

# Josephson Tunneling and Pairing Symmetry of High T<sub>c</sub> Superconductor

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## Abstract

The temperature dependent Josephson critical current  $J_c(T)/J_c(0)$  between high T<sub>c</sub> superconductors along the *a*-axis is theoretically studied. The interface influence on the wave functions of quasi-particles is included in the theory within the framework of the two-dimensional *t*-*J* model. It is found that the experimental results can be satisfactorily explained by the present model with *d* wave pairing symmetry.

*Keyword* : High T<sub>c</sub> Superconductors, *d* wave, Josephson tunneling

## I. Introduction

Since the discovery of superconductivity in the layered copper oxide materials [1], a number of microscopic models have been proposed [2-5]. For a proper understanding of high T<sub>c</sub> superconductivity, it is imperative to determine the correct symmetry of the superconducting order parameter. There have been conflicting experimental evidences whether the superconducting condensate state has *s*-, *d*-, *s*-*d*, or *s*\*-*d* mixed symmetries, where *s*\* is the extended *s*-wave [6-8]. Tunneling data along the *c*-axis of quasi-tetragonal copper oxides clearly show an *s*-wave character [9]. On the other hand, tunneling along the *a*- or *b*-axis of YBCO has indicated a *d*-wave character with some exceptions [10-13]. In order to explain these conflicting experiments, various theories including *s*+*id* and *s*+*id* symmetries were proposed [7,8,14,15]. In this paper, we include

the interface effect in theory and compare theories with various symmetries to the experiments.

## II. Theory

We develop a general theory which can accommodate various symmetries mentioned above. We employ the two-dimensional *t*-*J* model in order to describe tunneling for YBCO [16, 17]. The *t*-*J* Hamiltonian is given by

$$H = -t \sum_{\langle ij \rangle} (c_{i\sigma}^{\dagger} c_{j\sigma} + hc.) - \mu \sum_{i\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} - J \sum_{\langle ij \rangle} b_{ij}^{\dagger} b_{ij} \quad (1)$$

Where  $\langle ij \rangle$  means the nearest neighbor pairs  $b_{ij}^{\dagger} = c_{i\sigma}^{\dagger} c_{j\sigma}^{\dagger} - c_{j\sigma}^{\dagger} c_{i\sigma}^{\dagger}$  and  $\mu$  is chemical potential. Using the mean-field result of the 2-D *t*-*J* model with the order parameter  $\Delta_{ij} = 2 \langle c_{j\sigma} c_{i\sigma} \rangle$  and we obtain

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$$x_{ij} = \sum_{\sigma} \langle C_{i\sigma}^{\dagger} C_{j\sigma} \rangle = 2 \langle C_{i\uparrow}^{\dagger} C_{j\downarrow} \rangle,$$

$$\Delta_k = J \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{E_{k'}} \tanh \frac{\beta E_{k'}}{2}, \quad (2)$$

$$\delta = \frac{1}{N} \sum_k \frac{\varepsilon_k}{E_k} \tanh \frac{\beta E_k}{2} \quad (3)$$

and

$$x = \frac{1}{2N} \sum_k \gamma_k \left[ 1 - \frac{\varepsilon_k}{E_k} \tanh \frac{\beta E_k}{2} \right]. \quad (4)$$

where

$$\varepsilon_k = -(2t + xJ)\gamma_x - \mu,$$

$V_{k,k'} = 2[\cos(k_x - k_{x'}) + \cos(k_y - k_{y'})]$ ,  
 $\gamma_k = (\cos k_x + \cos k_y)$ ,  $\beta = 1/k_B T$  and energy spectrum  
 is  $E_k = \sqrt{\varepsilon_k^2 + J^2 |\Delta_k|^2}$  [17].  $\delta$  represents the average  
 hole doping concentration. In order to develop a  
 general theory, we follow Xu et al.'s approach and  
 assume an order parameter  $\Delta_k = \Delta_x + i\Delta_y(\cos k_x - \cos k_y)$

Here, we note that the simplest even parity expres-  
 sion for  $s^* + id$  has a form of  $\Delta_k = c_0 \Delta_x + i c_2 \Delta_y$ , where  
 $c_0 = \cos k_x + \cos k_y$  and  $c_2 = \cos k_x - \cos k_y$ . Other symme-  
 tries follow immediately by choosing  $c_0$  and  $c_2$   
 accordingly.

We substitute the above expressions into Eq.(2)  
 and solve for  $\Delta_k$ . In the calculation the  
 k-summation is confined to the first Brillouin zone  
 [17,18]. For numerical values, we use  $J \ni 0.124 \text{ eV}$   
 obtained from the NMR data [19]. The hopping  
 parameter,  $t$ , is estimated to be  $t \dots 0.19 \text{ eV}$  in  
 optimal doping [20]. At  $T = 0K$ , we obtain  
 $4(0) = 18 \sim 19 \text{ meV}$  and  $4(0)/k_B T_c = 2.32 \sim 2.52$  when the  
 ratio of  $s^*$  component to d-wave component is varied  
 from 0 to 1.

In analyzing the Josephson tunneling phenomena,  
 the pairing symmetry plays a crucial role. The space  
 is no longer homogeneous near the junction. We here  
 ignore inhomogeneity on the superconducting order  
 parameter. However, the existence of the interface is  
 expected to become a quasiparticles. The usiparticle  
 states are determined by the Bogoliubov-de Genes  
 (BdG) equations [21] which are obtained from the  
 mean-field result of the 2-D  $t$ - $J$  model.

$$\sum_j [-(\tilde{t} + \mu)u_k(j) - \Delta v_k(j)] = E_k u_k(i)$$

$$\sum_j [(\tilde{t} + \mu)v_k^*(j) - \Delta u_k^*(j)] = E_k v_k^*(i) \quad (5)$$

where  $\tilde{t} = t + xJ$ ,  $4_{ij} = 4$  and  $j$  is a nearest neighbor  
 of  $i$  along  $x$ (or  $y$ ) direction. Solutions to the above  
 equations are

$$u_k(j) = \zeta_+ \sin k_x j_x e^{ik_y},$$

$$u_k(j) = -\zeta_-(k) \text{sgn}(\Delta_k) \sin k_x j_x e^{ik_y}, \quad (6)$$

with  $\zeta_{\pm} = \frac{1}{\sqrt{N}} (1 \pm \frac{e_k}{E_k})^{1/2}$ . From Eq.(6), we note

that the quasiparticles are described by the standing  
 wave functions along the normal direction of the  
 interface.

We consider the Josephson tunneling in the  $a$ -axis  
 direction through a junction of a normal metal be-  
 tween two YBCO superconductors with perfectly flat  
 interfaces. To describe the electron transport process  
 in the junction, we use the Ambegaokar-Baratoff  
 theory [22]. The tunneling Hamiltonian given by

$$H_T = \sum_{ij} T_{ij} c_i^{\dagger} d_j + h.c. \quad (7)$$

where  $T_{ij}$  is the matrix element transferring elec-  
 trons through the junction from right-hand side to  
 left-hand side.  $c_i^{\dagger}$  and  $d_j$  are the creation and  
 annihilation operator in the left and right supercon-  
 ductor, respectively. It is expected that the largest  
 tunneling is between the nearest neighbors. Since the  
 coordinate system can be independently defined for  
 the left and right superconductors, we make the  
 choice that the nearest neighbors on the  $yz$ -planes  
 adjacent to the junction take the same value of  
 coordinate. Thus the  $ij$ -summation in above Hamilto-  
 nian is carried out with the conditions;  $i_x = j_x = l$ ,  
 $i_y = j_y$  and  $i_z = j_z$ . Using the standard perturbation  
 treatment, we obtains for the Josephson critical current [22]

$$J_c(T) = 4eN |T_0|^2 \frac{1}{\beta} \sum_{i,j} \sum_n F_{ij}^{\dagger}(i\nu_n) F_{ij}(i\nu_n) \quad (8)$$

where  $T_0$  is the matrix element of nearest neigh-  
 bors tunneling and  $N_{yz}$  is the total number of sites on  
 one of the interfaces.  $F_{ij}(i\omega_n)$  with  $\omega_n = \pi(2\nu^+ 1)/\downarrow$   
 is the Fourier transformation of the Gorkov's pair

Gorkov's pair correlation function,

$$F_{ij}(\tau) = -\langle T_c c_{i\uparrow}(\tau) c_{j\downarrow}(0) \rangle \quad (9)$$

Note that the Gorkov's pair correlation function is defined in the semi-infinite space :  $i_x \bullet I$ ,  $-I \bullet i_y \bullet I$  and  $j_x \bullet I$ ,  $-I \bullet j_y \bullet I$ . We evaluate the Gorkov's pair correlation function using the mean-field approximation,

$$F_{ij}(i\omega_n) = \sum_k \left[ \frac{u_k(i_x) v_k^*(j_x)}{i\omega_n - E_k} - \frac{u_k(j_x) v_k^*(i_x)}{i\omega_n + E_k} \right] \quad (10)$$

$$= -\frac{1}{N} \sum_k \frac{\Delta_k}{\omega_n^2 + E_k^2} \sin(k_x i_x) \sin(k_x j_x) e^{ik_j(i_y - j_y)}$$

### III. Results and discussions

Combining Eq.(6), Eq.(8), and Eq.(10) and taking a frequency summation, we obtain a formula for  $J_c(T)$  as

$$J_c(T) = \frac{4e|T_0|^2 N_{yz}}{N^2} \sum_{k,k'} \frac{\Delta_k \Delta_{k'}}{E_k E_{k'}} x(k,k') \sin^2 k_x \sin^2 k'_x \Big|_{k'_y=k_y} \quad (11)$$

with

$$x(k,k') = \frac{\tanh(\beta E_k / 2) + \tanh(\beta E_{k'} / 2)}{E_k + E_{k'}} - \frac{\tanh(\beta E_k / 2) - \tanh(\beta E_{k'} / 2)}{E_k - E_{k'}} \quad (12)$$

In Fig.1, we exhibit the calculated result for  $J_c(T)$  as a function of  $T$  for various symmetries and com

pared to experimental results[7,8,15]. Clearly, the d wave tunneling model with the interface effect describes the experimental results satisfactorily comparing to other symmetry waves. Our result demonstrates that  $s^*+id$  is not necessary to fit the experimental data contrary to the Xu et al's claim [14]. It should be noted that the main disagreement between previous d-symmetry theories and the present model arises mainly from neglect of the interface effect.

### IV. Conclusions

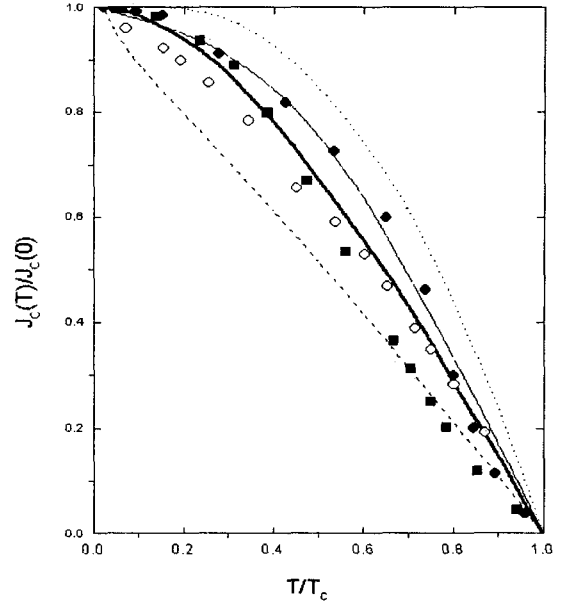


Fig. 1. Josephson critical current  $J(T)/J(0)$  as a function of temperature  $T$ . The solid bold line is the present result of pure d-wave symmetry. The experimental results (○: ref[7], ●: ref[8], ■: ref[15]) are also shown as well as a s-wave (dotted line), d-wave without interface effect (dashed line) and  $s^*+id$  with  $\phi_{4s^*} = 0.2\phi_{4d}$  (solid thin line) for the comparison.

In summary, using the 2-D  $t$ - $J$  model, we have investigated Josephson tunneling between YBCO superconductors. It is shown that the d-wave symmetry with the interface explains the experiment satisfactorily. We also have shown that the interface effect is critical in explaining the experimental results.

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