AN EFFICIENT METHOD OF THE SUSPENDED SEDIMENT-DISCHARGE MEASUREMENT USING ENTROPY CONCEPT

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Abstract: A method is presented which enables easily the computation of the suspended sediment discharge as the mean sediment concentration and mean flow velocity. This method has significant advantages over the traditional method, which principally depend on a set of measured concentration data. The method is based on both a new sediment concentration and mean sediment concentration equations which have been derived from the entropy concept used in statistical mechanics and information theory: (1) The sediment concentration distribution equations derived, are capable of describing the variation of the concentration in the vertical direction. (2) The mean concentration equation derived, is capable of calculating easily the mean concentration by using only one measured concentration in open channel.

The present study mainly addresses the following two subjects : (1) new sediment concentration and mean sediment concentration equations are derived from the entropy concept : (2) An efficient and useful method of suspended sediment-discharge measurements is developed which can facilitate the estimation of suspended sediment-discharge in open channel. Flume and laboratory data are used to carry out the research task outlined above.

An efficient method for determining the suspended sediment-discharge in the open channel has been developed. The method presented also is efficient and applicable in estimating the sediment transport in rivers and the sediment deposit in the reservoirs, and can drastically reduce the time and cost of sediment measurements.

Key Words: Entropy, Mean Concentration, Mean Velocity, Suspended Sediment Discharge.

1. Introduction

There are a lot of uncertainties in variables and model parameters involved in hydrology and water resources areas, such as mean velocity, sediment transport, and hydrological data. These uncertainties are due to both the inherent randomness and man's ignorance or inability to understand and cope with them. However, despite of advance made with the traditional approaches, such as deterministic or physical model, there are still major problems and barriers in many water resources areas. These problems can perhaps be evaluated using "Entropy Concept". The entropy concept has been successfully used in a variety of applications, such as hydrological processes(1,12-15), and hydraulics (2-7,10,22). The objective of this paper is to apply this concept to suspended sediment discharge which seems still to have many weak points, such as a reference concentration, mean concentration, and a lot of time and cost of sediment measurements. Finally, an effective method to estimate the suspended sediment discharge as the product of the mean velocity and the mean concentration, is proposed and verified, using flume and laboratory data.

2. Characteristic Parameters

Generally, the steady and uniform flow of water and sediment particles is defined by seven basic parameters : density of water (ρ), density of sediment(ρ_s), dynamic viscosity coefficient (μ), particle size (d), flow depth(D), channel slope(S), and acceleration of gravity (g). These seven basic parameters can be reduced to a set of four dimensionless parameters being a particle mobility parameter, a particle Reynolds' number, a depth-particle size ratio and a specific density parameter.

In this paper, it is assumed that the concentration at channel bed can be described by two dimensionless parameters only, being a dimensionless particle parameter(d_*) and a transport stage parameter(T). The introduction of these parameters is not new. Similar expressions have been used by Ackers- White and Yalin. The particle and transport stage parameters are defined by Rijn, Van(18) as follows:

$$d_* = d_{50}\left[\frac{(s-1)g}{\nu^2}\right]^{1/3} \qquad (1)$$

In which d_{50} = particle size, s = specific density, g = acceleration of gravity, ν = kinematic viscosity coefficient.

$$T = \frac{(u'_*)^2 - (u_{*,cr})^2}{(u_{*,cr})^2} \qquad (2)$$

In which u'_* is ($g^{0.5}\,\bar{u}/C'$) and $\bar{u}$ is a cross section or depth average flow velocity; and C' which is indicated as $18\log(12R_b/3d_{90})$, is Chezy-coefficient related to grains; R_b is hydraulic radius related to the bed according to Vanoni-Brooks(21), $u_{*,cr}$ is critical bed shear velocity according to Shield as given in analytical and graphical form in Fig. 1.

3. Representative Particle Size of Suspended sediment

Observations in flume and field conditions have shown that the sediment transported as bed and suspended load have different particle size distributions. Usually, the suspended sediment particles are considerably smaller than the bed load particles. Rijn(20) has proposed the following expression to obtain a better representation of the suspended load in the case of a graded bed material:

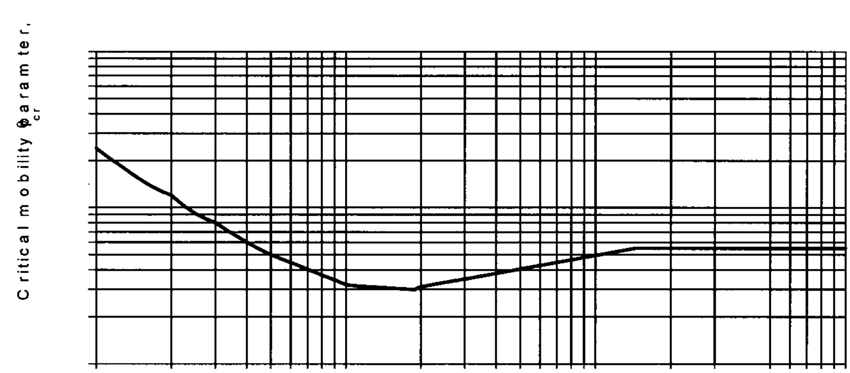

Fig. 1. Shield Curve (1981, Rijn, L. C. Van)

$$\frac{d_s}{d_{50}} = 1+0.011(\sigma_s - 1)(T-25) \tag{3}$$

In which $\sigma_s = 0.5(d_{84}/d_{50} + d_{16}/d_{50})$, (In practical, 1.5 and 2.5), d_s = representative diameter of suspended sediment.

4. Particle Fall Velocity

In a clear still fluid, the particle fall velocity of a solitary sand particle smaller than about 100 μm (Stokes-range) can be described by :

$$\omega_s = \frac{1}{18}\frac{(s-1)\,gd_s^2}{\nu} \tag{4}$$

For suspended sand particles in the range 100-1,000 μm, the following type of equation, as proposed by Zanke, can be used :

$$\omega_s = 10\frac{\nu}{d_s}\left[\left(1+\frac{0.01(s-1)gd_s^2}{\nu^2}\right)^{0.5} - 1\right] \tag{5}$$

For particles larger than about 1,000μm, the following simple equation, as proposed by Rijn(34), can be used :

$$\omega_s = 1.1\,[(s-1)\,gd_s]^{0.5} \tag{6}$$

In Eqs. 4-6, parameter d_s expresses the representative particle diameter of suspended sediment particle, which may be considerably smaller than d_{50} of the bed material.

5. Sediment Concentration Distribution (using Chiu's velocity)

The simplest differential equation that can describe the vertical distribution of sediment is :

$$\varepsilon_s\frac{dc}{dy} = -\varepsilon_s\frac{dc}{d\xi h_\xi} = \omega_s c \tag{7}$$

In which ε_s is customarily estimated through the diffusion coefficient for momentum transfer ε_m which is defined in the following expression for the turbulent shear :

$$\tau = \rho\varepsilon_m\frac{du}{dy} = \rho\varepsilon_m\frac{du}{d\xi h_\xi} = \tau_o\left(1-\frac{y}{D}\right) = \tau_o\left(1-\frac{\xi-\xi_o}{\xi_{max}-\xi_o}\right) \tag{8}$$

Where h_ξ is the scale factor or metric coefficient needed in the coordinate transformation between the y and ξ systems, which makes the product of h_ξ and d_ξ have the dimension of length, the same as *dy*.

Chiu's velocity equations are used for the velocity gradient as below :

$$\frac{du}{dy} = \frac{du}{d\xi h_\xi} = \frac{U_{nax}}{M}\frac{(e^M-1)}{(\xi_{max}-\xi_0)}\left[1+(e^M-1)\frac{\xi-\xi_0}{\xi_{max}-\xi_0}\right]^{-1} h_\xi \tag{9}$$

Fig. 2 shows velocity distribution on the y-axis(z=0) at M=2, in a wide range of h/D, simulated by applying Chiu's velocity distribution(2-7) as the following equations :

$$u = \frac{u_{max}}{M}\ln\left(1+(e^M-1)\frac{\xi-\xi_0}{\xi_{max}-\xi_0}\right) \tag{10}$$

(A) For wide-open channel (ξ = y/D)

$$u = \frac{u_{max}}{M}\ln\left(1+(e^M-1)\frac{y}{D}\right) \tag{10-A}$$

(B) If h > 0, $\xi_{max} = 1$, and $\xi_0 = 0$

$$u = \frac{u_{max}}{M}\ln\left(1+(e^M-1)\frac{y}{D-h}\exp(1-\frac{y}{D-h})\right) \tag{10-B}$$

(C) If h ≥ 0, $\xi_0 = 0$ and u_{max} occurs on the water surface but the channel is not wide.

$$u = \frac{u_{max}}{M} \ln\left(1+(e^M-1)\frac{y}{D}\exp\left(\frac{D-y}{D-h}\right)\right) \quad (10\text{-C})$$

In which ξ, along the y-axis with z=0, can be represented by

$$\xi = \frac{y}{D-h}\exp\left(1-\frac{y}{D-h}\right) \quad (11)$$

In which h = a parameter that mainly controls the slope and shape of velocity distribution curve near the water surface. When h is less than or equal to zero (between D and zero), its magnitude represents the actual depth of U_{max} below the water surface, as shown in Fig. 2. Shear stress distribution can also be obtained using the definition and expressions of ξ (Chiu and Chiou,1986). Fig. 2 also shows variation of shear distribution on the y-axis(z=0) at M=2, in a wide range of h/D, simulated by applying Eq. 8. and indicates that Eq. 8. is capable of simulating the shear stress profiles reasonably, relative to u_{max} occurring on or below the water surface.

Therefore, ε_m can be derived from Eqs. 8 and 9 ; and solution of Eq. 7, with ε_s represented by $\beta\varepsilon_m$, yields :

$$\frac{C}{C_0} = \left[\frac{\left\{1-\dfrac{\xi-\xi_0}{\xi_{max}-\xi_0}\right\}}{\left\{1+(e^M-1)\dfrac{\xi-\xi_0}{\xi_{max}-\xi_0}\right\}}\right]^{\lambda'} \quad (12)$$

In which C and C_0 are the sediment concentration at ξ and ξ_0, respectively; and

$$\lambda' = \frac{\omega_s UF(M)}{\beta U_*^2 e^M}$$

In which

$$F(M) = \frac{(e^M-1)(e^M-1)}{(Me^M - e^M+1)} = \frac{DgRS_f}{vU} \quad (13)$$

Which is a dimensionless function of M, and represents the entropy effect.

Eq. 12 is capable of describing the variation of concentration in both the vertical and transverse directions using a curvilinear coordinate system in which Chiu and Chiou(2) proposed. Some of the possible forms of these equations applicable along a vertical section (z=0) with ξ are :

(A) For wide-open channel (ξ = y/D)

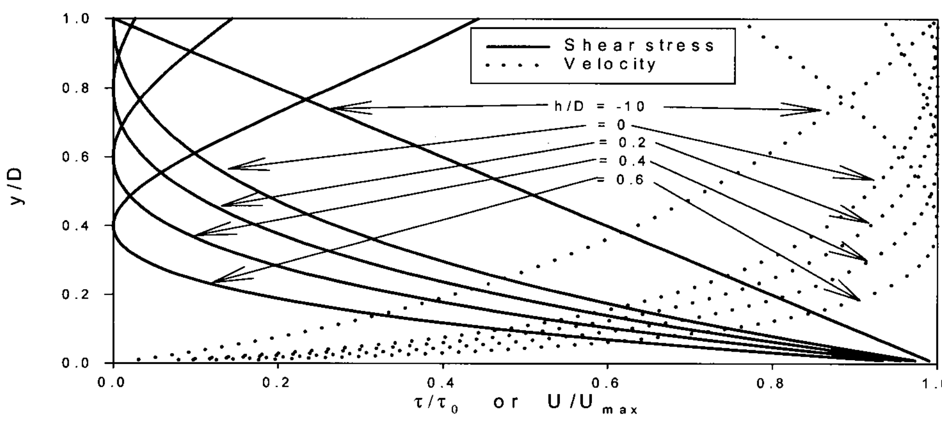

Fig. 2. Variation of Velocity & Shear stress Distribution with M=2

$$\frac{C}{C_0} = \left[\frac{\left\{1 - \frac{y}{D}\right\}}{\left\{1 + (e^M - 1)\frac{y}{D}\right\}} \right]^{\lambda'} \tag{14}$$

(B) If h > 0, $\xi_{max} = 1$, and $\xi_0 = 0$

$$\frac{C}{C_0} = \left[\frac{\left\{1 - \frac{y}{D-h}\exp\left(1 - \frac{y}{D-h}\right)\right\}}{\left\{1 + (e^M - 1)\frac{y}{D-h}\exp\left(1 - \frac{y}{D-h}\right)\right\}} \right]^{\lambda'} \tag{15}$$

(C) If $0 \geq h$, $\xi_0 = 0$ and u_{max} occurs on the water surface but the channel is not wide.

$$\frac{C}{C_0} = \left[\frac{\left\{1 - \frac{y}{D}\exp\left(\frac{D-y}{D-h}\right)\right\}}{1 + (e^M - 1)\frac{y}{D}\exp\left(\frac{D-y}{D-h}\right)} \right]^{\lambda'} \tag{16}$$

Figs. 3-6 are based on eqs. 14, 15 and 16, fitted to both flume and field data sets shown, in which M is used as the same one obtained from Eqs. 10-A, 10-B, and 10-C, respectively. Slight disagreement among them exists only below the data range near the bed. These figures show that concentration estimation at channel bed by Eq. 14 is higher than that by Eq. 15, specially at channel which maximum velocity occur below the surface. This difference can also be seen in Figs. 3-6. Since Eq. 14 is a part of Eq. 12 as shown in derivation. Therefore, it can be concluded that Eq. 12 instead of Eq. 14 should be used to estimate the vertical sediment concentration distribution, and can also be described the variation of concentration in 2-D with coordinate system.

(A) Using ($\xi = \xi_m$)the location at which the velocity is equal to the cross-sectional mean.

Chiu(1988) derived it from Eq. 10 by

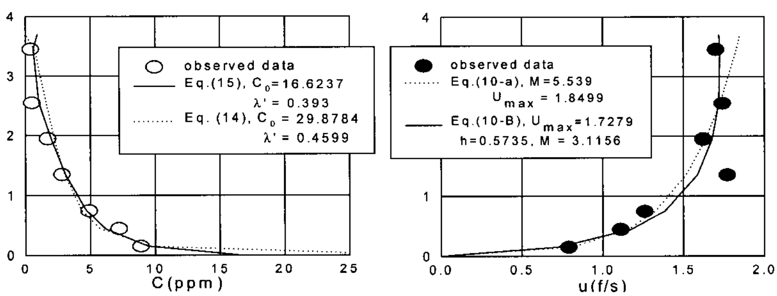

Fig. 3. Measured and computed concentration and flow velocity for Enoree River(E7-7-41)

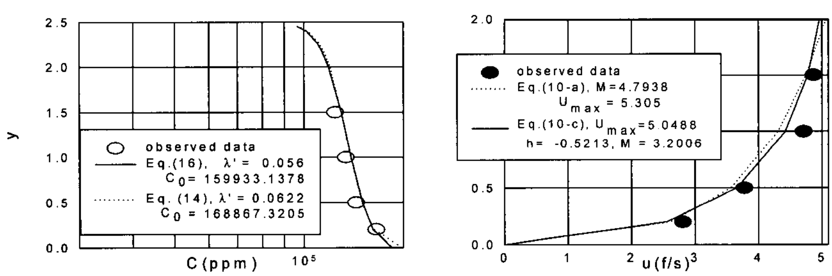

Fig. 4. Measured and computed concentration and flow velocity for Middle Rio Grande River near Bernalillo(june, 4, 1953)

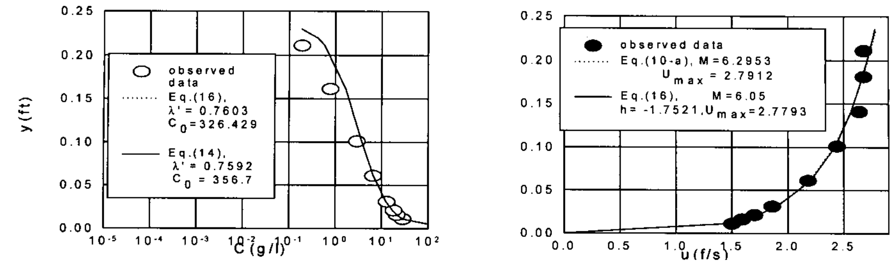

Fig. 5. Measured and computed concentration and flow velocity for Brooks Experiment(Run 21, 1954)

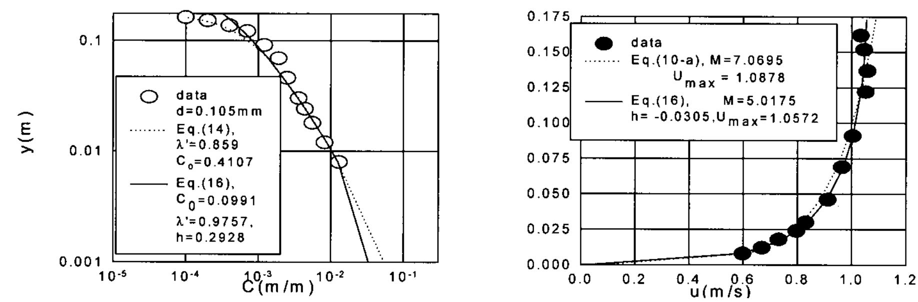

Fig. 6. Measured and computed concentration and flow velocity for Coleman Experiment(Run 12, 1981)

letting u = U(cross-sectional mean velocity) and $\xi = \xi_m$ as below :

$$\frac{\xi_m - \xi_0}{\xi_{max} - \xi_0} = \frac{(e^{M\Phi(M)} - 1)}{(e^M - 1)} = \frac{\left\{\exp\left(\frac{e^M - 1}{F(M)} - 1\right)\right\}}{\{e^M - 1\}} \tag{17}$$

(B) Substitution of Eq. 17 into Eq. 12 by letting C = Cu and $\xi = \xi_m$ gives :

$$\frac{C_U}{C_0} = \left[\frac{\left\{1 - \frac{\xi_m - \xi_0}{\xi_{max} - \xi_0}\right\}}{\left\{1 + (e^M - 1)\frac{\xi_m - \xi_0}{\xi_{max} - \xi_0}\right\}}\right]^{\lambda'} \tag{18}$$

$$= \left[\frac{e^M - \exp\left\{\frac{e^M - 1}{F(M)}\right\}}{(e^M - 1)\exp\left\{\frac{e^M - 1}{F(M)}\right\}}\right]^{\lambda'}$$

Eq. (18) is a very useful equation to obtain analytical relationship C_U / C_0 from M, λ' (as shown in Fig.7) which can be calculated by simple inputs, such as U(mean velocity), Manning's hydraulic radius and channel slope (Chiu.1989) without any concentration data. It is also a very useful equation that can be used with the linkage between the velocity and the sediment concentration. It will be discussed in later part in detail.

To obtain the sediment concentration at the channel bed (C_0), the following procedure is suggested: For given M and λ' obtained by simple inputs, use Eq. 18 to calculate C_U / C_0. Then, the sediment concentration at $\xi = \xi_m$ or $y=y_m$ is measured. The sediment concentration is the concentration at which the velocity is equal to the cross-sectional mean. C_0 can be obtained from dividing Cu by C_U / C_0. Besides, substitution of boundary condition into Eq. 12 by letting $C = C_{0.2}$, $\xi = \xi_{0.2}$, and $C = C_{0.5}$, $\xi = \xi_{md}$, and $C = C_{0.8}$, $\xi = \xi_{0.8}$ give, respectively :

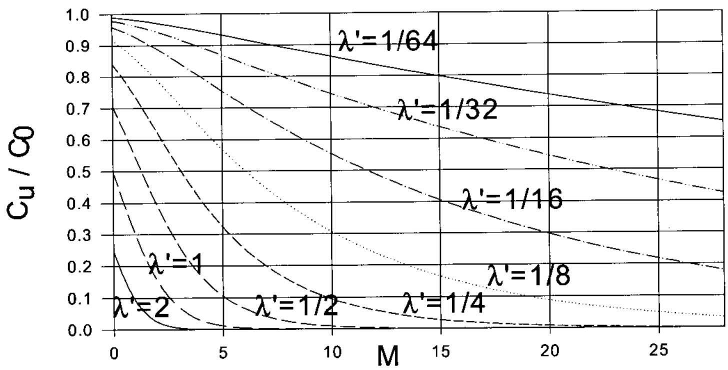

Fig. 7. Relation between C_u/C_0 and M, λ' using Eq. (18)

$$C_0 = C_{0.2} \left[1+\frac{e^M}{4}\right]^{\lambda'} \tag{19}$$

$$C_0 = C_{0.5} \left[1+ e^M\right]^{\lambda'} \tag{20}$$

$$C_0 = C_{0.8} \left[4+ e^M\right]^{\lambda'} \tag{21}$$

These relationship can be mainly used to describe the sediment concentration distribution for the practical purpose. The relationship between C_0 and C_D (sediment concentration at the water surface) can be obtained from substituting it into Eq. 12 as follows :

$$\begin{aligned} C_0 &= C_D\,[1+ e^M * 999.......999]^{\lambda'} \\ &= C_D\,[1+ e^M * 1\times 10^{10}] = C_D k \end{aligned} \tag{22}$$

This equation will be used to derive new another concentration equation.

6. Another concentration distribution (using Entropy concept)

Its derivation is similar to that of Chiu's velocity distribution.

The entropy under the present formulation is :

$$\mathrm{H(C)} = \int_{C_D}^{C_0} p(c)\ \ln p(c)dc \tag{23}$$

Subject to the constraints

$$1 = \int_{C_D}^{C_0} p(c)\ \ln p(c)dc \tag{24}$$

$$\overline{C} = \int_{C_D}^{C_0} cp(c)dc \tag{25}$$

The method of the calculus of variations can be applied to determine P(c) in Eq. 23 which has the general form :

$$\mathrm{H}(x) = \int_a^b I(x, p)dc \tag{26}$$

To maximize (or minimize) H(x) subject to a set of n constraints (2 constraints in this case)

$$\mathrm{E_i} = \int_a^b \Phi(x, p)dx\ ; \qquad \mathrm{I} = 1, 2 ...... \mathrm{n} \tag{27}$$

P(x) can be obtained by solving

$$\frac{\partial I(x,p)}{\partial p} + \sum_{i=1}^{n} \lambda_i \frac{\partial \Phi_i(x,p)}{\partial p} = 0 \tag{28}$$

According to the above procedure, P(c) can be obtained as

$$P(C) = \mathrm{Exp}^{\ b_1 + b_2 C} \tag{29}$$

Substitution of Eq. 28 into Eq. 24 yields

$$e^{b_1} = b_2 [e^{b_2 C_0} - e^{b_2 C_D}]^{-1} \quad (30)$$

The cumulative distribution function(yet to be identified) is

$$F(C) = \frac{D-y}{D} = 1 - \frac{y}{D} = 1 - \frac{\xi - \xi_0}{\xi_{max} - \xi_0} = \int_{C_D}^{C_0} P(c)dc \quad (31)$$

Substitution of Eq. 29 into Eq. 31 yields

$$C = \frac{C_0}{N} \ln\left[e^N - (e^N - e^{\frac{N}{K}}) \frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \quad (32)$$

In which $N = b_2 C_0$, a parameter called "entropy parameter for sediment transport" which is similar to M in the velocity distribution. $k = (C_0 / C_D)$ is a parameter which can describe the ratio of concentration at the bottom (C_0) to at the water surface (C_D).

7. Mean Concentration

From Eqs. 24, 25, and 29, The mean sediment concentration can be obtained as the expected value of C ;

$$\overline{C} = \int_{C_D}^{C_0} cp(c)dc = C_0 \Phi(N) \quad (33)$$

Where

$$\Phi(N) = \frac{e^N - \frac{1}{k} e^{\frac{N}{K}} - \frac{1}{N}\left[e^N - e^{\frac{N}{K}}\right]}{\left[e^N - e^{\frac{N}{K}}\right]} \quad (34)$$

There are two methods to calculate the mean concentration using the entropy concept. One method is to use Eq. 32 directly using the least square method to obtain the required(N, K), then Eq. 33 is used to calculate the mean concentration. The other one is to use Eq. 33 together with Eq. 12. In figure 8, mean concentration estimated by Eq. 33 are plotted together with measured values by Coleman(8,9). It indicates that those estimated by Eq. 33 agree with the observed one very well. Fig. 8 also compares the mean concentration measured by Coleman and those calculated by Eq. 33 with Eq. 32, and Eq. 33 with Eq. 12. It indicates that Coleman's values is constantly lower than those by two methods; Estimation by the first one is the highest ; and that by the

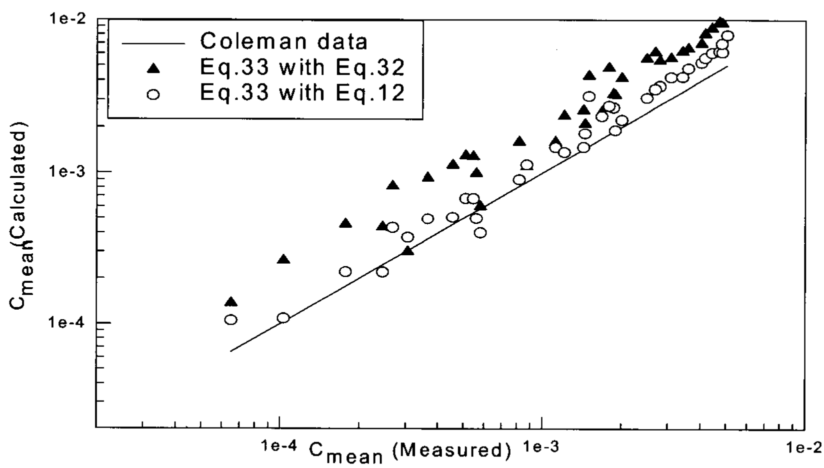

Fig. 8. Comprision of measured and calculated Mean Concentration

second one falls between those by the first one and Coleman.

8. Suspended Sediment Discharge

Usually, the suspended sediment discharge is computed by integration as :

$$Q_s = \int_0^d \int_0^w cu \, dydx \fallingdotseq \overline{U}\,\overline{C}\,A \quad (35)$$

Where $\overline{U}$ is a cross-sectional mean velocity from Eq. 10, and C is a cross-sectional mean concentration from Eq. 33. Using Eqs. 10, 12, 32, and 33, to describe the concentration profile and the velocity profile, the suspended sediment discharge is obtained from Eq. 35 resulting in :

$$\begin{aligned} Q_s &= \overline{U}\,\overline{C}\,A = Umax\ \Phi(M)\ C_0\ \Phi(N)\ A \\ &= \overline{U}\,A\ C_0\ \Phi(N) \end{aligned} \quad (36)$$

Similar to Φ(M) in the velocity distribution, Φ(N) is a very useful parameter that can be employed in many other application of sediment transport. Where the input data are cross sectional mean velocity ($\overline{U}$) and mean flow depth(D). Therefore, a proposed method can be easily applied the computation of the suspended sediment discharge as the cross sectional mean sediment concentration and cross sectional mean flow velocity as seen in Eq. (36).

9. Summary and Conclusion

The entropy principle, developed by Shannon(11), is very useful to evaluate the effects of flow system. It can also be expressed as a function of probability distribution as mentioned earlier. Chiu employed it for deriving velocity distribution in open channel and proved it very useful. Similar to that of Chiu's velocity distribution, the objective of this paper is also to apply it to sediment flow in hydraulics and to prove. The following conclusions are obtained through the research.

1. A new suspended sediment concentration-distribution equation(Eq.12) is used as an alternative to the existing Rouse equation for estimating sediment concentration and for studying sediment flow.

2. A newly-developed equation(Eq.18) with only one measured sediment concentration, can be utilized effectively to estimate a channel bed concentration which has been one of major uncertainties in sedimentation engineering, and used with the linkage between the velocity and the sediment concentration.

3. A newly-developed cross-sectional mean sediment concentration equation(Eq.33) is used effectively to evaluate the cross-sectional mean sediment concentration in open channel. This entropy based method can also be used to estimate the suspended sediment discharge as the product of the cross-sectional mean sediment concentration and the cross-sectional mean velocity.

4. This proposed method has significant advantages over the traditional methods, which principally depend on a set of measured concentration data, and can drastically reduce the time and cost of suspended sediment discharge measurements:
(1) Eq.18 can be used to obtain Analytical relationship C_U / C_0 from M, λ' which can be calculated by simple inputs, such as $\overline{U}$, R and S:
(2) A channel bed concentration(C_0) can be estimated by putting only one measured sediment concentration(C_U) into Eq.18:
(3) Parameters N and K can be estimated by putting C_U and C_0 into Eq.32:
(4) $\overline{C}$ can be estimated by using Eqs 33

and 34:

(5) Q_s can finally be obtained using Eq.36 effectively.

The probability-based entropy concept allows for new avenues of sedimentation researches. The entropy function and corresponding probability functions provide meaningful new hydraulic variables for characterizing sediment transport processes in open channels.

APPENDIX I. - REFERENCES

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APPENDIX II. - NOTATION

The following symbols are used in this paper:

C, C_0, C_D, C_U = sediment concentration at y, $y=0$, $y=D$, and $y=y_m$, respectively;

$\overline{C}$ = cross-sectional mean values of C;

D = water depth;

F = cumulative distribution function;

g = gravitational acceleration;

h = parameter in ζ;

h_ξ = scale factor on ζ coordinate ;

H = entropy;

K = the ratio of C_0 to C_D

M = "entropy parameter in clear water";

N = "entropy parameter in sediment process";

$\overline{U}$, U_{max} = cross-sectional mean and maximum values of u, respectively;

u_* = shear velocity;

w_s = settling velocity of a sediment particles;

p = probability mass or density function;

q_s = suspended sediment discharge per unit width of channel;

y_m, ζ_m = values of y or ζ at which u= U;

β = coefficient relating ε_m to ε_s;

ε_m, ε_s = diffusion coefficients for momentum and sediment transfer, respectively;

λ' = exponent in sediment distribution equation;

ζ = coordinate used in a velocity and concentration distribution equation;

ζ_0, ζ_{max} = minimum and maximum values of ζ, respectively;

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