지그재그 모델에 의한 복합샌드위치평판의 해석

Analysis of Sandwich Plates with Composite Facings based on Zig-Zag Models

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요　　약 : 본 연구는 지그재그모델에 근거하여 두꺼운 금속, 폴리머 복합재료 면재의 전단변형 효과를 고려해서 샌드위치평판의 휨해석 지배방정식을 유도하였으며, 판의 네 변은 단순지지 되어 있다고 가정하였다. 역대칭 적층면재의 화이버 각도와 면재의 적층수를 변화시켜가며 휨해석의 결과를 나타내었다. 해석방법의 정확성을 위해 일반 적층판이론의 값과 비교를 하여 검증을 하였다. 그 결과 적층판이론으로 구한 값보다 처짐에 있어서 더 큰 값을 나타냄을 알 수 있었는데, 그 이유로는 본 해석방법이 면재, 심재 모두 휨강성과 전단변형을 고려하였기 때문이라 사료된다. 강재, 폴리머 복합면재를 갖는 샌드위치평판을 설계할 때 제시된 정보를 유용하게 이용될 수 있을 것이다.

ABSTRACT : This study presents a governing equations of bending behavior of sandwich plates with thick metal, polymer composite facings. Based on zig-zag models for through thickness deformations, the transverse shear deformation of composite facings is included. All edges of plate are assumed to be simply supported. Results of the bending analysis under lateral loads are presented for the influence of various lay up sequences of antisymmetric angle-ply laminated facings. The accuracy of the approach is ascertained by comparing solutions from the sandwich plates theory with composite facings to the laminated plates theory. Since the present analysis considers the bending stiffness of the core and also the transverse shear deformations of the laminated facings, the proposed method showed higher than that calculated according to the general laminated plates theory. The information presented might be useful to design sandwich plates structure with metal, polymer matrix composite facings.

핵 심 용 어 : 복합면재, 지그재그모델, 샌드위치평판, 면재, 심재, 전단변형, 휨강성

KEYWORDS : composite facings, zig-zag models, sandwich plates, facings, core, shear deformations, bending stiffness

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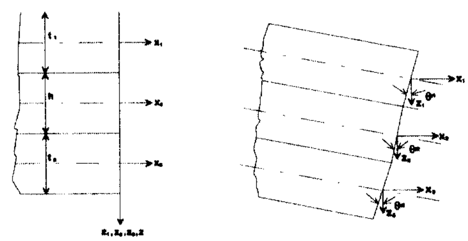
1. INTRODUCTION

The sandwich plate is constructed by combining two pieces of plate with high stiffness and one light one with a thick core material. The progress in material science and production techniques has led to wide application in the manufacturing of sandwich plate structures. Much research concerning the sandwich structure has been conducted since 1940(6). In the history of theoretical analysis, the face layers of the sandwich plate were first assumed to be membranes with no bending stiffness by Reissner(8), and the governing equation(7) was derived by him. For the face plate, the classical plate theory ignored the strain of the core in the direction of the thickness in order to derive the governing equation. The effect of transverse shear strain was further considered by Whitney(8) and Pagano(9) by employing the Mindlin Theorem. Owing to the existence of a vast literature on isotropic facings sandwich plates, this study is limited to sandwich plates having composite facings materials. Recently there has been interested in the analysis of sandwich plates having generally metal, polymer laminated facings. A series of such analyses using the energy method was conducted by Rao and Kaeser and Rao(7). However, they only considered shear deformation of the core. The objective of this study is to present a formulation of governing equation for a general sandwich plates with laminated facings including individual effects of transverse shear deformation and bending stiffness. The obvious choice to model sandwich plates with thick composite facings is a zig-zag deformation as was used by Allen(10).

The present model is a development of the Allen zig-zag model(10). By including transverse shear deformations of the composite facings, current method will be capable of analyzing thick laminated plates, general anisotropic sandwich plates. The results of the present model will be compared with analytical solution.

2. The DEFORMATIONS AND STRAINS

The deformations through the thickness of sandwich plates with thick composite facings using the zig-zag model(10) are shown in Fig.1, where $(x_1, z_1), (x_2, z_2), (x_3, z_3)$ are local coordinates of the upper face, core and lower face, respectively. From Fig.1, the displacement of the upper face, core and lower face can be expressed as follows:



(a) Geometry of a sandwich plate
(b) Deformations through the sandwich thickness with shear deformation by the zig-zag models(10)

Fig.1 Cross sections of assumed deformation of the sandwich plates with composite facings

For the upper face:

In the x-direction

$$u_1(x,y,z) = u_o(x,y) + \frac{h}{2}\theta_{x2} + \frac{t_1}{2}\theta_{x1} - z_1\theta_{x1} \tag{1.a-1}$$

and in the y-direction,

$$v_1(x,y,z) = v_o(x,y) + \frac{h}{2}\theta_{y2} + \frac{t_1}{2}\theta_{y1} - z_1\theta_{y1} \tag{1.a-2}$$

and in the z-direction

$$w_1(x,y,z) = w_o(x,y) \tag{1.a-3}$$

The same method can be used to calculate deformations in the core:

$$u_2(x,y,z) = u_o(x,y) - z_2\theta_{x2} \tag{1.b-1}$$

$$v_2(x,y,z) = v_o(x,y) - z_2\theta_{y2} \tag{1.b-2}$$

$$w_2(x,y,z) = w_o(x,y) \tag{1.b-3}$$

For the lower face:

$$u_3(x,y,z) = u_o(x,y) - \frac{h}{2}\theta_{x2} + \frac{t_2}{2}\theta_{x3} - z_3\theta_{x3} \tag{1.c-1}$$

$$v_3(x,y,z) = v_o(x,y) - \frac{h}{2}\theta_{y2} + \frac{t_2}{2}\theta_{y3} - z_3\theta_{y3} \tag{1.c-2}$$

$$w_3(x,y,z) = w_o(x,y) \tag{1.c-3}$$

where u, v, w are the displacement of the midsurface in the x,y,z axes, and θ_x, θ_y are the rotation angles of the xy and yz-plane caused by flexure and t, h are the thickness of composite facings and core, while the subscripts 1,2,3 denote upper face, core, and lower face, respectively.

The strains in terms of the displacements are given by the usual expressions:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x}; & \varepsilon_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \varepsilon_y &= \frac{\partial v}{\partial y}; & \varepsilon_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \varepsilon_z &= \frac{\partial u}{\partial x} = 0; & \varepsilon_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{aligned} \tag{2}$$

giving for the upper face

$$\begin{aligned} \varepsilon_{x1} &= \varepsilon_x^o + \frac{h}{2}\frac{\partial\theta_{x2}}{\partial x} + \frac{t_1}{2}\frac{\partial\theta_{x1}}{\partial x} - z_1\frac{\partial\theta_{x1}}{\partial x} \\ \varepsilon_{y1} &= \varepsilon_y^o + \frac{h}{2}\frac{\partial\theta_{y2}}{\partial y} + \frac{t_1}{2}\frac{\partial\theta_{y1}}{\partial y} - z_1\frac{\partial\theta_{y1}}{\partial y} \\ \gamma_{xy1} &= \gamma_{xy}^o + \frac{h}{2}\left(\frac{\partial\theta_{x2}}{\partial y} + \frac{\partial\theta_{y2}}{\partial x}\right) + \frac{t_1}{2}\left(\frac{\partial\theta_{x1}}{\partial y} + \frac{\partial\theta_{y1}}{\partial x}\right) \\ &\quad - z_1\left(\frac{\partial\theta_{x1}}{\partial y} + \frac{\partial\theta_{y1}}{\partial x}\right) \\ \gamma_{xz1} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} - \theta_{x1} \\ \gamma_{yz1} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\partial w}{\partial y} - \theta_{y1} \end{aligned} \tag{3}$$

Similar expressions are obtained for the lower face and core.

We assume that Hooke's Law is valid and the constitutive equations are given by:

$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \gamma_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{pmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{54} & \overline{Q}_{55} \end{bmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{pmatrix} \tag{4}$$

in which $\overline{Q}$ terms are the usual stiffness coefficients.

Integrating the stresses through the thickness of each component, we get the inplane forces and moment resultants for the upper face, core, and lower face in the form:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}_i = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^o_{x\,i} + \varepsilon^{mb}_i \\ \varepsilon^o_{y\,i} + \varepsilon^{mb}_i \\ \varepsilon^o_{xy\,i} + \varepsilon^{mb}_i \\ \chi^o_{x\,i} \\ \chi^o_{y\,i} \\ \chi^o_{xy\,i} \end{Bmatrix}_i \quad (5)$$

The shear stress resultants are given by:

$$\begin{Bmatrix} N_{xy} \\ N_{yz} \end{Bmatrix} = \begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \begin{bmatrix} A^{(i)}_{44} & A^{(i)}_{45} \\ A^{(i)}_{54} & A^{(i)}_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}_i = (A^{(i)}_{kl}) \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}_i \quad (6)$$

where [A] is the extensional stiffness matrix, [B] is the extensional-bending coupling stiffness matrix, [D] is the bending stiffness matrix.

$$A^{(i)}_{ij} = A^{(1)}_{ij} + A^{(2)}_{ij} + A^{(3)}_{ij} = \sum_{n=1}^{N} \int_{z_n}^{z_{n+1}} \overline{Q_{m}}_{ij}\, dz$$

$$B^{(i)}_{ij} = B^{(1)}_{ij} + B^{(2)}_{ij} + B^{(3)}_{ij} = \sum_{n=1}^{N} \int_{z_n}^{z_{n+1}} z\, \overline{Q_{m}}_{ij}\, dz$$

$$D^{(i)}_{ij} = D^{(1)}_{ij} + D^{(2)}_{ij} + D^{(3)}_{ij} = \sum_{n=1}^{N} \int_{z_n}^{z_{n+1}} z^2\, \overline{Q_{m}}_{ij}\, dz$$

$$(i,j=1,2,6)$$

$$A^{(i)}_{kl} = A^{(1)}_{kl} + A^{(2)}_{kl} + A^{(3)}_{kl} = K_i K_j \sum_{n=1}^{N} \int_{z_n}^{z_{n+1}} \overline{Q_{m}}_{ij}\, dz$$

$$(k,l=4,5) \quad (7)$$

where 1, 2, and 3 refer to upper face, core and lower face, respectively.

The strain energies of the upper face, core, and lower face are given by:

$$U_{fbi} = \frac{1}{2} \int\int_A [\, (\varepsilon^{mb} + \varepsilon^o)_i{}^T [A^{(i)}] (\varepsilon^{mb} + \varepsilon^o)_i + 2(\varepsilon^{mb} + \varepsilon^o)_i{}^T [B^{(i)}] (K)_i + (K)_i{}^T [D^{(i)}] (K)_i] dxdy \quad (8)$$

where i=1,2,3 refers to upper face, core, and lower face, respectively.

The corresponding shear strain energies are:

$$U_{fsi} = \frac{1}{2} \int\int_A (\varepsilon^s)_i{}^T [A^{(i)}{}_{kl}]_i (\varepsilon^s)_i\, dxdy \quad (9)$$

where U_{fbi} and U_{fsi} are bending and shear energies of upper face, core, and lower face. Substituting eqns (2) and (3) into eqns (8) and (9) we get a long expression for total strain energies of the upper face, core, and lower face.

The total internal energy is then

$$\Pi_i = \frac{1}{2} \int\int_A [\, (\varepsilon^{mb} + \varepsilon^o)_i{}^T [A^{(i)}] (\varepsilon^{mb} + \varepsilon^o)_i + 2(\varepsilon^{mb} + \varepsilon^o)_i{}^T [B^{(i)}] (K)_i + (K)_i{}^T [D^{(i)}] (K)_i + (\varepsilon^s)_i{}^T [A^{(i)}{}_{kl}]_i (\varepsilon^s)_i] dxdy \quad (10)$$

External energies due to lateral loads are given by the equation:

$$V_1 = -\int\int_A P(x,y)\, w\, dxdy \quad (11)$$

The total potential energy is the sum of internal energies and external energies:

$$\Pi = \sum_{i=1}^{3} \frac{1}{2} \int\int_A [\, (\varepsilon^o + \varepsilon^{mb})_i{}^T [A^{(i)}] (\varepsilon^o + \varepsilon^{mb})_i + 2(\varepsilon^o + \varepsilon^{mb})_i{}^T [B^{(i)}] (K)_i + (K)_i{}^T [D^{(i)}] (K)_i + (\varepsilon^s)_i{}^T [A_{kl}] (\varepsilon^s)_i] dxdy - \int\int_A P(x,y)\, w\, dxdy \quad (12)$$

3. GOVERNING EQUATION

Here we use the principle of virtual displacements to derive the governing equations appropriate for the displacement field in eqns (1) and constitutive equation in eqns (4). Therefore, the application of the principle of total potential energy gives nine governing equations.

The first of these is given below, the others being of similar form(4).

$$\delta U_o:\ [A_{11}^{(1)} + A_{11}^{(2)} + A_{11}^{(3)}]\frac{\partial^2 U_o}{\partial x^2}$$

$$+ [A_{66}^{(1)} + A_{66}^{(2)} + A_{66}^{(3)}]\frac{\partial^2 U_o}{\partial y^2}$$

$$+ [2A_{16}^{(1)} + 2A_{16}^{(2)} + 2A_{16}^{(3)}]\frac{\partial^2 U_o}{\partial x \partial y}$$

$$+ [A_{16}^{(1)} + A_{16}^{(2)} + A_{16}^{(3)}]\frac{\partial^2 V_o}{\partial x^2}$$

$$+ [A_{26}^{(1)} + A_{26}^{(2)} + A_{26}^{(3)}]\frac{\partial^2 V_o}{\partial y^2}$$

$$+ [A_{12}^{(1)} + A_{12}^{(2)} + A_{12}^{(3)} + A_{66}^{(1)} + A_{66}^{(2)} + A_{66}^{(3)}]\frac{\partial^2 V_o}{\partial x \partial y}$$

$$+ \left[A_{11}^{(1)}\left(\frac{t1}{2}\right) - B_{11}^{(1)}\right]\frac{\partial^2 \theta_{x1}}{\partial x^2}$$

$$+ \left[A_{66}^{(1)}\left(\frac{t1}{2}\right) - B_{66}^{(1)}\right]\frac{\partial^2 \theta_{x1}}{\partial y^2}$$

$$+ [A_{16}^{(1)}(t1) - 2B_{16}^{(1)}]\frac{\partial^2 \theta_{x1}}{\partial x \partial y}$$

$$+ \left[A16^{(1)}\left(\frac{t1}{2}\right) - B_{16}^{(1)}\right]\frac{\partial^2 \theta_{y1}}{\partial x^2}$$

$$+ [B_{26}^{(1)}]\frac{\partial^2 \theta_{y1}}{\partial y^2}$$

$$+ \left[A_{12}^{(1)}\left(\frac{t1}{2}\right) + A_{66}^{(1)}\left(\frac{t1}{2}\right) - B_{12}^{(1)} - B_{66}^{(1)}\right]\frac{\partial^2 \theta_{y1}}{\partial x \partial y}$$

$$+ \left[A_{11}^{(1)}\left(\frac{h}{2}\right) - A_{11}^{(3)} - B_{11}^{(2)}\right]\frac{\partial^2 \theta_{y1}}{\partial x^2}$$

$$+ \left[A_{66}^{(1)}\left(\frac{h}{2}\right) - B_{66}^{(2)} - A_{66}^{(3)}\left(\frac{h}{2}\right)\right]\frac{\partial^2 \theta_{x2}}{\partial y^2}$$

$$+ [A_{16}^{(1)}(h) - 2B_{16}^{(2)} - A_{16}^{(3)}(h)]\frac{\partial^2 \theta_{x2}}{\partial x \partial y}$$

$$+ \left[A_{16}^{(1)} - B_{16}^{(2)} - A_{16}^{(3)}\left(\frac{h}{2}\right)\right]\frac{\partial^2 \theta_{x2}}{\partial y^2}$$

$$+ \left[-A_{26}\left(\frac{t_2}{2}\right)\right]\frac{\partial^2 \theta_{y2}}{\partial y^2}$$

$$+ \left[A_{12}^{(1)}\left(\frac{h}{2}\right) - A_{12}^{(3)}\left(\frac{h}{2}\right) + A_{66}^{(1)}\left(\frac{h}{2}\right) - A_{66}^{(3)}\left(\frac{h}{2}\right) - B_{12}^{(2)} - B_{66}^{(2)}\right]\frac{\partial^2 \theta_{y2}}{\partial x \partial y}$$

$$+ \left[-A_{11}^{(3)}\left(\frac{t_2}{2}\right) - B_{11}^{(3)}\right]\frac{\partial^2 \theta_{x3}}{\partial x^2}$$

$$+ \left[-A_{66}^{(3)}\left(\frac{t_2}{2}\right) - B_{66}^{(3)}\right]\frac{\partial^2 \theta_{x3}}{\partial y^2}$$

$$+ [-A_{16}^{(3)}(t_2) - 2B_{16}^{(3)}]\frac{\partial^2 \theta_{x3}}{\partial x \partial y}$$

$$+ \left[-A_{16}^{(3)}\left(\frac{t_2}{2}\right) - B_{16}^{(3)}\right]\frac{\partial^2 \theta_{y3}}{\partial x^2}$$

$$+ \left[-A_{26}\left(\frac{t_2}{2}\right) - B_{26}^{(3)}\right]\frac{\partial^2 \theta_{y3}}{\partial y^2}$$

$$+ \left[-A_{12}^{(3)}\left(\frac{t_2}{2}\right) - A_{66}^{(3)}\left(\frac{t_2}{2}\right) - B_{12}^{(3)} - B_{66}^{(3)}\right]\frac{\partial^2 \theta_{y3}}{\partial xx \partial y} = 0 \tag{13}$$

We will analyze a rectangular plate with length a and width b. The edges of the plate are assumed to be simply-supported such that shear deformations are prevented in the cross-sectional planes around the edges.

The boundary conditions of such a plate are idealized as:

$$W = M_x = \theta_{y_1} = \theta_{y_2} = \theta_{y_3} = 0 \text{ at } x=0,\ a$$
$$W = M_y = \theta_{x_1} = \theta_{x_2} = \theta_{x_3} = 0 \text{ at } y=0,\ b \tag{14}$$

For essential boundary conditions given

eqn (14) above, assumed displacements are chosen in the form :

$$u_0 = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} U^1_{mn} \cos\alpha x \ \sin\beta y,$$

$$v_0 = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} V^2_{mn} \sin\alpha x \ \cos\beta y$$

$$w_0 = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} W^3_{mn} \sin\alpha x \ \sin\beta y,$$

$$\theta_{x1} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} X^4_{mn} \cos\alpha x \ \sin\beta y$$

$$\theta_{y1} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} Y^5_{mn} \sin\alpha x \ \cos\beta y,$$

$$\theta_{x2} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} X^6_{mn} \cos\alpha x \ \sin\beta y$$

$$\theta_{y2} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} Y^7_{mn} \sin\alpha x \ \cos\beta y,$$

$$\theta_{x3} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} X^8_{mn} \cos\alpha x \ \sin\beta y$$

$$\theta_{y3} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} Y^9_{mn} \sin\alpha x \ \cos\beta y \qquad (15)$$

where

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b}$$

and a and b are length and width of the plate, respectively, and m and n are the half-wavelength integers.
Substituting eqn (14) into eqn (12), and collecting the coefficients, one obtains:

$$[K]\{\delta\} = \{F\} \qquad (16)$$

Matrix [K] is the coefficient matrix, $\{\delta\}$ and {F} refer to the displacement vector and load vector.

4. NUMERICAL EXAMPLE

The material properties of facings and core are indicated in Table 1. The problem was solved using a series which exactly satisfied the boundary conditions.

For simply supported antisymmetric angle-ply faces sandwich plate as shown in Fig. 2, plate stiffness are given by:

$$A_{16} = A_{26} = A_{45} = B_{16} = B_{26} = D_{16} = D_{26} = 0.$$

Results are illustrated in Table 2, Fig.3 for square laminated plates with two layers [0/90]. The accuracy of the approach is ascertained by comparing solutions from the sandwich plates theory with composite facings based on 'zig-zag models' to the laminated plates theory. The solution in

Table 1. Material properties of faces and core for analysis model of anisotropic sandwich plates with composite facings (Gpa)

FACE (Graphite/epoxy)	E_1	E_2	V_{12}	G_{12}
	2070	5.17	0.25	5.17
CORE (Glass fabric honeycomb)	E_z	G_x	G_y	$E_x=E_y=G_{xy}$
	0.300	0.241	0.117	0

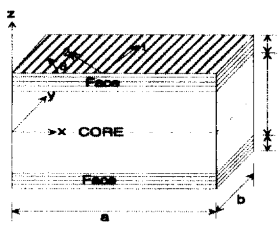


Fig. 2 Geometry of a sandwich plates with composite facings

the present paper is compared in Table 2.

The present results considering the bending stiffness of the core and also the transverse shear deformations of the laminated facings are higher than that calculated according to the first order shear deformation theory, higher order shear deformation theory

Table 2. Comparison of normalized central deflection with side to thickness ratio (a/h) of a laminated plates, [0/90]

a/h \ theory	CPT	FSDT	HSDT	PRESENT
4	1.3257	5.1275	7.6697	7.8850
5	1.3257	5.1275	7.6697	7.8850
10	1.3257	2.3398	3.1180	3.1820
20	1.3257	1.5848	1.7934	2.1466
25	1.3257	1.4920	1.6266	1.6694
100	1.3257	1.3362	1.3447	1.3522

shown in Table 2, Fig. 3; the difference are particularly significant for ratio a/h < 5.

Results are illustrated in Fig. 4 for sandwich plates with composite facings.

$$[\, w_z E_2 t^3 10^2/(q_z a^4)\,]$$

The present results are higher than that calculated according to the first order shear deformation theory , higher order shear deformation theory shown in Fig. 4. Table 3, Fig. 5 show the comparison between the present theory and classical sandwich theory. The classical sandwich theory considers only transverse shear deformations of the core. The bending stiffness of the core is excluded, since it was assumed that the core is very flexible.

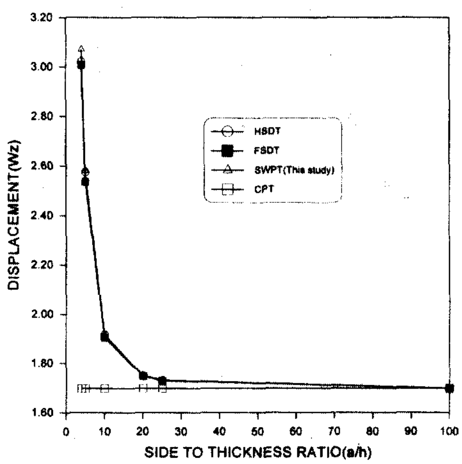


Fig. 3 Comparison of normalized central deflection according to a first order shear deformation theory, high order shear deformation theory, and classical plate theory with side to thickness ratio (a/h) of a general laminated plates, [0/90]

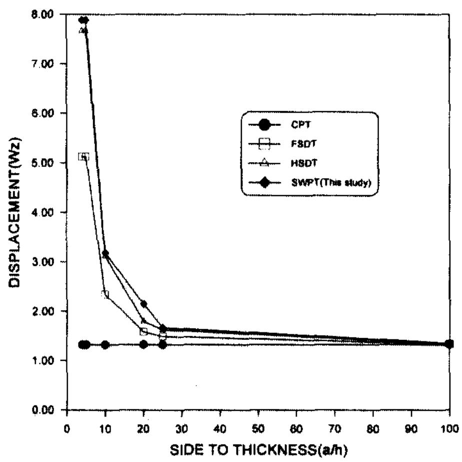


Fig. 4 Comparison of central deflection according to a first order shear deformation theory, high order shear deformation theory and classical plate theory with side to thickness ratio(a/h) of a sandwich plates with composite facings, [0/core/0]

Table 3. Comparison of central deflection according to modified sandwich plates theory with side to thickness ratio(a/h) of a sandwich plates with composite facings including shear deformation of face, and core, [0/core/0].

해석방법 / a/h	Ref.(4)	present
4	11.7349	7.8850
5	11.7349	7.8850
10	4.4247	3.1820
20	2.2161	2.8466
25	1.9321	1.6694
100	1.4482	1.3522

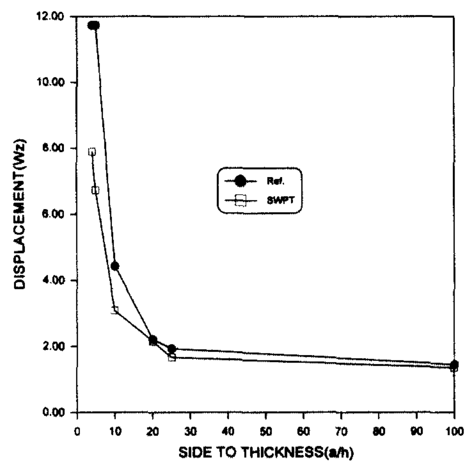


Fig. 5 Comparison of central deflection according to modified sandwich plates theory with side to thickness ratio (a/h) of a sandwich plates with composite facings including shear deformation of face, core, [0/core/0]

Fig. 6 shows the influence of antisymmetric angle-ply laminates with ply orientations [$\theta/\theta/\theta/-\theta$/core/$\theta/-\theta/-\theta/-\theta$]. Results are presented for facings of four. The maximum value of normalized displacement is seen to occur at 0/0/0/-90/core/90/0/0/0.

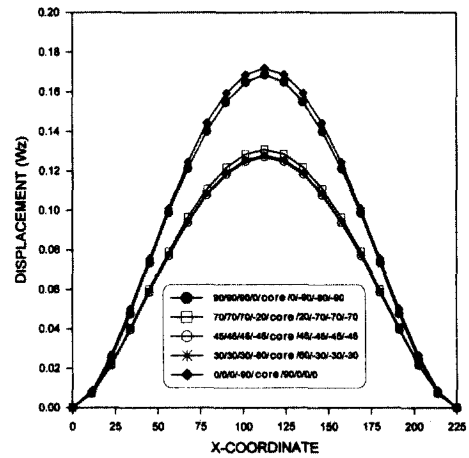


Fig. 6 Central deflection of typical hybrid sandwich plates with laminated facings [$\theta_3^{gr/ep}/(90-\theta)^{gl/ep}$/core/$(\theta-90)^{gl/ep}/\theta_3^{gr/ep}$]

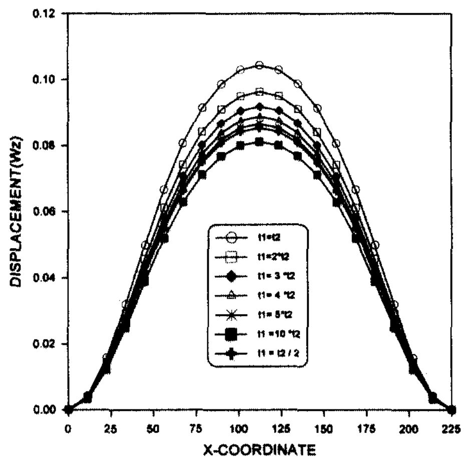


Fig. 7 Comparison of central deflection according to variation of upper & lower face in a sandwich plates with composite facings

Fig. 7 shows the central deflection related to thickness of upper face t1 and the lower face t2. Sandwich plates having symmetric angle-ply faces are superior compared with ones having anisotropic faces.

[$w_z E_2 t^3 10^2/(q_z a^4)$]

The [45/-45/core/-45/45] configuration yields the largest value of moment M_x. plots are given in Fig. 8.

The [90/-90/core/-90/90] configuration yields the largest value of moment M_y. Plots are given in Fig. 9.

Fig. 10 shows the influence of antisymmetric angle-ply laminates with ply orientations [$\theta/\theta/\theta/-\theta$/core/$\theta/-\theta/-\theta/-\theta$]. Results are presented for facings of four. The maximum value of moment M_{xy} is seen to occur at 0/0/0/-90/core/90/0/0/0.

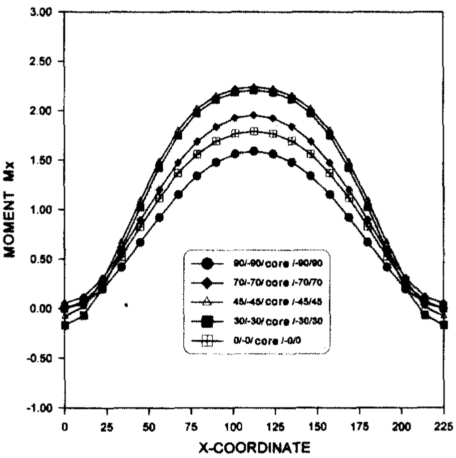


Fig. 8 Moment(Mx) in y-direction with variation of fiber angle

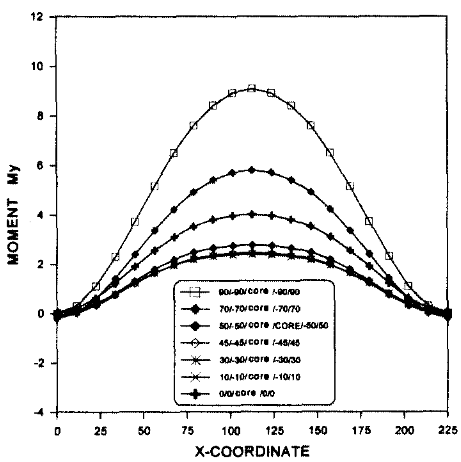


Fig. 9 Moment(My) in x-direction with variation of fiber angle

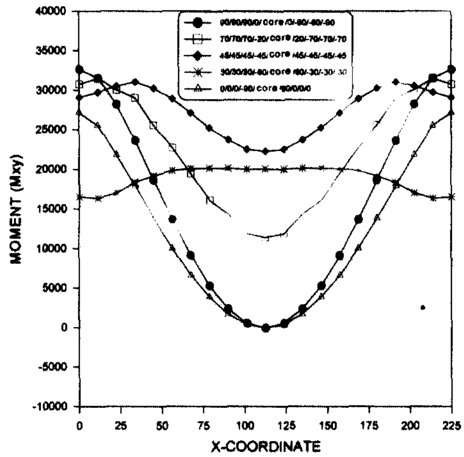


Fig. 10 Moment(Mxy) in x-direction with variation of fiber angle

5. CONCLUSIONS

The governing equations for analysis of bending of sandwich plates with thick laminated facings based on zig-zag models are derived. Results of the bending analysis under lateral uniform loads are presented for square laminate plates and sandwich plates with composite facings.

For the laminated plates: the present analysis considering the bending stiffness of the core and also the transverse shear deformations of the laminated facings show higher than first order shear deformation, smaller than higher order shear deformation theory in laminated plates with two layers[0/90].

For the sandwich plates with composite facins: It is shown that the influence of antisymmetric angle-ply laminates with ply orientations [$\theta/\theta/\theta/-\theta$/core/$\theta/-\theta/-\theta/-\theta$]. Results are presented for composite facings with four layers. The maximum value of moment M_{xy} is seen to occur at 0/0/0/-90/core/90/0/0/0.

The (90/-90/core/-90/90) configuration yields the largest value of moment (M_y) and (45/-45/core/-45/45) yields the largest value of moment (M_x). Since the present analysis considers the bending stiffness of the core and also the transverse shear deformations of the laminated facings, it is expected that the analysis is capable to analyze general laminated plates with shear deformations. Applying the present method to investigate the bending behavior of laminated plates, it is necessary to divide the plate's thickness into three components, namely upper face, core, and lower face. The thickness of each component is arbitrary.

The information presented might be useful to design sandwich plates structure with metal, polymer matrix composite facings.

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