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Twin-Image Noise Effects in Optical scanning Holography

Kyu Bong Doh and Hwangsuk Lee

Department of telecomunication and Information Engineering,
Hankuk Aviation University, Koyang-shi 411-791, KOREA
E-mail : kdoh@mail.hangkong.ac.kr

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In Optical Scanning Holography(OSH), 3-D holographic information of an object is generated by 2-D active optical scanning. The optical scanning beam can be a time-dependent Gaussian apodized Fresnel zone plate. In this technique, the holographic information manifests itself as an electric signal which can be sent to an electron-beam-addressed spatial light modulator for coherent image reconstruction. In this paper, we briefly review optical scanning holography and analyze the resolution achievable with the system. We also present mathematical expressions of real and virtual images which are responsible for holographic image reconstruction. We then show the twin-image noise effect on the reconstruction in conjunction with the size of the Fresnel zone pattern through computer simulation.

I. INTRODUCTION

Holography has been an important tool for scientific and engineering studies, it has found a wide range of application.[1-10,12] Electronic holography is a technique in which electronic processing is used in the context of holography, allowing holographic recording to be performed in real time, bypassing the use of films for recording.[6-14] As one type of many electronic holographic technique, optical scanning holography(OSH) is a technique in which three-dimension information of an object can be recorded by two-dimensional scanning.[13-16] The technique involves optically scanning the three-dimensional object by a so-called time-dependant Fresnel zone-lens plate(TDFZP) [12]. The TDFZP is created by the superposition of a plane wave and a spherical wave of different temporal frequencies. When the object is optically scanned, a photo-detector collects the scattered light and delivers a heterodyne current as output. The current is then mixed down to become a demodulated signal. The demodulated signal is synchronized with the x-y optical scanning system and fed to a display. A hologram or Fresnel zone-lens plate coded information of the object being scanned is displayed. To decode the information optically in real time, the holographic information could be transferred to a spatial light modulator (SLM) for coherent reconstruction. Digital reconstruction is also possible by convolving the hologram with a free-space impulse response. Whereas the work of optical scanning holography has been concentrated in on-axis techniques, which inherently produce a twin-image upon reconstruction (or decoding). It is known that twin-image noise has been the subject of great interest for many researchers. In the next section, we introduce the concept of optical scanning holography and mathematical expressions of real and virtual images which are responsible for holographic image reconstruction. In the section after, we demonstrate computer simulation results for the validity of the idea.

II. FRESNEL ZONE PLATE

The spherical wave is produced by a point source and is assumed to be incident on the plate. The wavefront at a given instant of time may be divided into a number of concentric zones in such a way that each zone is one-half of a wavelength farther away from point source. If the distance from the wave front to the source is S, the distances from successive boundaries between zones to the source are given by

$$s+\frac{\lambda}{2},\ s+\frac{2\lambda}{2},\ s+\frac{3\lambda}{2},\ \ldots,\ s+\frac{m\lambda}{2} \qquad (1)$$

The boundaries between zones have radii that are proportional to the square root of the integer number,

$$R_m = R_1\sqrt{m}, \qquad m = 1, 2, 3, \ldots \qquad (2)$$

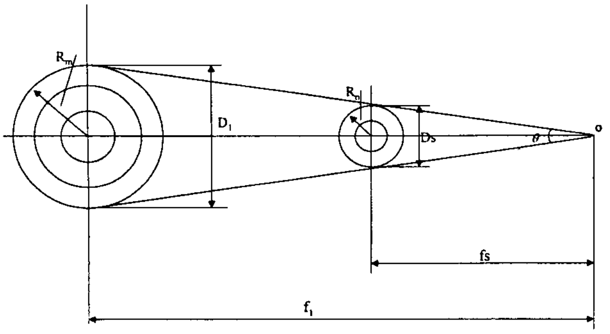

FIG. 1. Fresnal Zone Plate

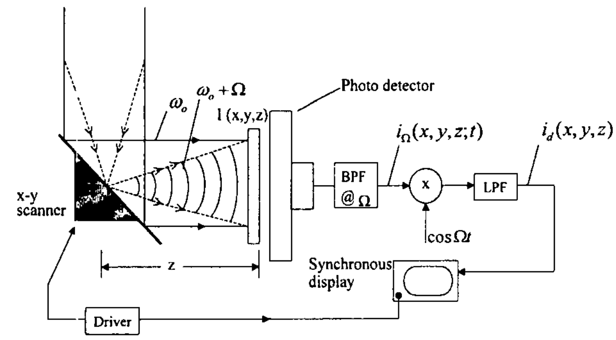

FIG. 2. Optical Scanning System

where$R_1$and R_2 are the radii of the innermost boundary and the m-th boundary respectively. From Eq.1

$$S^2 = (S + m\frac{\lambda}{2})^2 - R_m^2, \qquad R_m^2 \approx mS\lambda \tag{3}$$

Since λ is small compared with S, the term $\frac{\lambda}{S}$ may be neglected. However S is also the focal lengths of the zone plate because of its focusing properties. Thus

$$f = \frac{R_m^2}{m\lambda} \tag{4}$$

As shown in Fig. 1, Fresnel zone plates have R_m and R_n as radii of the boundary and f_l and f_s are their focal lengths respectively. We can have

$$f_l = \frac{R_m^2}{M\lambda}, \qquad f_s = \frac{R_n^2}{N\lambda} \tag{5}$$

where M, N are the integers. The resolution limit is defined as $R \approx \frac{\lambda}{\theta}$. In order to make the two zone lens with the same resolution on image reconstruction, we need to have

$$(R)_l = (R)_s, \qquad \frac{f_l}{D_l}\lambda = \frac{f_s}{D_s}\lambda, \qquad \frac{M}{N} = \frac{R_m}{R_n} \tag{6}$$

which shows that the ratio of the number of zones is inversely proportional to that of the radius of the zone lens plate.

III. OPTICAL SCANNING HOLOGRAPHY

In optical scanning holography, an object is two-dimensionally scanned by a time-dependent Fresnel zone plate(TDFZP) created by the superposition of a plane wave and a spherical wave of different temporal frequencies to generate a hologram of the object, as shown in Fig. 2. We model the scanning pl ane wave and the spherical wave by two Gaussian beams of a broad and a narrow waist, respectively. In practice, a broad and a narrow Gaussian beam are combined with a beam-splitter.[13-15,17] One of the Gaussian beams is frequency-shifted so that the two beams interfere to form a temporally modulated Gaussian apodized Fresnel zone plate $I(x,y,z;t)$ at the location of the object $P(x,y;z)$

$$I(x,y,z;t) = b(x,y,z) + \frac{2}{\pi^2\omega_u(0)\omega_u(z)\omega_v(0)\omega_v(z)} \exp\left[-\left(\frac{x^2+y^2}{\omega(z)^2}\right)\right] \cdot \sin\left[\frac{k_0}{2R(z)}(x^2+y^2) + \Omega t)\right] \tag{7}$$

where $b(x,y,z)$ is $|u(x,y,z)|^2 + |v(x,y,z)|^2$ and k_0 is the wave number of the light and z is the distance measured away from the waist of the narrow beam to the location of the object $P(x,y;z)$. $b(x,y,z)$ is the DC term and the AC term modulated at temporal frequency Ω is the temporally modulated Gaussian apodized Fresnel zone plate. Ω denotes the temporal frequency shift between the narrow Gaussian beam and the board Gaussian beam. For details of the other variables, interested readers may refer to the cited reference [13], [17]. The photo-detector PD collects the scattered light after the modulated zone plate interacts with the objects, $P(x,y;z)$, to give a current

$$\begin{aligned} i(x,y,;z;t) &\sim \int\int_A P(x',y';z)I(x(t)-x',y(t)-y';z;t)dx'dy = I(x,y,;z;t) * P(x,y;z) \\ &= b(x,y,;z) * P(x,y;z) + P(x,y;z) * C(z)\exp\left[-(\frac{x^2+y^2}{\omega(z)^2})\right] \cdot \sin\left[\frac{k_0}{2R(z)}(x^2+y^2) + \Omega t\right] \end{aligned} \tag{8}$$

where $C(z)$ is $2/\pi^2\omega_u(0)\omega_u(z)\omega_v(0)\omega_v(z)$. Note that the integration is over the photo-detector area A; $x(t)$ and $y(t)$ are determined by the xy -scanner's motion. Therefore, the convolution operation $*$ is effected by the physical scanning. The AC term can be separated by a bandpass filter (BPF) tuned to a frequency to give the signal which can be demodulated by mixing it with $\cos\Omega t$ and lowpass filtered to give a demodulated current i_d :

$$i_d(x,y;z) \sim P(x,y;z)*C(z)\exp\left[-\frac{(x^2+y^2)}{\omega(z)^2}\right] \times \sin\left[\frac{k_0}{2R(z)}(x^2+y^2)\right] \quad (9)$$

$i_d(x,y;z)$ contains the holographic information of the object being scanned. In fact it is zone plate coded information of the object. If this holographic information needs to be displayed or stored on films, a constant bias can be added to it to give a hologram of an amplitude transmittance $t(x,y;z)$

$$t(x,y,z) = b + i_d = b + P(x,y;z) * C(z)\exp\left[-\frac{(x^2+y^2)}{\omega(z)^2}\right]\cdot\sin\left[\frac{k_0}{2R(z)}(x^2+y^2)\right] \quad (10)$$

Note that the space-variant term $b(x,y;z)$ in $i(x,y;z;t)$ has been filtered out by the band-pass filter. Therefore, in computer simulations to be performed later, we will plot the scanning beam profile as

$$f_s(x,y;z) = I(x,y;z;t=0) - b(x,y;z) = C(z)\exp\left[-\frac{(x^2+y^2)}{\omega(z)^2}\right]\cdot\sin\left[\frac{k_0}{2R(z)}(x^2+y^2)\right] \quad (11)$$

To reconstruct the hologram $t(x,y;z)$ optically, we can illuminate it with a plane wave. Let us now concentrate on a point object reconstruction. We, therefore, let $P(x,y;z) = \delta(x,y)$,; located at $z = z_0$ away from the waist of the narrow beam, the point-object hologram, according to Eq. (11), is

$$t_\delta(x,y;z_0) = b + \exp(-\frac{(x^2+y^2)}{\omega(z_0)^2}) \times\sin\left[\frac{k_0}{2R(z_0)}(x^2+y^2)\right] \quad (12)$$

Upon plane wave illumination, the complex field distribution at a distance z away from the point-object hologram is given by

$$\psi(x,y;z) = t_\delta(x,y;z) * h(x,y;z) \quad (13)$$

where $h(x,y;z) = \exp(-jk_0(x^2+y^2)/2z)$ is the free space impulse response [10,11], neglecting some constants. By expanding Eq.13, we have

$$\psi(x,y;z)\sim b*\exp(-j\frac{k_0}{2z}(x^2+y^2)) + \frac{1}{2j}\exp(-\frac{(x^2+y^2)}{\omega(z_0)^2})\exp(j\frac{k_0}{2R(z_0)}(x^2+y^2))*\exp(-j\frac{k_0}{2z}(x^2+y^2)) - \frac{1}{2j}\exp(-\frac{(x^2+y^2)}{\omega(z_0)^2})\exp(-j\frac{k_0}{2R(z_0)}(x^2+y^2))*\exp(-j\frac{k_0}{2z}(x^2+y^2)) \quad (14)$$

At a distance $z = z_0$, the second term can be evaluated and gives rise to a real image reconstruction:

$$\psi_r = \pi\frac{\omega(z_0)^2}{4}\exp(-a^2\omega(z_0)^2(x^2+y^2)) \times\exp(-ja(x^2+y^2)) \quad (15)$$

where $a = k_0/2R(z_0)$. Hence, a point-object gives rise to a Gaussian distribution with its waist given by $1/a\omega(z_0)$, i.e., the resolution of the reconstructed point depends on the size of the scanning Gaussian beam, as well as the number of zones within the beam on the scanned object, i.e.,$\sim a$. The third term in Eq.15 gives a twin-image ψ_t in the $z = z_0$ plane:

$$\psi_t = \frac{\pi\omega(z_0)^2}{4(1+4a^2\omega(z_0)^4)^{\frac{1}{2}}}\exp(-\frac{a^2\omega(z_0)^2(x^2+y^2)}{1+4a^2\omega(z_0)^4}) \times\exp\left[-ja(x^2+y^2)\left(\frac{1+2a^2\omega(z_0)^4}{1+4a^2\omega(z_0)^4}\right)\right] \quad (16)$$

The first term in Eq.15 gives a constant background π/a. To find the intensity distribution on the real-image reconstruction plane, we calculate

$$P(x,y;z) = \psi(x,y;z)\psi^*(x,y;z) = |\pi/a + \psi_r + \psi_t|^2 \quad (17)$$

IV. COMPUTER SIMULATION

We now want to investigate the effect of the parameters $k_0/2R(z_0)$ and $\omega(z_0)$ on reconstruction based on the above equation. It is evident that the resolution is given by the waist of the reconstructed point and is equal to $1/[(\frac{k_0}{2R(z_0)})\omega(z_0)]$. Referring to Eq.7, the larger the scanning beam waist, $\omega(z_0)$, the better is the resolution if $k_0/2R(z_0)$ remains the same. Also, a larger number of zones within the Gaussian beam, i.e., the larger the $k_0/2R(z_0)$, will give a better resolution if the beam waist is the same. For a given $[k_0/2R(z_0)]\times\omega(z_0)$, the resolution of the reconstructed image remains the same. We choose different sizes of the Gaussian waist of the scanning beam and observe the difference in the reconstruction. In Fig.3, we plot the 1-D scanning beam profile $f_s(x;z)$, according to Eq (.6), with $\frac{R_m}{R_n}=\frac{M}{N}=3$, that is, the scanning beam R_m is three times larger than the scanning beam R_n and the number of zones in beam R_m is three times that in beam R_n . Note that the two scanning beams will gives the same resolution on image reconstruction.

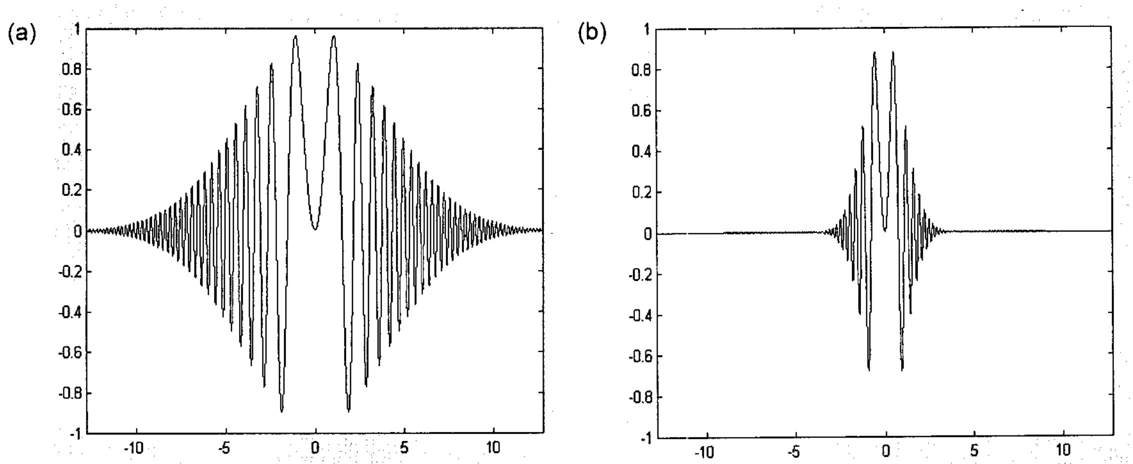

FIG. 3. (a) Scanning beam Characterized by R_n, (b) Scanning beam Characterized by R_m.

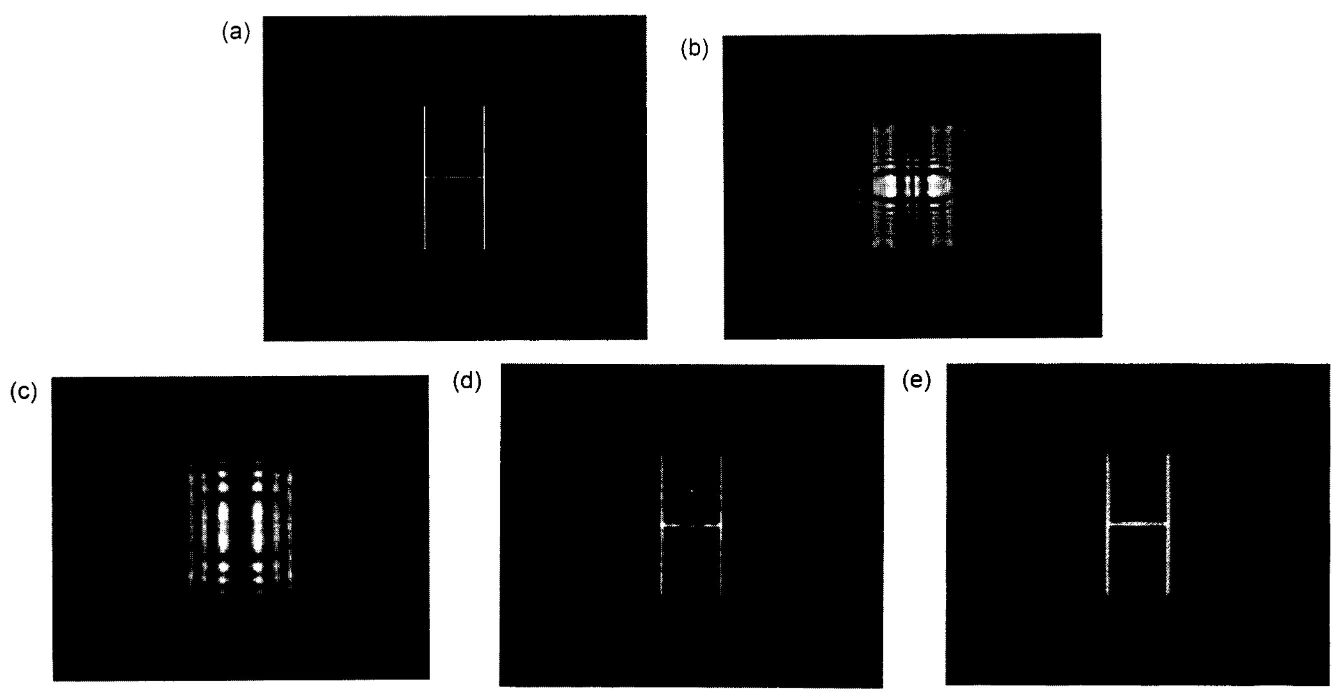

FIG. 4. (a) 2-D image of object 'H', (b) Computer-generated-hologram of the object using the scanning beam 3(a), (c) Computer-generated-hologram of the object using the scanning beam 3(b), (d) Digital reconstruction of the hologram shown in 4(b), (e) Digital reconstruction of the hologram shown in 4(c)

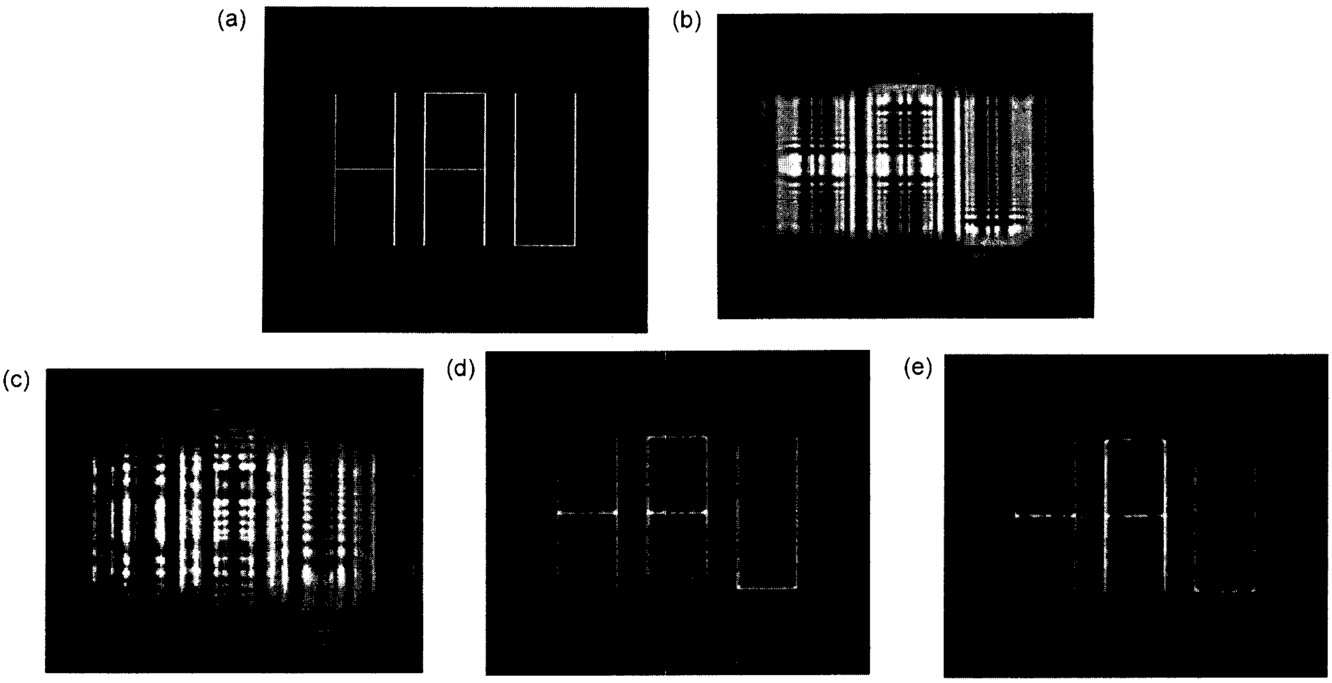

FIG. 5. (a) 2-D image of object 'HAU', (b) Computer-generated-hologram of the object using the scanning beam 3(a), (c) Computer-generated-hologram of the object using the scanning beam 3(b), (d) Digital reconstruction of the hologram shown in 5(b), (e) Digital reconstruction of the hologram shown in 5(c)

Fig. 4(a) shows the computer-generated 2-D object 'H' image. Figs. 4(b) and 4(c) display the coded image computed by small and large 2-D Gaussian apodized Fresnel zone scanning beams respectively. Figs. 4(d) and 4(e) show the real-image reconstruction when the two beams, shown in Figs. 4(d) and 4(e), respectively, are used to generate reconstructed image. Fig. 5 shows the same type of simulations as those shown in Fig. 4, but with a more complicated object. By comparing Figs. 4(d) and 4(e), we clearly distinguish the twin-image noise effect from the in-focus real image. This effect proves to be more noticeable if the object is complicated. It may be that when the size of the scanning zone plate is larger than that of the object so that the scanning beam always illuminates the entire object, we have a situation reminiscent of coherent holographic recording since the hologram is the simultaneous superposition of the individual hologram of each point within the object.

V. CONCLUDING REMARKS

We have discussed Optical Scanning Holography (OSH). The principle of OSH is based on optical heterodyning and scanning and therefore is an electro-optical hybrid system that is realtime in nature. It has been shown that by scanning an object with a temporally modulated Gaussian apodized Fresnel zone plate to acquire holographic information, the resolution achievable is directly proportional to the size of the beam and the number of zones within the beam. If the object is in the near field, and as the object become more and more complicated, it is advantageous to use a smaller beam rather than a larger beam even though the two beams may lead to the same nominal resolution on image reconstruction. One may argue that scanning with a larger beam causes twin-image noise to interact with a broader part of the image. This effect may prove to be more important if the object is contaminated with noise. It may be that when the size of the scanning zone plate is larger than that of the object

so that the scanning beam always illuminates the entire object, we have a situation reminiscent of coherent holographic recording since the hologram is the simultaneous superposition of the individual hologram of each point within the object. In contrast, if the zone plate size is much smaller than that of the object, the zone plate overlaps only a small part of the object at any instant of the scanning process, a situation reminiscent of a partially coherent recording.

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