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Pulse Code Signal Recognition using Integra-Normalizer

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Abstract – A scheme is proposed for measuring similarities between the binary pulse signals in the pulse-code modulation using the Integra-Normalizer. The Integra-Normalizer provides a better interpretation of the relationship between the pulse signals by removing redundant codes, which maps all possible observed signals to one of the codes to be received with relative similarities between each pair of compared signals. The proposed method provides better error tolerance than L^2 metric, such as Hamming distance, since the distances between pulse signals are measured not only by the distance in L^2 but also by the shape of the waveforms. This method also contains a potential that may be useful for the time-delay detection in the pulse-code modulation.

Key Words : PCM (Pulse Code Modulation), Signal space, Metric, Integra-Normalizer

1. Introduction

This paper focuses on developing a scheme that better interprets the relationship between pulse code waveforms, so that we can improve the error correction rate in the pulse-code modulation (PCM) by reducing the number of redundant codes that are not mapped into any of the source codes. Generally, the system of transmission called (binary) pulse-code modulation uses only two digits, 0 and 1, where a set of N binary digit sequences can represent $M=2^N$ number of signals which we call codes. The binary digits are represented by electrical pulses in order to transmit the codes over a communication channel. The waveform generated by PCM consists of a sequence transitions between two levels where the pulse patterns, the sequence of waveforms, are the binary waveforms that would be transmitted to the receiver.

At the receiver, however, we must be able to identify the information on the received signals. The set of pulse signals, which is a subset of the binary waveforms $M=2^N$ where an N binary digit sequence, needs to be selected at the source and recognized at the receiver by being mapped to each of the pulse codes of source. Recognition of a received signal is established by being mapped to the only one signal out of M waveforms with the minimum distance based on the metric we employ, such that the received signal is recognized even there are several bits that are different from the bits of the waveform to be recognized.

Usually, in a band-pass data transmission system, the messages in the source are mapped into a symbol every T second and the received signal at a detector is transferred to a vector receiver that maps the data into a message to be recognized. In the source, the task of transforming an incoming message m_i, $i=1,2,\cdots,M$, into a modulated wave $s_i(t)$ needs to be done by putting apart as possible as it can be. Any set of M energy signals, $s_i(t)$, is represented as a linear combinations of N orthonormal basis functions, where $N\le M$. That is to say, we may represent the given set of real-valued signals in the form

$$s_i(t)=\sum_{j=1}^{N}s_{ij}\phi_j(t),\quad t\in[0,T],\qquad i=1,2,\cdots,M$$

where the coefficients of the expansion are defined by

$$s_{i,j}=\int_0^T s_i(t)\phi_j(t)dt,\quad i,j=1,2,\cdots,M,$$

and the real-valued basis functions $\phi_1(t),\phi_2(t),\cdots,\phi_N(t)$ are orthonormal to each other, by which we mean

$$\int_0^T\phi_i(t)\phi_j(t)dt=\begin{cases}1.0 & \text{if } i=j\\ 0, & \text{if } i\neq j\end{cases}.$$

The distance between signals in a source signal space is $\|s_i-s_k\|^2=\int_0^T[s_i(t)-s_k(t)]^2dt$ for a pair of signals represented by the signal vectors $s_i(t)$ and $s_k(t)$. In the similar way, the received signals $r(t)$ defined in the

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interval $[0, T]$ is detected as one of the possible source signals using a L^2 metric usually such as

$$\|r(t)-s_i\|^2=\int_0^T [r(t)-s_i(t)]^2 dt$$

where $s_i(t)=\sum_{j=1}^{N} s_{ij}\phi_i(t),\ t\in[0,T],\ i=1,2,\cdots,M$. However, the received signal is one of the 2^N number of codes instead of being mapped into one of the source codes $s_i(t)$ where $2^N > M$ usually. Since the possible number of the received signal is much greater than the number of source signal, there is a possibility that the received signal $r(t)$ can be detected to be mapped to several source codes, while the error correction can be established using the property $2^N > M$.

Suppose that the basis functions for N dimensional signal space in both of the source and the receiver are defined as

$$\phi_i(t)=\begin{cases}1.0, & t\in[\frac{(i-1)T}{N}, \frac{(i)T}{N}]\\ 0, & elsewhere,\end{cases}$$

where $i=1, 2, 3, 4,\cdots, N$. For example when the number of source signals is three, such as $s_1(t)=\phi_1(t)$, $s_2(t)=\phi_2(t)$ and $s_3(t)=\phi_3(t)$, which are the basis functions of $N=3$, the distances $d(s_1(t), s_2(t))$, $d(s_2(t), s_3(t))$, and $d(s_1(t), s_3(t))$ are all equal to $\frac{\sqrt{2}T}{3}$. The geometrical interpretation of signals $s_1(t)$, $s_2(t)$, and $s_3(t)$ in the three-dimensional space quite agree with the geometrical interpretation of the signals defined in $[0, T]$. In this case, a received code $r(t)=[1\,1\,0]$ $[\phi_1(t)\ \phi_2(t)\ \phi_3(t)]^T$ is in the same distance from both $s_1(t)$ and $s_2(t)$ such as $d(r(t), s_1(t))=d(r(t), s_2(t))$, which implies that $r(t)$ yields a piece of useless information. The problem in this result is that a received signal is recognized as two waveforms that have two different shapes of waveforms.

In order to remove the redundancy of mapping to several codes being in the same minimum distance in L^2 metric sense, the shape of waveforms is considered such that those waveforms in a similar shape yield a high relative similarity. The weakness of using L^2 metric for measuring distance between waveforms is that the information on the waveforms may not be considered. Instead, the global difference between waveforms is used as a tool of measuring distance between waveforms. The distance in the signal space, especially when the signals considered are functions of time, may not represent the difference between signals where the distance is defined by the Euclidean metric in L^2. For instance, when the number of basis is three, the eight source signals $s_{000}, s_{001},\cdots, s_{111}$ represent precisely the geometrical relationship between s_{ijk}'s, where there are some elements in R^3 with no difference between the waveforms in L^2 metric space (For example, $s_{010}, s_{111}, s_{100}$ have the same L^2 metric distance with respect to s_{110}. This implies that the set of three different waveforms $s_{010}, s_{111}, s_{110}$ are in the same distance by being recognized as the same signal to s_{110}). As the dimension of signal space increases, the expansion for N dimensional signal space constructed with $\phi_1, \phi_2, \cdots, \phi_N$ which are orthonormal to each other, it becomes clear that the distance measured by the L^2 metric may disagree with human's intuitive understanding. When $N=8$, $s_1(t)=[1\ 1\ 1\ 0\ 0\ 0\ 0\ 1]$, $s_2(t)=[1\ 1\ 1\ 0\ 0\ 0\ 1\ 0]$, and $s_3(t)=[1\ 1\ 1\ 1\ 0\ 0\ 0\ 0]$ defined in a unit interval, have the same distance 0.25 in the L^2 metric space where two pulses are different.

Intuitively, without loss of generality, we see that $s_1(t)$ is closer to $s_2(t)$ than to $s_3(t)$ with respect to the waveform shape. This is one of the counter-examples of measuring distance defined using L^2 metric. This conventional method that measures distances between the signals, fails to measure the distance between the waveforms because the distance is defined globally without considering local information. This limits the number of waveforms to be classified, especially in the pulse code modulation. However, when we consider the shape of the waveform, the difference becomes a function of time in $t\in[0, T]$.

The information on a signal's shape is important in demodulator where a received signal is detected and recognized by comparison with another signal with respect to the time variable $t\in[0, T]$. The information on the shape of waveform and the metric defined with it, yields a better recognition than a L^2 metric. It is one of the withdraws of the least square error method that a mathematical method and human's intuitive understanding do not meet [1, 2, 3]. In this paper, human-oriented interpretation of distance between signals is added to the mathematical scheme by considering the shape of a signal using the Integra-Normalizer.

2. Integra-Normalizer : An Operator for Waveform Comparison

Without loss of generality, let X be a space of real-valued functions of bounded variation [6], defined in the unit interval I. Let a distance between two signals in X be denoted as $E_{lse}(f, g)=\int_I (f(t)-g(t))^2 dt$, which is known as the Least Square Error (LSE), that is the metric in L^2 metric space [4, 5, 7]. There are some cases that the LSE method of measuring the distance between signals does not interpret the relationship between functions as human's intuition does [1, 2, 3]. In order to remove restriction on LSE, an operator defined as the Integra-Normalizer is developed [2].

With $f(t)$ as an integrable function over the unit interval I, a monotonically increasing function can be generated by an n-times integration over I with $0 \le f(t) \le 1$. Let $g(t)$ also be an integrable function with variable t in the unit interval I. Let Φ^n be n times integration and Γ^n be the quotient $\frac{\Phi^n}{Max(\Phi^n)}$, so that, for $n=k$, $f^k = \Gamma^m(f)$, $g^k = \Gamma^n(g)$ become continuous monotonically increasing functions. Let's define Γ^n as the Integra-Normalizer.

The relative similarity is measured by $Similarity = e^{-E_{lse}(f^k, g^k)}$ for the waveforms $f, g \in X$, such that the relative similarity can be mapped into $[0, 1]$ by letting the waveform interval $T = 1.0$. One of the good properties of the Integra-Normalizer is that it interprets the relation between functions more closely to human's intuition than the least square error method does because the Integra-Normalizer measures the relative signal energy as a function of a variable, t.

The procedure of measuring distance between waveforms is illustrated in the Fig.2, where a metric is defined using the Integra-Normalizer.

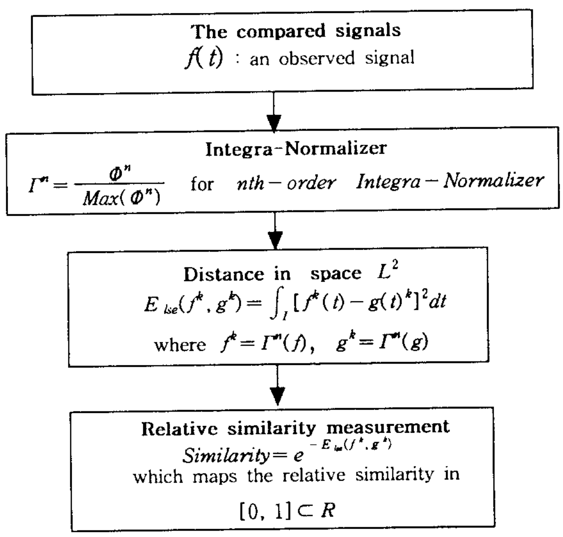

Fig. 1 Block Diagram

3. Experimental Results

Let s_1, s_2, and $s_3 \in X$ be the waveforms as shown in Fig.2. The values of $E_{lse}(s_1, s_2)$, $E_{lse}(s_2, s_3)$, and $E_{lse}(s_1, s_3)$ are equal to $0.25T$, such that the three signals of the $N=8$ dimensional signal space are located in the same distance to each other respectively.

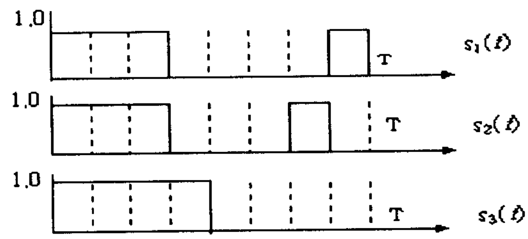

Fig. 2 A counter example of LMS (Least Square Error)

In L^2 metric space, the waveforms s_1 and s_2 are in the same category with respect to s_3, so that the degree of similarity between the two signals s_1, s_2 turns out to be the same, which is not the way human's intuition understands. This ambiguity can be avoided by the Integra-Normalizer described in Fig.1. The results obtained through the 1st order Integra-Normalizer show that s_2 is closer to s_3 than to s_1, which agrees with human's intuitive understanding when we consider the shape of wavefrms. The relation presented in Table.1 agrees with human's intuition, which is an advantage of the Integra-Normalizer to the LSE.

Table 1 Relative Similarity Measurement Using Integra-Normalizer

Similarity	S1	S2	S3
S1	1.0	0.9716	0.9793
S2	0.9716	1.0	0.9947
S3	0.9793	0.9947	1.0

As it is shown in Fig. 1, the waveforms are applied into the Integra-Normalizer such that the waveforms which are not separable in L^2 metric become separable. For example, for $N=4$, there are $2^4 = 16$ number of recognizable waveforms. However, we eliminate one waveform of no energy, $x = [0\ 0\ 0\ 0]$ from the procedure. In Table.2, four sub-groups are generated such as $G_1 = [x_1, x_2, x_3, x_4]$, $G_2 = [x_5, x_6, x_7, x_8, x_9, x_{10}]$, $G_3 = [x_{11}, x_{12}, x_{13}, x_{14}]$, $G_4 = [x_{15}]$. The values of energy contained in the sub-groups are 0.25, 0.5, 0.75, and 1.0 for G_1, G_2, G_3, and G_4 respectively.

Table 2 Relative Similarity Measurement Using Integra-ormalizer w.r.t. x=[1 0 0 0]

Waveforms	Similarity in IN	Distance in IN	Least Square Error
x_1 = [1 0 0 0]	1.0	0.0	0.0
x_2 = [0 1 0 0]	0.8645	0.1667	0.5
x_3 = [0 0 1 0]	0.6592	0.4167	0.5
x_4 = [0 0 0 1]	0.5134	0.6667	0.5
x_5 = [1 1 0 0]	0.9592	0.0417	0.25
x_6 = [1 0 1 0]	0.9006	0.1047	0.25
x_7 = [1 0 0 1]	0.8456	0.1677	0.25
x_8 = [0 1 1 0]	0.7788	0.3122	0.75
x_9 = [0 1 0 1]	0.7361	0.2500	0.75
x_{10} = [0 0 1 1]	0.6073	0.4887	0.75
x_{11} = [1 1 1 0]	0.8948	0.1111	0.5
x_{12} = [1 1 0 1]	0.8697	0.1398	0.5
x_{13} = [1 0 1 1]	0.8637	0.2222	0.5
x_{14} = [0 1 1 1]	0.8007	0.3333	1.0
x_{15} = [1 1 1 1]	0.7165	0.1875	0.75

As it is shown in Table 2, the similarity of each waveforms to $x=[10\ 0\ 0]$ is measured based on the shape of the distribution of signal's energy. The waveforms that are in the same distance in the signal space of L^2, are separated using the Integra-Normalizer. In general, for the N-dimensional signal space, we can compare 2^N number of waveforms with N-number of basis in PCM.

4. Conclusion

In this work, a metric that measures similarities between the binary pulse-code signals has been introduced, which improves the performance of pulse code signal recognition via measuring similarities between pulse-code signals (waveforms) using the Integra-ormalizer.

The Integra-Normalizer scheme also contains the property that might be applied to the time-delay detection for the pulse-code signals (waveforms). Moreover, the scheme introduced in this work can be used for further theoretical development and practical applications in communication, control, and other related areas.

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