Econ. Environ. Geol.
Vol. 33, No. 2 p.111-116, 2000.

Wavenumber Correlation Analysis of Satellite Geopotential Anomalies

Jeong Woo Kim*, Won Kyun Kim and Hye-Yun Kim*

ABSTRACT: Identifying anomaly correlations between data sets is the basis for rationalizing geopotential interpretation and theory. A procedure is presented that constitutes an effective process for identifying correlative features between the two or more geopotential data sets. Anomaly features that show direct, inverse, or no correlations between the data may be separated by applying filters in the frequency domains of the data sets. The correlation filter passes or rejects wavenumbers between co-registered data sets based on the correlation coefficient between common wavenumbers as given by the cosine of their phase difference. This study includes an example of Magsat magnetic anomaly profile that illustrates the usefulness of the procedure for extracting correlative features between the data sets.

INTRODUCTION

Interpreting geopotential field anomalies is commonly based on recognizing detailed correlations between the data sets. Correlation analysis involves both qualitative and quantitative components with deceptively simple principles that often belie the difficulties of their implementation. These difficulties commonly lead to wasted effort and a proliferation of output that can excessively complicate interpretation (e.g., von Frese *et al.*, 1982). In this study, we present a method for analyzing geopotential data sets for their correlative features.

To explore the interpretational possibilities that the correlation coefficient between data sets may represent, wavenumber correlation filters can be designed and applied in the frequency domain representations of the data sets (von Frese *et al.*, 1997a). Inverse transforming the spectra of data sets that have been modified by wavenumber correlation filtering yields predictions at all coordinates of the data sets.

To illustrate the implementation and performance of the procedure, an example is presented. The example considers the problem of separating static lithospheric anomalies from dynamic extraneous signals in satellite magnetometer observations obtained by Magsat mission. Recognizing the utility of satellites for mapping regional magnetic anomalies of the lithosphere, NASA launched the Magsat satellite as the first mission specifically designed for mapping lithospheric anomalies in 1979.

In the developments presented below, we assume the existence of two or more digital data sets in 1, 2, ..., m independent variables (i.e., orders or dimensions). The data sets are co-registered and uniformly gridded in each of their dimensions, and hence their corresponding Fourier transforms are also gridded in common wavenumber coordinates. In particular, we will consider the correlation analysis of co-registered signals X and Y, each of which is represented by a gridded m-dimensional array of amplitudes or coefficients, i.e., $X=(\chi_1, \chi_2, ..., \chi_m)$ and $Y=(y_1, y_2, ..., y_m)$, where χ_i and y_i are the gridded amplitudes.

THEORETICAL BACKGROUND

Correlation Coefficient (CC)

In general, the correlation coefficient (CC) between signals X and Y is given by

$$CC(X,Y)=\frac{\sigma^2_{x,y}}{\sqrt{\sigma^2_x\sigma^2_y}} , \qquad (1)$$

*Department of Earth Sciences and Research Institute of Geoinformatics & Geophysics, Sejong University, 98 Kunja-Dong, Kwangjin-Ku, Seoul, 143-747, Korea

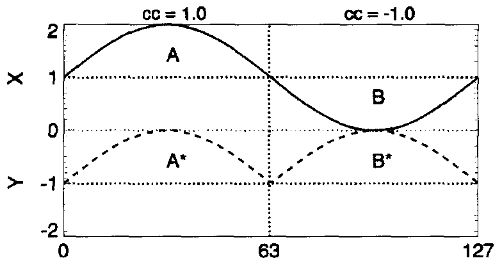

Fig. 1. Correlation Coefficient (CC) between two coegistered signals X and Y. The overall CC of X and Y is zero, but they have strong local correlations. Part A and A* are correlated directly (i.e., CC=1.0), while Part B and B* are inversely correlated (i.e., CC=-1.0).

where $\sigma^2_{x,y}$ represents covariance of the signal X and Y (Davis, 1986). The signals X and Y should be co-registered where each is represented by a gridded m-dimensional array (i.e., $X=(\chi_1, \chi_2 ..., \chi_m)$ and $Y=(y_1, y_2 ..., y_m)$, where χ_i and y_i are the gridded amplitudes). CC varies between -1 and +1, where +1 means perfect correlation of the two signals X and Y, and -1 means perfect inverse correlation. When the absolute value of CC is close to zero or perfect zero, it is generally said that there is little or no correlation between two signals. But, in fact, this is only one of many possible interpretations. For example, in Fig. 1, the features **A** and **A*** are directly correlated, while **B** and **B*** are inversely correlated, so that the overall CC is almost zero. Even though the overall CC between signals X and Y is very close to zero, there are locally significant correlative features between the two signals.

Wavenumber correlation analysis

To resolve anomaly feature correlations between co-registered data sets, a procedure is required to estimate the wavenumber correlation coefficient CC_k for each wavenumber k. Such a procedure is evident if we consider the transforms at any given wavenumber as vectors in the complex plane as shown in Fig. 2. These wavevectors can be represented in polar coordinates as follows

$$\overline{X}(k)=|\overline{X}(k)|e^{-j\theta\overline{X}(k)},\ \overline{Y}(k)=|\overline{Y}(k)|e^{-j\theta\overline{Y}(k)}, \quad (2)$$

where for the transforms corresponding to wavenumber k, $|\overline{X}(k)|$ and $|\overline{Y}(k)|$ are the amplitudes; and

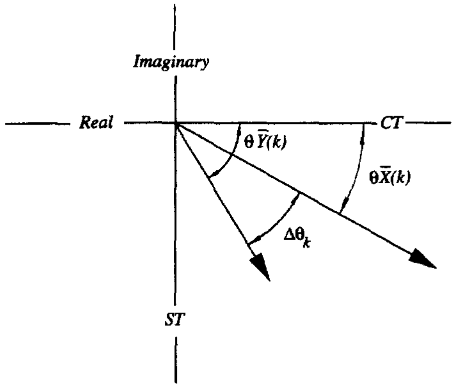

Fig. 2. Exponential representation and polar plot of wavevectors at a common wavenumber, k, for transforms rm overline X and rm overline Y of co-registered signals X and Y, respectively. The complex plane of projection is defined by the imaginary or sine transform (ST) axis and the real or cosine transform (CT) axis. The correlation coefficient between the wavevectors is given by the cosine of their phase difference [$\cos(\Delta\theta_k)$].

$\theta\overline{X}(k)$ and $\theta\overline{Y}(k)$ are the phase angles; so that $\Delta\theta_k=(\theta\overline{Y}(k)-\theta\overline{X}(k))$ is the phase difference; and $j=\sqrt{-1}$. The CC between two vectors is simply their normalized dot product so that the correlation spectrum is given by

$$CC_k=\cos(\Delta\theta_k)=\frac{\overline{X}(k)\cdot\overline{Y}(k)}{|\overline{X}(k)||\overline{Y}(k)|}. \quad (3)$$

In other words, the correlation coefficient between k-th wavenumber components of X and Y is given simply by the cosine of the shift or difference in the phase of these components.

This result has been widely used to extract static lithospheric components of the satellite magnetometer observations (Kim, 1996; Arkani-Hamed, 1988; Alsdorf *et al.*, 1994). Jones (1988) extended the use of equation (3) in the equivalent form given by Arkani-Hamed and Strangway (1986)

$$CC_k=\frac{Re[\overline{X}(k)\overline{Y}^*(k)]}{[\overline{X}(k)\overline{X}^*(k)][\overline{Y}(k)\overline{Y}^*(k)]}, \quad (4)$$

to a method for isolating and filtering individual wavenumbers directly in terms of the complete spectrum of possible correlations. This study verified the effectiveness of designing notch filters based on the correlation spectrum to isolate positively and negatively correlative features, as well as non-correlative features between data sets. Such wavenumber correlation filters (WCF) have been implemented to study the correlative features between the complex gravity and magnetic fields of Ohio (Jones, 1988; von Frese *et al.*, 1997b).

To implement the WCF, the correlation spectrum between the two signals X and Y are determined from either equations (3) or (4). Based on the correlation spectrum, notch filters are applied so that only those wavenumber components of X and Y are inverse transformed which correspond to the feature correlations desired. As with any spectral filtering application, the filtered output must be compared against the input signals to judge the reasonableness of the results and to establish the most effective values of the CC to use in any investigation.

EXAMPLE OF APPLICATION

An example is presented to demonstrate the implementation of wavenumber correlation analysis. The example demonstrates the use of wavenumber correlation filters to extract common features between a pair of parallel profiles of satellite magnetic anomaly observations. For the example, the depth to the geologic sources of the anomalies is large compared to the distance between the pair of magnetic profiles so that, with respect to the geologic anomalies, the profiles in the pair may be considered roughly coincident. Accordingly, the WCF method can be applied to extract the common features between the profiles to enhance the geologic signal-to-noise ratios of the satellite magnetic observations.

In the example, which is given by Fig. 3, we consider the problem of extracting polar lithospheric anomalies from orbital satellite magnetic data contaminated by highly dynamic external fields from auroral electrojets, field-aligned currents, large-scale ring currents, and other effects. Polar external fields are extremely variable in space and time and at present cannot be modeled with sufficient accuracy to extract the relatively weak magnetic signals of the lithosphere. Accordingly, effective separation of the polar anomaly fields is best approached as a statistical problem which exploits the coherent or static properties of lithospheric anomalies (Alsdorf *et al.*, 1994).

The example involves mostly Arctic orbital magnetic observations collected by NASA's Magsat mission from the northeastern tip of Greenland the southwestern most coast of Finland at about 330 km altitude. Here, the two subparallel orbits are separated by an average distance of slightly less than 7 km, which is small compared to the distance of these orbits from magnetic sources in the lithosphere. Accordingly, these orbits should exhibit similar lithospheric signals that may be extracted by the WCF method. Fig. 3A shows the raw data for orbits 1848 and 2833 after removal of a core field model using the adjustment procedures of Alsdorf *et al.* (1994). Non-correlative data features include auroral external field effects, measurement errors, and other non-lithospheric sources.

Analysis of the correlation spectrum (Table 1) between the signals of Fig. 3A shows that all the wavenumbers except the second one (i.e., k=2, which is marked with * in Table 1) are relatively well correlated. Accordingly, a cutoff value ($CC_k \geq 0.5$) is chosen to estimate the lithospheric anomaly components from the dusk orbits as shown in Fig. 3B. The anomaly components corresponding to the second wavenumber that are rejected by this application of WCF are given in Fig. 3C. These rejected components are partly coherent and long wavelength trends that appear to be related more readily to external field effects, induced currents in the mantle, and errors in the core field reduction than to magnetic variations of the underlying lithosphere.

In summary, this analysis suggests that the satellite magnetometer observations of the example in Fig. 3A are essentially made up of high coherent components in Fig. 3B that presumably are caused by magnetic sources of the lithosphere, and partly coherent components in Fig. 3D that probably are related to non-lithospheric effects. Accordingly, a least-squares estimate of the lithospheric anomalies in the satellite magnetic observations can be obtained by averaging point-by-point the coherent signals of Fig. 3B as shown in 3D. The differences between the coherent signals in Fig. 3B are also presented as point-by-point RMSEs in Fig. 3D to constrain interpretations of the averaged lithospheric anomaly estimates.

Of course, results such as in Fig. 3D hold only to the degree that the assumptions underlying the analysis are met. Additional correlation tests are required to help identify non-lithospheric compo-

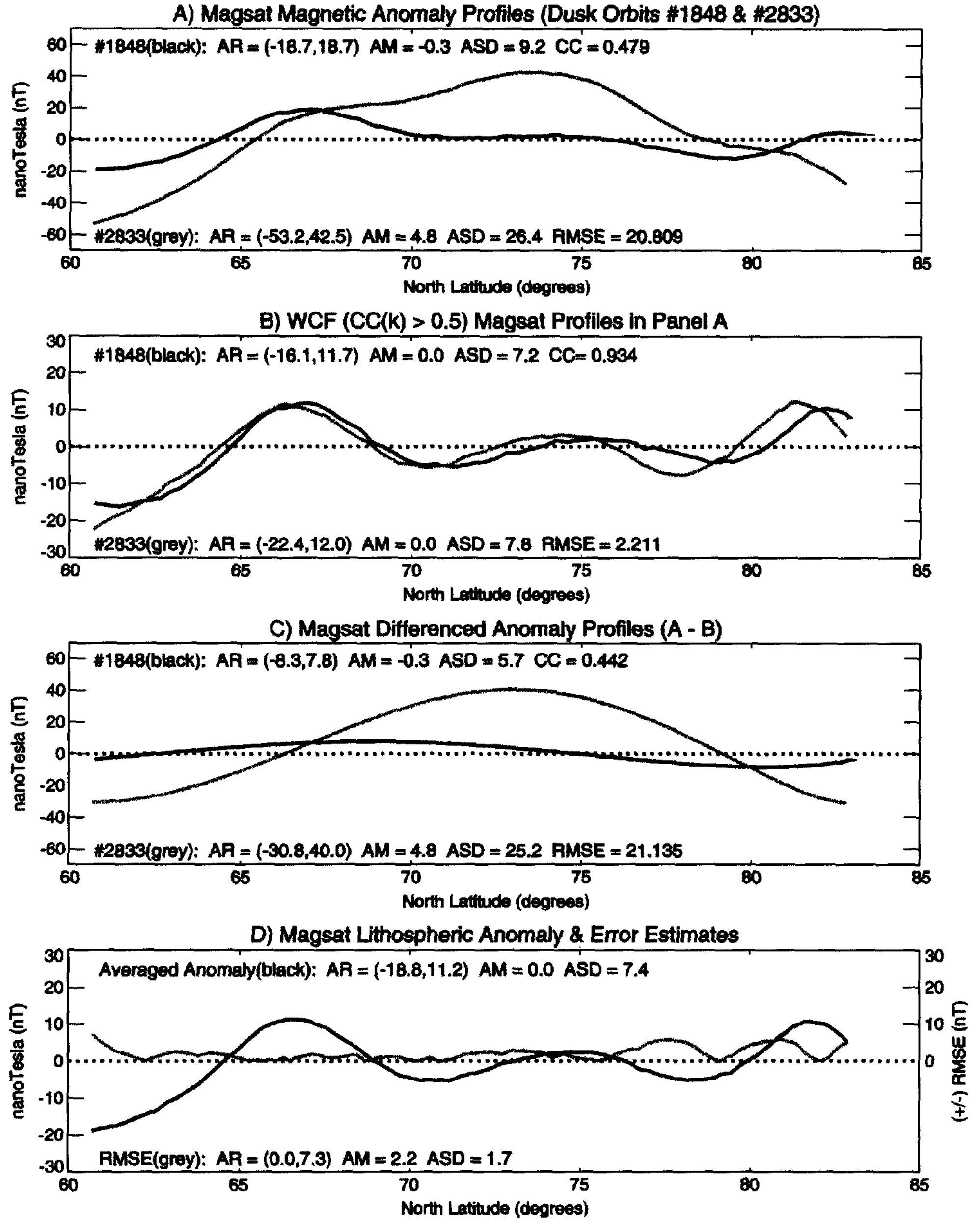

Fig. 3. Wavenumber correlation analysis for spatially adjacent dusk orbits #1848 and #2833 at about 330 km altitude across the Arctic from northern Greenland to southern Finland. Note the changes in amplitude scale between panels.

nents which are coherent between the passes. Hence, we also implements the WCF method on co-registered maps produced from the pass-to-pass correlations at different local magnetic times (e.g., dawn and dusk for the Magsat data) and at different elevations (Alsdorf *et al.*, 1994). To help identify possible non-lithospheric components with periods longer than a mission lifetime (e.g., about six months for Magsat) requires correlation tests between data of different missions. Accordingly, independent data sets from past (e.g., POGO) and on-going missions (e.g., the joint Danish/US/French

Table 1. Tabulated correlation spectrum between the first 32 wavenumbers (k) of dusk orbits #1848 and #2833. Tabulated values include the real and imaginary wavenumber components, as well as the correlation coeffi- cient between the corresponding wavenumbers. Note that only the CC of the second wavemumber (i.e., CC_2) is less than 0.5 (marked with *) and this component will be eliminated by wavenumber correlation filtering with cut-off CC of 0.5.

k	Re(X)	Im(X)	Re(Y)	Im(Y)	CC_k
1	+0.000	+0.000	+0.000	+0.000	1.000
2	-1.732	+3.626	-17.777	+0.218	0.442*
3	-1.352	-1.311	-1.221	-1.884	0.975
4	-0.454	-4.112	-2.109	-4.132	0.935
5	+0.435	-1.528	-0.126	-0.944	0.917
6	+0.000	-0.534	-0.108	-0.432	0.970
7	-0.088	-0.514	-0.228	-0.662	0.987
8	+0.014	-0.522	-0.123	-0.471	0.961
9	+0.015	-0.461	-0.009	-0.568	0.999
10	+0.026	-0.408	-0.003	-0.489	0.998
11	+0.102	-0.379	+0.011	-0.383	0.973
12	+0.065	-0.359	-0.017	-0.361	0.975
13	+0.034	-0.318	-0.001	-0.330	0.994
14	+0.042	-0.275	+0.024	-0.301	0.998
15	+0.012	-0.329	+0.016	-0.218	0.999
16	+0.057	-0.253	+0.024	-0.234	0.993
17	+0.049	-0.230	+0.033	-0.224	0.998
18	+0.052	-0.187	-0.003	-0.274	0.960
19	+0.030	-0.196	+0.063	-0.254	0.996
20	+0.056	-0.208	+0.043	-0.195	0.999
21	+0.076	-0.185	+0.043	-0.217	0.981
22	+0.048	-0.128	+0.052	-0.224	0.992
23	+0.036	-0.203	+0.047	-0.171	0.996
24	+0.040	-0.138	+0.021	-0.157	0.988
25	+0.039	-0.119	+0.018	-0.152	0.981
26	+0.057	-0.151	+0.053	-0.153	1.000
27	+0.034	-0.119	+0.025	-0.157	0.992
28	+0.036	-0.119	+0.045	-0.145	1.000
29	+0.025	-0.113	+0.032	-0.145	1.000
30	+0.050	-0.129	+0.037	-0.132	0.996
31	+0.064	-0.100	+0.048	-0.136	0.973
32	+0.026	-0.133	+0.028	-0.134	1.000

Oersted mission) are critical for obtaining further insight on the basic statistical problem of extracting lithospheric anomalies from satellite magnetometer observations.

CONCLUSIONS

A procedures is presented that constitutes an effective process for extracting correlative features between the two or more geopotential data sets. Feature correlations between data sets may be isolated by the application of correlation filters in the wavenumbers between co-registered data sets based on the correlation coefficient between common wavenumbers as given by the cosine of their phase difference. The presented Wavenumber Correlation Filtering procedure can be implemented to obtain improved estimates of the lithospheric anomaly components from satellite magnetic observations.

ACKNOWLEDGMENTS

We thank Prof. Ralph von Frese at the Ohio State University for his guidance during this study. Elements of this study were produced with support from 1999 National R & D Program of Korean Ministry of Science and Technology.

REFERENCES

Alsdorf, D.E., R.R.B. von Frese, J. Arkani-Hamed, and H. Noltimier (1994) Separation of lithospheric, external, and core components of the polar geomagnetic field at satellite altitude. J.Geophys. Res., v.99, p.4655-4667.

Arkani-Hamed, J. (1988) Differential reduction-to-the-pole of regional magnetic anomalies, Geophysics, v.53, p.1592-1600.

Arkani-Hamed, J. and D.W. Strangway (1986) Magnetic susceptibility anomalies of the lithosphere beneath Europe and Middle East. Geophysics, v.51, p.1711-1724.

Davis, J.C. (1986) Statistic and data analysis in geology. John Wiley & Sons, Inc., 464p.

Jones, M.B. (1988) Correlative analysis of the gravity and magnetic anomalies of Ohio and their geological significance. M.Sc. Thesis, Department of Geological Sciences, The Ohio State University, Columbus, Ohio, 155p.

Kim, J.W. (1996) Spectral Correlation of satellite and airborne geopotential field measurements for lithospheric analysis. Ph.D. Dissertation, Department of Geological Sciences, The Ohio State University, Columbus, Ohio, 171p.

von Frese, R.R.B., M.B. Jones, J.W. Kim, and J.-H. Kim (1997a) Analysis of anomaly correlations. Geophysics, v.62(1), p.342-351.

von Frese, R.R.B., M.B. Jones, J.W. Kim, and W.S. Li (1997b) Spectral Correlation of Magnetic and Gravity Anomalies of Ohio. Geophysics, v.62(1), p.365-380.

von Frese, R.R.B., W.J. Hinze, and L.W. Braile (1982) Regional North American gravity and magnetic anomaly correlation. Geophys. J. Roy., Astr. Soc. 69, p.745-761.

2000년 1월 3일 원고접수, 2000년 3월 28일 게재승인.

파동수대비법을 이용한 인공위성 포텐셜 이상 연구

김정우* · **김원균** · **김혜윤**

요 약 : 합리적 이론을 바탕으로 중자력포텐셜 이상을 해석하기 위해서는 그들 사이의 상관관계를 연구하는 것이 기본이다. 따라서 본 연구에서는 포텐셜 이상값 사이에서 서로 연관되어 나타나는 유사한 성분들을 효율적으로 찾아낼 수 있는 방법을 제시하였다. 이상값들은 주파수영역에서 개발된 필터를 이용하면 그들 사이의 다양한 상관관계에 따라, 즉 정이나 역으로 대비되는, 아니면 전혀 대비가 되지 않는 성분들로 따로 분류될 수 있다. 이러한 필터의 원리는 대비되는 데이터들 사이에 내재하는 같은 파장, 다시 말해 동일한 파동수를 갖는 성분들의 상관계수를 이용하는 것으로, 이는 위상차의 코사인함수로 구할 수 있다. 본 연구에서는 인공위성에서 측정된 지구자기이상값을 예로 들어, 이들 사이에서 서로 연관되어 나타나는 성분들을 어떻게 효과적으로 분류할 수 있는지를 보여준다.