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FUZZY ALMOST PRECONTINUOUS MAPPINGS

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ABSTRACT. The purpose of this paper is to introduce a new type of weakend fuzzy continuity called fuzzy almost precontinuous and investigate properties of it

1. Preliminaries

Let X be a set and I be the closed unit interval [0,1]. In [14], a fuzzy set $\mu \in X$ is defined to be a mapping $\mu : X \to \mathbf{I}$ and we will denote it by $\mu \in \mathbf{I}^X$ The fuzzy null set 0 and the fuzzy whole set $1 \in \mathbf{I}^X$ are fuzzy sets such that $0(x) = 0$ and $1(x) = 1$ for all $x \in X$, respectively. For a class $\{\lambda_\alpha \in \mathbf{I}^X : \alpha \in \Lambda\}$, the union $\bigvee_{\alpha\in\Lambda}\lambda_\alpha$ and the intersection $\bigwedge_{\alpha\in\Lambda}\lambda_\alpha$ are, respectively, defined by $\sup_{\alpha\in\Lambda}\{\lambda_\alpha\}$ and $\inf_{\alpha\in\Lambda}\{\lambda_\alpha\}$.

Let λ, $\mu \in \mathbf{I}^X$. Then λ is said to be *contained* in μ, denoted by $\lambda \leq \mu$, if $\lambda(x) \leq \mu(x)$ for every $x \in X$. The complement of λ, denoted by $1 - \lambda$, is defined by $(1 - \lambda)(x) = 1 - \lambda(x)$ for each $x \in X$. A fuzzy point x_β of X is a fuzzy set in X which is taking the value 0 for all $y \in X$ except for x and taking β at x A fuzzy point x_β of X is said to be *contained* in a fuzzy set λ, denoted by $x_\beta \in \lambda$, if $\beta \leq \lambda(x)$

DEFINITION 1.1. [9] Let λ, $\mu \in \mathbf{I}^X$. Then

(1) A fuzzy point x_β is said to be *quasi-coincident* with λ, denoted by x_β Q λ, if $\beta + \lambda(x) > 1$,

(2) λ is said to be *quasi-coincident* with μ, denoted by λ Q μ, if there exists a point $x \in X$ such that $\lambda(x) + \mu(x) > 1$.

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REMARK 1.1. It is shown in [9] that for any λ,

(1) $\mu \in \mathbf{I}^X$, $\lambda \leq \mu$ if and only if λ and $1 - \mu$ are not quasi-coincident,
(2) $x_\beta \in \lambda$ if and only if x_β is not quasi-coincident with $1 - \lambda$.

Let $f : X \to Y$ be a mapping and let $\lambda \in \mathbf{I}^X$, $\mu \in \mathbf{I}^Y$. Then $f(\lambda) \in \mathbf{I}^Y$ such that $f(\lambda)(y) = \bigvee_{x \in f^{-1}(y)} \lambda(x)$ if $f^{-1}(y) \neq \emptyset$ and 0, otherwise. And $f^{-1}(\mu) \in \mathbf{I}^X$ such that $f^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in X$.

We use in this paper the definition of fuzzy topology on a set X in the sense of [5], denote it by $\tau(X)$, and the ordered pair $(X, \tau(X))$ is called a fuzzy topological space (fts, for short). $\mu \in \tau(X)$ is called fuzzy open in X and the complement $(1 - \mu)$ is called fuzzy closed in X.

DEFINITION 1.2.

(1) $\text{Int}(\lambda) = \bigvee\{\mu : \mu \leq \lambda, \mu \in \tau(X)\}$ is called the *interior* of λ.
(2) $\text{Cl}(\lambda) = \bigwedge\{\mu : \lambda \leq \mu, (1 - \mu) \in \tau(X)\}$ is called the *closure* of λ.

Let $(X, \tau(X))$ be an fts. Then $\mu \in \mathbf{I}^X$ is called a Q-*neighborhood* (shortly, Q-*nbd*) of a fuzzy point x_β [9] (resp. pre Q-nbd [8, 12]) if there exists a $\mu \in \tau(X)$ (resp. $\lambda \in \text{FPO}(X)$) such that x_β Q $\lambda \leq \mu$.

DEFINITION 1.3. [1, 2, 10] Let x be an fts. Then $\lambda \in \mathbf{I}^X$ is said to be:

(1) fuzzy regular open if $\lambda = \text{Int}(\text{Cl}(\lambda))$,
(2) fuzzy feebly open ($\equiv \alpha$-open)if $\lambda \leq \text{Int}(\text{Cl}(\text{Int}(\lambda)))$.
(3) fuzzy preopen if $\lambda \leq \text{Int}(\text{Cl}(\lambda))$.
(4) fuzzy semi open if $\lambda \leq \text{Cl}(\text{Int}(\lambda))$,
(5) fuzzy regular closed if $\lambda = \text{Cl}(\text{Int}(\lambda))$,
(6) fuzzy feebly closed ($\equiv \alpha$-closed)if $\text{Cl}(\text{Int}(\text{Cl}(\lambda))) \leq \lambda$,
(7) fuzzy preclosed if $\text{Cl}(\text{Int}(\lambda) \leq \lambda$,
(8) fuzzy semi cloesd if $\text{Int}(\text{Cl}(\lambda) \leq \lambda$.

In this paper, we will denote the family of all fuzzy open (resp. fuzzy regular open, fuzzy α-open, fuzzy pre-open, fuzzy semi-open and fuzzy regular closed) sets in an fts X by $\tau(X)$ (resp. FRO(X), FαO(X), FPO(X), FSO(X) and FRC(X)).

REMARK 1.2. For an fts $(X, \tau(X))$, the following holds:
$\mathrm{FRO}(X) \subset \tau(X) \subset \mathrm{F}\alpha\mathrm{O}(X) \subset \mathrm{FPO}(X)$ (or, $\mathrm{FSO}(X)$)

DEFINITION 1.4. [1, 2, 5, 10, 12] Let $\lambda \in \mathbf{I}^X$. Then

(1) $\alpha\mathrm{Int}(\lambda) = \bigvee\{\mu : \mu \leq \lambda, \mu \in \mathrm{F}\alpha\mathrm{O}(X)\}$ is called the α-interior of λ (feeble interior $f\mathrm{Int}(\lambda)$),
(2) $p\mathrm{Int}(\lambda) = \bigvee\{\mu : \mu \leq \lambda, \mu \in \mathrm{FPO}(X)\}$ is called the preinterior of λ,
(3) $s\mathrm{Int}(\lambda) = \bigvee\{\mu : \mu \leq \lambda, \mu \in \mathrm{FSO}(X)\}$ is called the semi interior of λ,
(4) $\alpha\mathrm{Cl}(\lambda) = \bigwedge\{\mu : \lambda \leq \mu, (1-\mu) \in \mathrm{F}\alpha\mathrm{O}(X)\}$ is called α-closure of λ (feeble closure $f\mathrm{Cl}(\lambda)$).
(5) (4) $p\mathrm{Cl}(\lambda) = \bigwedge\{\mu : \lambda \leq \mu, (1-\mu) \in \mathrm{FPO}(X)\}$ is called preclosure of λ.
(6) $s\mathrm{Cl}(\lambda) = \bigwedge\{\mu : \lambda \leq \mu, (1-\mu) \in \mathrm{FSO}(X)\}$ is called semi closure of λ.

REMARK 1.3. Let $\lambda, \mu \in \mathbf{I}^X$. Then

(1) $\lambda \in \mathrm{FPO}(X)$ if and only if for every fuzzy points $x_\beta \in \lambda$, there exists $\delta \in \mathrm{FPO}(X)$ such that $x_\beta \in \delta \leq \lambda$ [11],
(2) $\lambda \in \mathrm{FPO}(X)$ if and only if $\lambda = p\mathrm{Int}(\lambda)$ [12],
(3) $1 - \lambda \in \mathrm{FPO}(X)$ if and only if $\lambda = p\mathrm{Cl}(\lambda)$ [12].

DEFINITION 1.5. [1, 2, 4, 5, 8, 10, 11] A mapping $f : X \to Y$ is said to be:

(1) fuzzy continuous if $f^{-1}(\lambda) \in \tau(X)$ for each $\lambda \in \tau(Y)$
(2) fuzzy feebly continuous (fuzzy α-continuous, fuzzy strongly semi continuous) if $f^{-1}(\lambda) \in \mathrm{F}\alpha\mathrm{O}(X)$ for each $\lambda \in \tau(Y)$.
(3) fuzzy precontinuous if $f^{-1}(\lambda) \in \mathrm{FPO}(X)$ for each $\lambda \in \tau(Y)$.
(4) fuzzy M-precontinuous (fuzzy pre-irresolute) if $f^{-1}(\lambda) \in \mathrm{FPO}(X)$ for each $\lambda \in \mathrm{FPO}(Y)$,
(5) fuzzy almost continuous if $f^{-1}(\lambda) \in \tau(X)$ for each $\lambda \in \mathrm{FRO}(Y)$.

REMARK 1.4. In the above Definition 1.5,

(1) fuzzy M-precontinuity of [7] and fuzzy pre-irresoluteness of [8] are the same mappings on any fts, and

(2) fuzzy feeble continuity of [2] and fuzzy strongly semicontinuity of [10] are also the same mappings and in [4] it was also renamed by fuzzy α-continuity. So, from now on, we will call it a fuzzy α-continuous mapping.

2. Fuzzy almost precontinuous mappings

DEFINITION 2.1. Let X and Y be fts's. A mapping $f : X \to Y$ is said to be *fuzzy almost precontinuous* (written as $f.a.p.C.$) if $f^{-1}(\lambda) \in \mathrm{FPO}(X)$ for each $\lambda \in \mathrm{FRO}(Y)$.

REMARK 2.1. Every fuzzy precontinuous and fuzzy almost continuous mapping are $f.a.p.C.$ But the converses may not be true, as shown by the following examples.

EXAMPLE 2.1. Let $X = \{a, b\}$, $Y = \{x, y\}$ and let $\lambda \in \mathbf{I}^X$, $\mu \in \mathbf{I}^Y$ such that $\lambda(a) = 0.4$, $\lambda(b) = 0.3$, and $\mu(x) = 0.5$, $\mu(y) = 0.6$, and let $\tau(X) = \{0, \lambda, 1\}$ and $\tau(Y) = \{0, \mu, 1\}$. Define $f : (X, \tau(X)) \to (Y, \tau(Y))$ by $f(a) = x$ and $f(b) = y$. Then f is $f.a.p.C.$, but not fuzzy precontinuous

EXAMPLE 2.2. Let $X = \{\mathrm{a, b}\}$, $Y = \{\mathrm{x, y}\}$ and let $\lambda \in \mathbf{I}^X$, $\mu \in \mathbf{I}^Y$ such that $\lambda(a) = 0.5$, $\lambda(b) = 0.4$, and $\mu(x) = 0.4$, $\mu(y) = 0.4$, and let $\tau(X) = \{0, \lambda, 1\}$ and $\tau(Y) = \{0, \mu, 1\}$. Define $g : (X, \tau(X)) \to (Y, \tau(Y))$ by $g(a) = x$ and $g(b) = y$. Then g is $f.a.p.C.$, but not fuzzy almost continuous.

In general, the composition of $f.a.p.C.$ mappings may be not $f.a.p.C.$, as shown by the following.

EXAMPLE 2.3. Let $f : (X, \tau(X)) \to (Y, \tau(Y))$ be the mapping defined in Example 2.1, then f is $f.a.p.C.$ Let $Z = \{v, w\}$ and $\eta \in \mathbf{I}^Z$ such that $\eta(x) = 0.4$, $\eta(y) = 0.4$ and let $\tau(Z) = \{0, \eta, 1\}$. Define $g : (Y, \tau(Y)) \to (Z, \tau(Z))$ by $g(x) = v$ and $g(y) = w$. Then g is also $f.a.p.C.$ However, the composition $g \circ f : (X, \tau(X)) \to (Z, \tau(Z))$ is not $f.a.p.C.$, because $\mathrm{Int}(\mathrm{Cl}((g \circ f)^{-1}(\eta))) < (g \circ f)^{-1}(\eta)$ and thus $(g \circ f)^{-1}(\eta) \notin \mathrm{FPO}(X)$ for $\eta \in \mathrm{FRO}(Z)$. Note that f is not fuzzy precontinuous.

THEOREM 2.1. *Let $f : X \to Y$ is fuzzy precontinuous and $g : Y \to Z$ is f.a.p.C., then $g \circ f : X \to Z$ is f.a.p.C.*

PROOF. Let $\lambda \in \text{FRO}(Z)$. Then $g^{-1}(\lambda) \in \tau(X)$, because g is $f.a.p.C$. Since f is fuzzy precontinuous, $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda) \in \text{FPO}(X)$. Hence $g \circ f$ is $f.a.p.C$.

THEOREM 2.2. *Let $f : X \to Y$ is fuzzy M-precontinuous and $g : Y \to Z$ is f.a.p.C., then $g \circ f : X \to Z$ is f.a.p.C*

PROOF. Let $\lambda \in \text{FRO}(Z)$, then $f^{-1}(\lambda) \in \text{FPO}(Y)$. Thus $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda) \in \text{FPO}(X)$. Thus $g \circ f$ is $f\ a.p.C$.

THEOREM 2.3. *Let f be a mapping from an fts X to an fts Y, then the following are equivalent:*

(1) *f is f.a.p.C.*
(2) *$(1 - f^{-1}(\mu)) \in FPO(X)$ for each $\mu \in FRC(Y)$.*
(3) *$f^{-1}(\lambda) \leq pInt(f^{-1}(Int(Cl(\lambda))))$ for each $\lambda \in \tau(Y)$.*
(4) *$pCl(f^{-1}(Cl(Int(\mu)))) \leq f^{-1}(\mu)$ for each $(1 - \mu) \in \tau(Y)$.*
(5) *for each fuzzy point x_β of X and $\mu \in FRO(Y)$ containing $f(x_\beta)$, there exists $\lambda \in FRO(X)$ such that $x_\beta \in \lambda$ and $\lambda \leq f^{-1}(\mu)$.*
(6) *for each fuzzy point x_β of X and $\mu \in FRO(Y)$ containing $f(x_\beta)$, there exists $\lambda \in FPO(X)$ such that $x_\beta \in \lambda$ and $f(\lambda) \leq \mu$.*
(7) *for each fuzzy point x_β of X and $\mu \in FRO(Y)$ with $f(x_\beta)\ Q\ \mu$, there exists $\lambda \in FPO(X)$ such that $x_\beta\ Q\ \lambda$ and $f(\lambda) \leq \mu$.*
(8) *for each fuzzy point x_β of X and $\mu \in FRO(Y)$ with $f(x_\beta)\ Q\ \mu$, there exists $\lambda \in FPO(X)$ such that $x_\beta\ Q\ \lambda$ and $\lambda \leq f^{-1}(\mu)$.*

PROOF. (1) $\Leftrightarrow$ (2). The proofs are obvious.

(1) $\Rightarrow$ (3). Let $\lambda \in \tau(Y)$, then $\lambda \leq \text{Int}(\text{cl}(\lambda))$ and hence $f^{-1}(\lambda) \leq f^{-1}(\text{Int}(\text{Cl}(\lambda)))$. From [1. Theorem 5.6-(b)]. $\text{Int}(\text{Cl}(\lambda)) \in \text{FRO}(Y)$ Thus $f^{-1}(\text{Int}(\text{Cl}(\lambda))) \in \text{FPO}(X)$ since f is $f.a\ p.C$. So. $f^{-1}(\lambda) \leq f^{-1}(\text{Int}(\text{cl}(\lambda))) = \text{pInt}(f^{-1}(\text{Int}(\text{Cl}(\lambda))))$ from Remark 1.3.

(3) $\Rightarrow$ (1). Let $\lambda \in \text{FRO}(Y)$, then $f^{-1}(\lambda) \leq \text{pInt}(f^{-1}(\text{Int}(\text{Cl}(\lambda)))) = \text{pInt}(f^{-1}(\lambda))$ by (3). So $f^{-1}(\lambda) = \text{pInt}(f^{-1}(\lambda))$. Hence $f^{-1}(\lambda) \in \text{FPO}(X)$.

(2) $\Rightarrow$ (4): Let $(1 - \mu) \in \tau(Y)$, then $\text{Cl}(\mu) = \mu$. Thus $\text{Cl}(\text{Int}(\mu)) \leq \mu$ and so $f^{-1}(\text{Cl}(\text{Int}(\mu))) \leq f^{-1}(\mu)$. From [1, Theorem 5.6-(a)], $\text{Cl}(\text{Int}(\mu)) \in \text{FRC}(Y)$. So $(1 - f^{-1}(\text{ClInt}\mu) \in \text{FPO}(X)$ by (2). Thus $p\text{Cl}(f^{-1}(\text{ClInt}\mu)) = f^{-1}(\text{ClInt}(\mu)) \leq f^{-1}(\mu)$.

(4) $\Rightarrow$ (2): Let $\mu \in \text{FRC}(Y)$, then $p\text{Cl}(f^{-1}(\mu)) = p\text{Cl}(f^{-1}(\text{ClInt}(\mu))) \leq f^{-1}(\mu)$. Thus $p\text{Cl}(f^{-1}(\mu)) = f^{-1}(\mu)$. So $(1-f^{-1}(\mu)) \in \text{FPO}(X)$.

(1) $\Rightarrow$ (5): Let x_β be a fuzzy point of X and $\mu \in \text{FRO}(Y)$ with $f(x_\beta) \in \mu$. Putting $\lambda = f^{-1}(\mu)$, then by (1) $\lambda \in \text{FPO}(X)$, $x_\beta \in \lambda$ and $\lambda \leq f^{-1}(\mu)$.

(5) $\Rightarrow$ (6). Let x_β be a fuzzy point of X and $\mu \in \text{FRO}(Y)$ containing $f(x_\beta)$. Then by (5) there exists $\lambda \in \text{FPO}(X)$ such that $x_\beta \in \lambda$ and $\lambda \leq f^{-1}(\mu)$. So $x_\beta \in \lambda$, $f(\lambda) \leq f(f^{-1}(\mu)) \leq \mu$.

(6) $\Rightarrow$ (1): Let $\mu \in \text{FRO}(Y)$ and let x_β be a fuzzy point of X such that $x_\beta \in f^{-1}(\mu)$. Then $f(x_\beta) \in f(f^{-1}(\mu)) \leq \mu$. So by (6) there exists $\lambda \in \text{FPO}(X)$ such that $x_\beta \in \lambda$ and $f(\lambda) \leq \mu$, that is, $x_\beta \in \lambda \leq f^{-1}(\mu)$. Thus by Remark 1.3-(1), $f^{-1}(\mu) \in \text{FPO}(X)$. So f is *f.a.p.C*

(1) $\Rightarrow$ (7): Let x_β be a fuzzy point of X and $\mu \in \text{FRO}(Y)$ such that $f(x_\beta)$ Q μ. Then $f^{-1}(\mu) \in \text{FPO}(X)$ by (1) and x_β Q $f^{-1}(\mu)$. Taking $\lambda = f^{-1}(\mu)$, then $\lambda \in \text{FPO}(X)$, x_β Q λ and $f(\lambda) = f(f^{-1}(\mu)) \leq \mu$.

(7) $\Rightarrow$ (8): Let x_β be a fuzzy point of X and $\mu \in \text{FRO}(Y)$ such that $f(x_\beta)$ Q μ, then by (7) there exists $\lambda \in \text{FPO}(X)$ such that x_β Q λ and $f(\lambda) \leq \mu$. Thus we have $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(\mu)$.

(8) $\Rightarrow$ (1): Let $\mu \in \text{FRO}(Y)$. To show $f^{-1}(\mu) \in \text{FPO}(X)$ we use Remark 1.3-(1). let x_β be a fuzzy point of X such that $x_\beta \in f^{-1}(\mu)$. Then $f(x_\beta) \in \mu$. Choosing the fuzzy point $x_{(1-\beta)}$, then $f(x_{(1-\beta)}$ Q μ. So by (8) there exists $\delta \in \text{FPO}(X)$ such that $x_{(1-\beta)}$ Q δ and $f(\delta) \leq \mu$. Now $x_{(1-\beta)}$ Q δ implies that $x_{(1-\beta)}(x) + \lambda(x) = 1 - \beta + \mu(x) > 1$. It follows that $x_\beta \in \lambda \leq f^{-1}(\mu)$. So by Remark 1.3-(1). $f^{-1}(\mu) \in \text{FPO}(X)$. Thus f is *f.a.p.C.*

DEFINITION 2.2. [6] An fts X said to be *fuzzy semi regular* if for each $\lambda \in \tau(X)$ and for each fuzzy point x_β of X with x_β Q λ, there exists $\mu \in \tau(X)$ such that x_β Q μ and $\mu \leq \text{Int}(\text{Cl}(\mu)) \leq \lambda$.

THEOREM 2.4. *Let $f : X \to Y$ be a mapping from an fts X to a fuzzy semi regular space Y, then f is f.a p.C.s if and only if f is fuzzy precontinuous.*

PROOF. Necessity: Let x_β be a fuzzy point of X and $\lambda \in \tau(Y)$ such that $f(x_\beta)$ Q λ. Since Y is fuzzy semi regular, there exists $\mu \in \tau(Y)$ such that $f(x_\beta)$ Q μ and $\mu \leq \text{Int}(\text{Cl}(\mu)) \leq \lambda$. Since $\text{Int}(\text{Cl}(\mu)) \in \text{FRO}(Y)$ and f is $f.a.p.C.$, by Theorem 2.3-(7) there exists $\mu_1 \in \text{FPO}(X)$ such that x_β Q μ_1 and $f(\mu_1) \leq \text{Int}(\text{Cl}(\mu))$. Thus $\mu_1 \in \text{FPO}(X)$ such that x_β Q μ and $f(\mu_1) \leq \lambda$. So by [11, Theorem 3.4] f is fuzzy precontinuous.

Sufficiency is obvious and is thus omitted.

THEOREM 2.5. *Let f be a mapping from an fts X to an fts Y. If the graph mapping $G_f : X \to X \times Y$ of f is f.a.p.C., then f is f.a.p.C.*

PROOF. Let $\mu \in \text{FRO}(Y)$, then $f^{-1}(\mu) = 1 \bigwedge f^{-1}(\mu) = (G_f)^{-1}(1 \times \mu)$. Since $1 \times \mu = 1 \times \text{Int}(\text{Cl}(\mu)) = \text{Int}(1 \times \text{Cl}(\mu)) = \text{Int}(\text{Cl}(1 \times \mu))$, $1 \times \mu \in \text{FRO}(X \times Y)$. Since G_f is $f.a.p.C.$, $f^{-1}(\mu) = (G_f)^{-1}(1 \times \mu) \in \text{FPO}(X)$. Hence f is $f.a.p.C.$

From the above, We have the following implication diagram:

$$\begin{array}{ccccc} & & f.m.p.C & & \\ & & \Downarrow & & \\ f.\alpha.C. & \Rightarrow & f.p.C & \Rightarrow & f.a.p.C. \\ \Uparrow & & & & \Uparrow \\ f.C & & \Rightarrow & & f.a.C. \end{array}$$

where $f.m.p.C.$, $f.\alpha.C.$, $f.p.C.$, $f.a.p.C.$, $f.C.$ and $f.a.C.$ denote fuzzy M-precontinuous [7, 8], fuzzy α-continuous [2,4,10], fuzzy precontinuous [10], *fuzzy almost precontinuous*, fuzzy continuous [5] and fuzzy almost continuous [1].

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