1}{\left(\left(\frac{n_1}{\Delta l_1}\right)^2\kappa+\frac{\eta}{nZ_t}\right)^2}\left(\frac{\theta_4}{\alpha^2+14\pi^2\Theta_1^2}+\theta_5U_1\left[\frac{1}{\alpha^2+14\pi^2\Theta_1}-\frac{1}{\beta^2+14\pi^2\Theta_1}\right]\right)$$
$$+\sum_{n_2\neq 0}\frac{1}{\left(\kappa\left(\frac{n_2}{\Delta l_2}\right)^2+\frac{\eta}{nZ_r}\right)^2}\left(\frac{\theta_4}{\alpha^2+14\pi^2\Theta_2^2}+\theta_5U_2\left[\frac{1}{\alpha^2+14\pi^2\Theta_2}-\frac{1}{\beta^2+14\pi^2\Theta_2}\right]\right)$$
$$+\sum_{n_1\neq 0}\sum_{n_2\neq 0}\frac{1}{\left(\kappa\left(\left(\frac{n_1}{\Delta l_1}\right)^2+\left(\frac{n_2}{\Delta l_2}\right)^2\right)+\frac{\eta}{nZ_t}\right)^2}\left(\frac{\theta_4}{\alpha^2+14\pi^2\Theta_4^2}+\theta_5U_3\left[\frac{1}{\alpha^2+14\pi^2\Theta_4}-\frac{1}{\beta^2+14\pi^2\Theta_4}\right]\right)\Bigg] \qquad (27)$$

By inserting Eqs. (15) and (22) into Eq. (3), the MSE for satellite design can be written as

$$\varepsilon_S{}^2 = \frac{\sigma^2}{TL_1L_2}\sum_{n_3\neq 0}\frac{\left(\frac{1}{nZ_t}\right)^2}{\left(\frac{\eta}{nZ_r}\right)^2+\left(\frac{n_3}{\Delta t}\right)^2}\left(\frac{\theta_4}{\alpha^2+14\pi^2\Theta_3^2}+\theta_5U_4\left[\frac{1}{\alpha^2+14\pi^2\Theta_3}-\frac{1}{\beta^2+14\pi^2\Theta_3}\right]\right) \tag{28}$$

The variance of spatial averages $\sigma_A{}^2$ is needed to estimate the percentage error and $\sigma_A{}^2$ can be written as

$$\sigma_A{}^2 = \iiint G^2(\pi\nu_1L_1)\,G^2(\pi\nu_2L_2)\,S(\nu_1,\nu_2,f)d\nu_1d\nu_2df \tag{29}$$

The expressions for the percentage errors are

$$E_k = \sqrt{\frac{\varepsilon_k{}^2}{\sigma_A{}^2}}\times 100 \tag{30}$$

where $k=S$ represents the estimator using satellite design and $k=G$ represents the estimator using ground gage design and $k=D$ represents the estimator using ground gage design with discrete sampling in time.

4.2 Parameter Estimation

The parameters of the soil moisture and precipitation model were estimated from the SGP '97 experimental data and the Oklahoma Mesonet data. The WGR model shows good stochastic representation of rainfall events in space and time and the parameters of the WGR model represent the physical features of the observed characteristics of rainfall fields in mesoscale precipitation (Waymire *et al.*, 1984). But it has a complex framework and the large number of parameters and the non-linearities make it difficult to estimate the parameters of the WGR model. Valdés *et al.* (1990) estimated the parameters for different fields through nonlinear optimization techniques, which minimized the sum of the square errors. The first step in computing estimates of the WGR model parameters is to estimate the first- and second-order sample moments of the precipitation data, for several aggregation periods T from the historic records. Then use the analytical expression of the first- and second-order moments to develop a system of nonlinear equations with the parameters of the WGR models as unknowns. Finally estimate the unknown coefficients by using a nonlinear optimization procedure. The parameters were estimated using a technique by using temporally aggregated record.

The Oklahoma Mesonet has 115 stations distributed across the Oklahoma. There are 11 Oklahoma Mesonet sites within the SGP '97 experimental area or close to its boundaries. All sites were equipped to monitor meteorological data such as relative humidity 1.5 m above the surface, air temperature 1.5 m above the surface,

Table 1. Parameters of the WGR Model and the WGR Parameter Estimates from June to August 1997.

Parameters	*Definition*	*Unit*	*Range*
D	Rainfall intensity spatial attenuation parameter	km	1.0-5.0
σ	Cell location parameter	km	-
u	Rain band speed relative to ground	km/hr	30
ρ	Mean density of cluster potential	cluster/km^2	0.01-0.001
ν	Mean number of cells per cluster	-	2.0-8.0
β	Cellular birth rate	1/hr	0.06-6 0
α	Mean cell age	1/hr	0.6-6.0
λ	Mean rain band arrival	1/hr	0.06-0.0006

Parameter set	λ (storms/hr)	ρ (CPCs/km^2)	ν (cells/CPC)	α (1/hour)	β (cells/hour)	i_0 (mm/hr)	SSQ
Little Washita							
1	0.0188	0.0017	5.00	4.25	0.9996	162.4	0.3563
2	0.0291	0.0013	4.25	4.19	0.9996	162.4	0.3803
3	0.0151	0 0032	5.25	4.83	0.8308	124.9	0.2490
4	0.0153	0.0027	5.00	4.90	0.8167	153.1	0.2623
5	0.0155	0.0021	5 00	4.42	0.9996	162.4	0.2339
6	0.0166	0.0021	4.94	4.41	0.9996	157.7	0.2551
El Reno							
7	0.0117	0.0056	7.81	5.28	0.7183	76.1	0.0432
8	0.0153	0.0055	6.50	4.90	0 7886	71.8	0.0429
9	0.0152	0.0011	4.81	4 00	0.9996	199 9	3.5930
10	0.0152	0.0012	4.92	4.00	0.9996	199.9	3.7970
11	0.0093	0.0094	7.00	5.33	0.7034	63 3	0.1368
12	0.0093	0.0094	7.00	5.33	0.7034	63.3	0 1366
Central facility							
13	0.0155	0.0044	8.00	5.42	0.677	87.4	0.6267
14	0.0151	0.0055	5.00	5.86	0.3808	124 9	0.5399
15	0.0155	0 0044	8 00	5 37	0.6963	87.4	0.3409
16	0.0168	0.0027	6.00	5.00	0.4792	150.7	0.4351
17	0.0166	0.0032	5.00	5 65	0.9292	132.0	0.5186
18	0.0168	0.0032	4 75	4.64	0.9292	139.0	0.6494

average wind speed and direction at 10 m, accumulated rainfall, solar radiation, and soil temperatures 10 cm below the surface. Three of these sites (ACME, ELRE, and BREC; near the Little Washita, El Reno, and Central facility) were used to estimate the WGR model parameters. In this study, the spatial spread parameter σ and the spatial attenuation factor D were estimated to be 3.5 and 2.0 km, respectively. The cell and rain band speed u is 14 km/hr which was estimated from the wind speed data of the Oklahoma Mesonet during the SGP '97 experimental period and during June to August, 1997. The definition and general range of the WGR

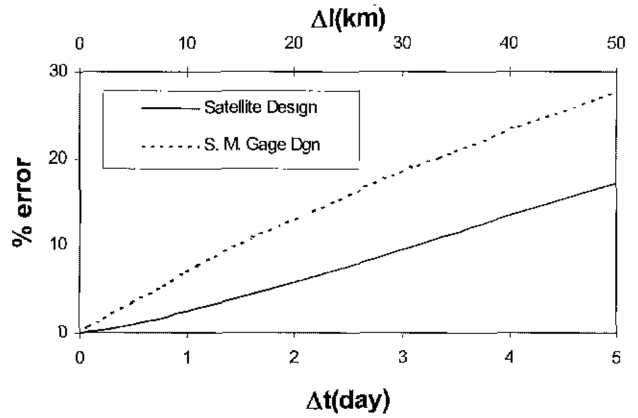

Fig. 5. Sampling Error for Satellite and Soil Moisture Gage Design Using Parameters from the SGP '97 Data

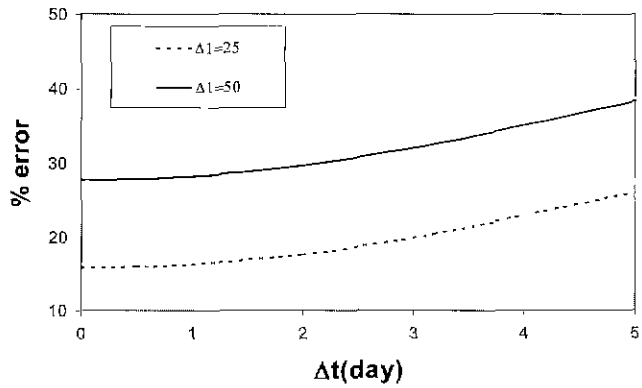

Fig. 6. Sampling Error for Discrete Sampling in Space and Time Using Parameters from the SGP '97 Data (Δl in km)

model parameters and the parameter estimates for the WGR model were provided in Table 1. The parameters with the lowest sum of square of error (SSQ) value parameters were used to compute the sampling error analysis.

The loss coefficient, η=0.03m/day with porosity n=0.46 was estimated from the SGP '97 data. The variations of soil moisture profile decrease with increase of depth. If the large variations in the soil moisture profile occur within the top 50 cm of the soil column, active soil depth could be assumed to be Z_r=0.5 m same as Entekhabi and Rodriguez-Iturbe (1994). The soil moisture field behavior is dominated by precipitation forcing and decay process caused by evapotranspiration and infiltration. The regression analysis shows that the spatial interaction represented by the diffusion terms κ is very small. The diffusion coefficient was applied a value, κ=0.001m^2/hr, as suggested by Entekhabi and Rodriguez-Iturbe (1994).

Figs. 5 and 6 are the results of sampling error analysis for several sampling designs with the above parameters. From these graphs, the sampling errors are showing power increments by increasing Δt and Δl. The sampling error for the SGP '97 remotely sensed soil moisture data was around 2.4% for daily sampling with 1 km × 1 km pixel. The 25 km square point measurement ground probe network will have 16% sampling error in the SGP '97 experimental domain.

5. SUMMARY AND CONCLUSION

The North and Nakamoto formalism (1989) was reviewed to evaluate the sampling strategy for soil moisture measurements. The formalism factorized the measurement design (spectrum filter) and physical process (frequency and wave number spectrum) separately. The physical process, in this work soil moisture process, can be divided into soil moisture dynamics (gain function) and rainfall forcing (rainfall spectrum). The fact that measurement design, rainfall forcing and soil moisture dynamics can be separated, makes it easy to evaluate the sampling strategy and to understand the relationship between physical process and sampling design. A sampling filter was developed to estimate the sampling error caused by spatial and temporal gap in soil moisture measurements. This sampling filter has an advantage in its general form applicable for all kinds of designs such as satellite over pass and point measurement gage network. The limit of extensive temporal measurement of the two-dimensional soil moisture field makes it difficult to estimate the soil moisture spectrum directly from the filed data. In the SGP '97 soil moisture data, just 16 days' two-dimensional soil moisture fields are available for 29 days 6 days is the longest period in which soil moisture was continuously monitored. Those missing data were caused by calibration problem and no flew with bad weather. It is not sufficient to estimate the soil moisture spectrum directly from field data. Therefore, we used the space-time soil moisture spectra derived by a stochastic model. The idealized soil moisture spectrum was computed with estimated model parameters from the SGP '97 soil moisture data and the Oklahoma Mesonet data.

This study has presented the sampling error for various sampling designs on a soil moisture field. The sampling error for the remotely sensed passive micro wave soil moisture data during the SGP '97 experiment was around 2.4% for daily sampling with 1 km × 1 km pixel. Fig. 5 shows that the sampling error of soil moisture estimation with satellite over pass and point measurement ground probe under the climate condition between June and August 1997 and soil properties of the SGP '97 experimental area. The sampling error caused by the spatial gap is greater than that of the temporal gap. Fig. 6 shows that the ground truth design with 25 km and 50 km spatial gap and the temporal gap from zero to 5 days. The 25 km square point measurement ground probe network will have 16% sampling error with timely continuous measurement in the SGP '97 experimental domain.

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