BRAGG RESONANT REFLECTION OF OBLIQUELY INCIDENT WATER WAVES

Yong-Sik Cho

Department of Civil and Environmental Engineering, Sejong University, Seoul, Korea

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Abstract: The Bragg reflection of obliquely incident monochromatic water waves propagating over a sinusoidally varying topography is theoretically investigated in this study. The eigenfunction expansion method is first employed to calculate reflection coefficients of water waves due to depth changes. A reasonable agreement is observed. Obtained reflection coefficients of normally incident waves are compared with laboratory measurements. Reflection coefficients of obliquely incident waves are then calculated. The wavenumber providing the Bragg reflection agrees well with analytical predictions.

Key Words: wave, Bragg reflection, diffraction, eigenfunction expansion method, evanescent modes

1. INTRODUCTION

The water waves, traveling from offshore to the nearshore zone, experience many important phenomena resulted primarily from combined effects of irregular bottom topographies, interferences with man-made coastal structures and nonlinear interactions between different harmonic components. Among those phenomena, the Bragg resonant reflection of obliquely incident water waves caused by bottom topographical changes is investigated theoretically in this study.

The Bragg resonant reflection, briefly Bragg reflection, is believed to play a significant role in formation of multiple offshore ripples frequently observed on the mildly sloping beaches of the Great Lakes, Japanese, Danish and many other open coasts (Mei and Liu, 1993). These multiple offshore ripples may be responsible for the change of coastal morphology.

As confirmed in many previous studies, the Bragg reflection occurs when the wavelength of incident water wave is approximately equal to twice of that of the corrugated bottom topography. Thus, the spacing between two adjacent ripples may be in the range of several tens of meters for offshore bars believed to be formed due to wind waves. On the other hand, the spacing reaches to several hundreds of meters for offshore bars believed to be formed due to long waves generated by wave groups (Liu and Cho, 1993). Following to Mei and Liu (1993), a group of offshore sandbars consists of 3 to 17 sandbars and the spacing between two adjacent bars is about 10 to 480 meters.

Although many studies have been reported on the Bragg reflection of the monochromatic water waves (e.g., Davies and Heathershaw, 1984; Mei *et al.*, 1988; O'Hare and Davies, 1992; Rey et al., 1992; Liu and Cho, 1993; Cho and Lee, 2000), the study on the Bragg reflection of obliquely incident waves is still rare. However, the waves approaching from offshore are not always normal to the coastline. To make a more realistic calculation of reflection coefficients it should be taken obliquely as well as normally incident waves into consideration.

In the following section, the diffraction of incident monochromatic waves is briefly described for a completeness. A short explanation of the eigenfunction expansion method is also given. In section 3, numerical computations of reflection coefficients for waves over a sinusoidally varying topography are carried out. Obtained reflection coefficients are compared with laboratory measurements. A detailed discussion on the oblique incidence of waves is also described. Finally, concluding remarks are made in section 4.

2. DIFFRACTION OF INCIDENT WAVES

In this section, the diffraction of monochromatic waves by a depth discontinuity is first given simply for completeness. A bottom topography is first divided into a finite number of small steps similarly to O'Hare and Davies (1992). The variation of the bottom topography is limited to x-direction and thus, the wavenumber in y-direction remains constant due to the Snell's law.

Although described in Cho and Lee (2000), the velocity potentials satisfying the Laplace equation and the boundary conditions within each region are reiterated as

$$\Phi_m = \left\{ A^l_m e^{+il_m x} \cosh k_m (h_m + z) + \sum_{n=1}^{\infty} B^l_{m,n} e^{+\lambda_{m,n} x} \cos K_{m,n} (h_m + z) \right\} e^{i(k_y y - \omega t)} \tag{1}$$

for the right-going and evanescent modes and

$$\Phi_m = \left\{ A^l_m e^{-il_m x} \cosh k_m (h_m + z) + \sum_{n=1}^{\infty} B^l_{m,n} e^{-\lambda_{m,n} x} \cos K_{m,n} (h_m + z) \right\} e^{i(k_y y - \omega t)} \tag{2}$$

for the left-going wave and evanescent modes. In equations (1) and (2), A^l_m, $B^l_{m,n}$, A^r_m and $B^r_{m,n}$ are complex amplitude functions to be determined, subscripts m and n denote the number of regions and number of evanescent modes to be implemented, respectively and θ is the incident angle. The superscripts r and l represent the right- and left-going components, respectively.

In equations (1) and (2), l_m and $\lambda_{m,n}$ are wavenumber components of x-direction for propagating and evanescent modes, respectively. They are calculated from the relations given by

$$l_m = (k_m^2 - k_y^2)^{1/2}, \quad \lambda_{m,n} = (K_{m,n}^2 + k_y^2)^{1/2} \tag{3}$$

in which k_y is the wavenumber component of y-direction. The wavenumbers k_m and $K_{m,n}$ are calculated from the linear dispersion relationships given as

$$\omega^2 = g k_m \tanh k_m h_m, \quad \omega^2 = -g K_{m,n} \tan K_{m,n} h_m \tag{4}$$

If waves are normally incident, $k_y = 0$ and equation (3) is simplified as

$$l_m = k_m, \quad \lambda_{m,n} = K_{m,n} \tag{5}$$

To solve equations (1) and (2) at each region two matching conditions are required at each depth discontinuity. The first condition is expressed as

$$\frac{\partial \Phi_i}{\partial x} = \frac{\partial \Phi_{i+1}}{\partial x}, \quad x = x_i, \; -h_i \le z \le 0 \qquad (6)$$

which ensures the continuity of horizontal flux in x-direction. The second matching condition is given by

$$\Phi_i = \Phi_{i+1}, \quad x = x_i, \; -h_i \le z \le 0 \qquad (7)$$

which guarantees the continuity of pressure.

By substituting equations (1) and (2) into matching conditions (6) and (7), a linear system of matrix can be obtained. In the eigenfunction expansion method, the orthogonality of trigonometric and hyperbolic functions is employed to reduce the number of unknowns in the system. Details of the eigenfunction expansion method is well described in Kirby and Dalrymple (1983).

3. NUMERICAL COMPUTATION AND DISCUSSION

In this section, the model is first used to calculate reflection coefficients of normally incident waves propagating over a sinusoidally varying topography. The occurrence of Bragg reflection is investigated. Variation of reflection coefficients is then examined for waves with different incident angles. Although the evanescent modes may not play a significant role in reflection coefficients if the size of each small step is properly controlled (O'Hare and Davies, 1992), they are considered in this study.

By solving equations (1) and (2) with matching conditions (6) and (7) the reflection coefficients can be expressed in terms of the amplitude of incident wave as

$$R = \frac{|A_1^r|}{|A_1^i|} \qquad (8)$$

In this study, $A_1^i = 1$ has been used for simplicity. Thus, equation (8) is reduced to

$$R = |A_1^r| \qquad (9)$$

As shown in Fig. 1, the bottom topography consists of two constant depth regions and a sinusoidally varying region. In this study, water depths of two regions connected to the sinusoidally varying region are assumed to be same for simplicity. Thus, the bottom topography is described as

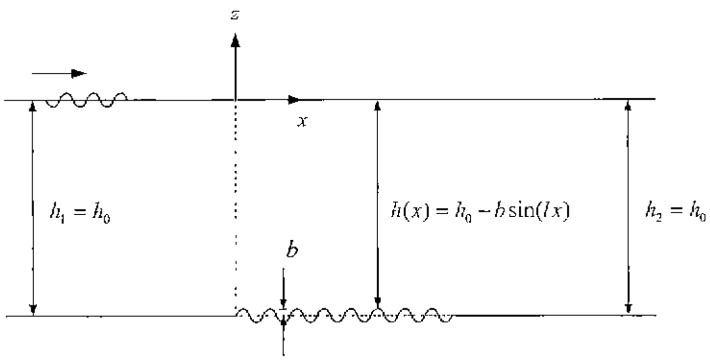

Fig. 1. A Definition Sketch of a Sinusoidally Varying Topography

$$h(x) = h_0, \quad x < 0$$
$$h(x) = h_0 - b\sin(lx), \quad 0 < x < 2\pi m/l \qquad (10)$$
$$h(x) = h_0, \quad x > 2\pi m/l$$

in which h_0 is the constant water depth, b is the amplitude, m is the number of seabed ripples and l is the wavenumber of the seabed ripple.

Fig. 2 displays a schematic sketch of a replacement of a sinusoidally varying topography by a finite number of steps. The waves are incident with an angle of θ and

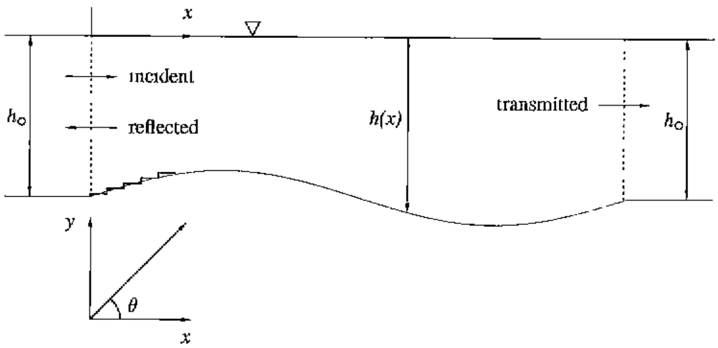

Fig. 2. Schematic Sketch of a Representation of a Sinusoidally Varying Topography with a Finite Number of Small Steps

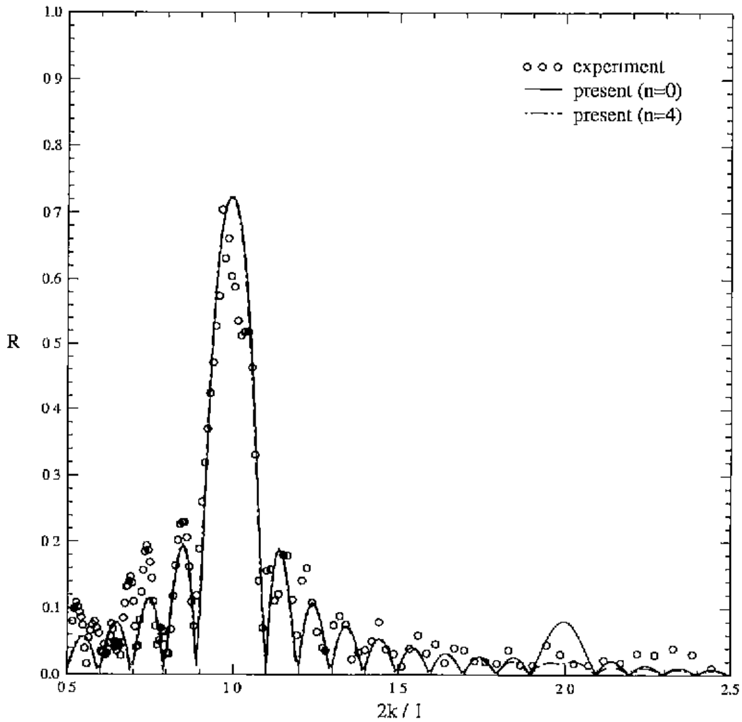

Fig. 3. Distribution of Reflection Coefficients for Waves Propagating over the Topography

the topography varies in x-direction only. The parameters used by O'Hare and Davies (1992) are used again in this study to verify and compare the calculated results with laboratory measurements.

In the first example, the Bragg reflection of water waves is re-examined to demonstrate the performance of the eigenfunction expansion method. Fig. 3 shows distribution of reflection coefficients for waves propagating over the topography defined by equation (10). In the figure, the number of ripples is $m=10$, the incident angle is $\theta=0^{o}$ and the relative amplitude of ripple is $b/h_o=0.16$. In spite of slight discrepancies in shallow zone, the overall agreement is very good. In special, the present method predicts the Bragg reflection accurately.

In Fig. 4, distributions of reflection coefficients for waves with different incident angles are compared. In the figure, the number of ripples is $m=2$ and the relative amplitude of the ripple is $b/h_o=0.32$. The peak decreases and moves rightward slightly as θ increases. According to Dalrymple and Kirby (1986), the ratio of wavenumbers providing the Bragg reflection for obliquely incident waves is given by $2k\cos\theta/l=1$. Thus, $2k/l=1$ for $\theta=0^{o}$ and $2k/l=1.15$ for $\theta=30^{o}$ for peaks. The present method produces agreeable results for both $\theta=0^{o}$ and 30^{o}. The number of evanescent modes is 4 in this figure.

Finally, reflection coefficients for different incident angles are also compared in Fig. 5.

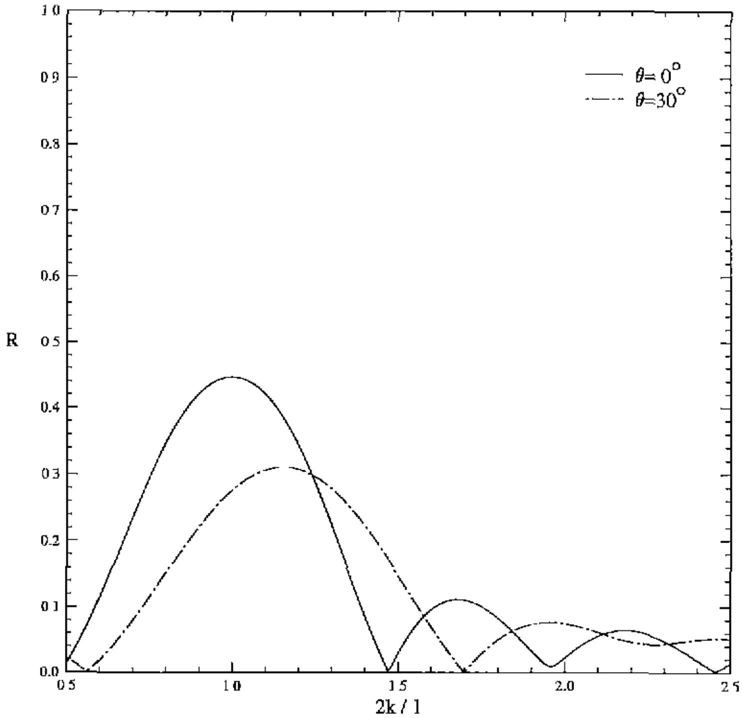

Fig. 4. Distributions of Reflection Coefficients over a Sinusoidally Varying Topography

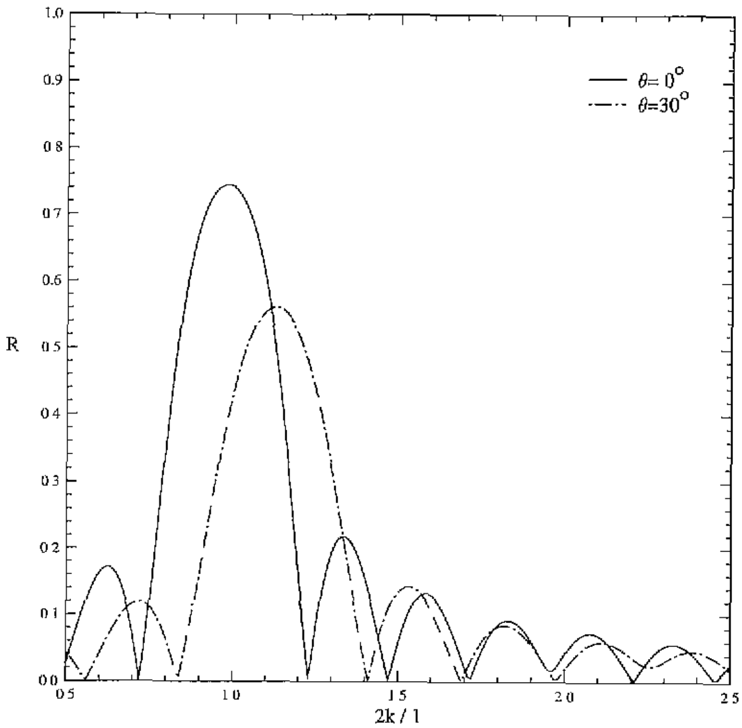

Fig. 5. Distributions of Reflection Coefficients over a Sinusoidally Varying Topography

In the figure, the number of ripples is $m=4$ and the relative amplitude of ripple is $b/h_0=0.32$. Similarly to Fig. 4, the peak decreases and moves rightward as θ increases. The eigenfunction expansion method also gives agreeable results for both $\theta=0°$ and $30°$.

4. CONCLUDING REMARKS

In this study, a theoretical model describing the diffraction of monochromatic waves by abrupt depth changes has been introduced. The effects of the evanescent modes as well as the propagating mode are included in the model. Furthermore, the incident angle of waves is not limited to the normal.

Although the replacement of a sinusoidally varying topography by a finite number of small steps requires a little more computational efforts and computing time, it is worth to calculate the reflection coefficients over an slowly and fast varying topography as accurately as possible because of its direct practical applications in designing of coastal structures and preventing of unwanted beach erosion and deposition.

The concept of the present study may be used as a powerful design guide for a train of the submerged breakwaters which can protect coastal structures and unwanted beach erosion by reflecting a significant amount of incident wave energy.

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REFERENCES

Cho, Y.-S. and Lee, C. (2000). "Resonant reflection of waves over sinusoidally varying topographies." *Journal of Coastal Research*, in press.

Dalrymple, R.A. and Kirby, J.T. (1986). "Water wave over ripples." *Journal of Waterway, Port, Coastal and Ocean Engineering*, ASCE, Vol. 112, pp. 309-319.

Davies, A.G. and Heathershaw, A.D. (1984). "Surface-wave propagation over sinusoidally varying topography." *Journal of Fluid Mechanics*, Vol. 144, pp. 419-443.

Kirby, J.T. and Dalrymple, R.A. (1983). "Propagation of obliquely incident water waves over a trench." *Journal of Fluid Mechanics*, Vol. 133, pp. 47-63.

Liu, P.L.-F. and Cho, Y.-S. (1993). "Bragg reflection of infragravity waves by sandbars." *Journal of Geophysical Research*, Vol. 98, pp. 22733- 22741.

Mei, C.C., Hara, T. and Naciri, M. (1988). "Note on Bragg scattering of water waves by parallel bars on the seabed." *Journal of Fluid Mechanics*, Vol. 186, pp. 147-162.

Mei, C.C. and Liu, P.L.-F. (1993). "Surface waves and coastal dynamics." *Annual Review of Fluid Mechanics*, Vol. 25, pp. 215-240.

O'Hare, T.J. and Davies, A.G. (1992). "A new model for surface-wave propagation over undulating topography." *Coastal Engineering*, Vol. 18, pp. 251-266.

Rey, V., Belzons, M. and Guazzelli, E. (1992). "Propagation of surface gravity waves over a rectangular submerged bar." *Journal of Fluid Mechanics*, Vol. 235, pp. 453-479.

Yong-sik Cho, Department of Civil and Environmental Engineering, Sejong University, 98 Gunja-Dong, Gwangjin-Gu. Seoul, 133-747, Korea.
(email: yscho@sejong.ac.kr)

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