

Analysis of the Behavior of Bolt Jointed Wood Connections by Applying Semi-Rigid Theory*¹

Gwang-Chul Kim*² and Jun-Jae Lee*²

ABSTRACT

Attempts were made to analyze the behavior of single and multiple-bolted connections through theoretical methods such as European yield theory, empirical approaching method, and semi-rigid theory instead of many experimental methods that have been actually inefficient and non-economical.

In the case of a single-bolted connection, if accurate characteristic values of a material could be guaranteed, it would be more convenient and economical to perform the behavior analysis using a model based on the semi-rigid theory, instead of the existing complex yield model, or the empirical formula which produces errors, giving different results from the actual ones. If the variables of equation determining the load and deformation could be appropriately controlled, the analytical method in conjunction with a semi-rigid theory could be effectively applied to obtain the desirably predicted value, considering that the appropriate solution could be derived through a simpler equation using a less difficult method compared to the existing yield model.

It is concluded that analytical method with semi-rigid theory can be used in the behavior analysis of bolted connection because our developed method showed excellent analysis ability of behavior until number of bolt is two.

Although our analytical method has the disadvantage that the number of bolt is limited to two, it is concluded that it has the advantage than numerical method which complicated and time-consuming.

Keywords : Semi-rigid theory, bolted connection, yield model, behavior analysis, analytical method

1. INTRODUCTION

To execute the connection design, analysis or estimation for the behavior of connection should first proceed. Behavior analysis from experiment with practical connection is needed for safe design, however, economical point, complexities of reflection method for parameters that affect

the connection performance, and difficulties of the appropriate type constitution for the connection have obstructed the efficiency, resulting in the necessity of the theoretical methods on the estimation of connection behaviors.

For calculating convenience, the connection was assumed to behave very simply though such an assumption is not an actual condition. In the

*1 Received on November 6, 2000, accepted on December 11, 2000

*2 College of Agriculture & Life Sciences, Seoul National University, Suwon 441-744, Korea

most part, it has been considered to be an ideal pin connection or completely rigid connection. However, such conditions never occur in wood and steel structures. In addition, stress distribution within frames is heavily influenced by the connection behavior, whereby requiring a more practical modeling to obtain efficient consumption of material.

Among many estimation methods for the connection behavior, European yield model accepted widely has produced underestimated values than experimental values. In addition, to model the connections, complex equations are required, and the number of equations vary according to the type of fasteners or the number of shear planes; This is why a consistent estimation of connection behavior is difficult to obtain using the yield model (McLain & Thangjitham, 1983). Moreover, only the usual connection types and not the many recently developed connection types can be estimated through this. Consequently, this study was carried out to solve these problems. The semi-rigid model was used to develop a new analysis method that can estimate the behaviors of all used connection types and employ convenient equations. Behaviors of a single bolted connection were analyzed and estimated using the semi-rigid model.

2. MATERIALS and METHODS

2.1 Theoretical study of semi-rigid model

The spring model was used to explain the behavior of semi-rigid connection, since the relative internal force and deformation of connection could be determined through the spring element. In this study, we used three springs modeling to describe the behavior of connection: a rotational spring that can control the relative rotation, and

two translational springs that can control the relative slips in two mutually orthogonal directions.

In general, semi-rigid connections in wooden members show a linear load-slip relationship at low load levels, while showing a nonlinear relationship at high load levels.

K , stiffness of the nonlinear spring, is composed of a constant term, K_0 , and a dependent term on deformation, K_N . Deformation, u , induced by load, P , is given as follow (Cook *et al.*, 1989):

$$(K_0 + K_N) \cdot u = P \dots \dots \dots (1)$$

where, $K_N=f(u)$.

Idealized connection modeling with internal force and moment for mechanically connected timber joint is presented in Figure 1.

Connection was assumed as a semi-rigid joint with nonlinear load-slip behavior. Partial stiffness effect within the connection was modeled through the introduction of the nonlinear elastic spring as shown in Figure 1 (More informations on semi-rigid approaching method are given reference Kim, 1999).

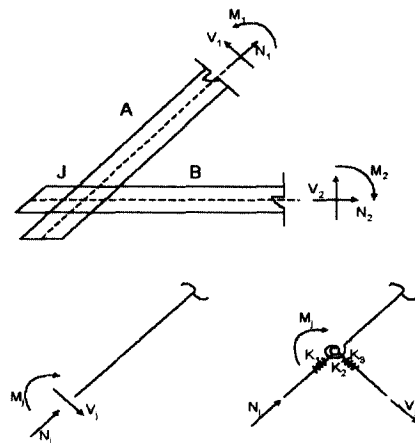


Fig. 1. Mechanically connected timber joint under action and idealized elastic restraint modeling of joint action.

Table 1. Characteristics of connections.

Type	Materials	Size (mm)	Sp. Gr.	MC (%)	Fes* (kg/cm ²)
Wood-to-Steel	Main member (Glulam)	130 × 200 × 210	0.45 (0.15)*1	12 (0.8)*1	-
	Side member (Steel)	9 × 200 × 300	-	-	4060
Wood-to-Wood	Main member (Japanese Larch)	55 × 100 × 500	0.45 (0.10)	12 (0.9)	-
	Side member (Japanese Larch)	50 × 100 × 500	0.45 (0.10)	12 (0.9)	-

*Fes: Dowel bearing strength of steel plate. Ultimate tensile strength is generally assumed as the dowel bearing strength.

*1: Standard deviation

2.2 Materials

Wood-to-steel connection was constituted of steel plate and structural glued-laminated timber made by Japanese Larch (*Larix leptolepis*). Wood-to-wood connection, on the other hand, was constituted of only Japanese Larch (*Larix leptolepis*) lumber.

2.2.1 Materials

The configurations of connection and the characteristics of each material are presented in Table 1 and Figure 2.

2.2.1.1 Members

Main members of the connection were structural glulam for wood-to-steel plate connection and structural lumber for wood-to-wood connection. For wood-to-steel connection, structural glulam was selected on the basis of the large scale connection of commercial building, and, for the wood-to-wood connection, lumber was chosen according to the previous reference (Soltis *et al.*, 1986) because a peculiar size of member are not suggested.

Side member of wood-to-steel connection was a A36 steel plate with 9mm thickness. Bolt locations for the main and side members are shown in Figure 5.

2.2.1.2 Bolts

Three types of bolts were used to manufacture the connection. The bolt diameter and bolt-hole sizes are shown in Table 2. The yield strength of bolts is generally calculated as (yield tensile strength + ultimate tensile strength)/2, and 3150 cm² is commonly used as a yield strength for A307 bolt. In this study, we used A307 bolt, and since Wilkinson (1992) also executed experiment with A307 bolt, his results were adopted as the yield strengths of the bolt without laboratory self-test.

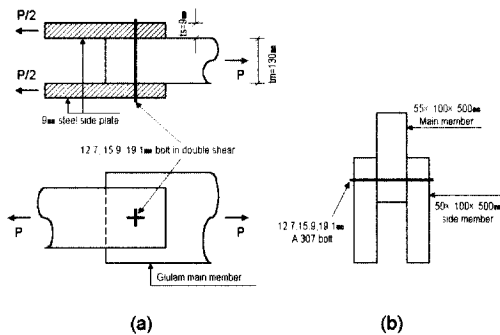


Fig. 2. Configuration of bolted connections for wood-to-steel connections (a) and wood-to-wood connections (b).

Table 2. Characteristics of bolts.

Bolt diameter (mm)	Bolt hole (mm)*1	Bending yield strength of bolt (Fyb, kg/cm ²)*2
12.7	13.5	3160
15.9	17.5	2890
19.1	20.7	2590

*1 ASTM D 1761

*2 Values referred from Wilkinson T. L. (1992)

2.2.1.3 Dowel bearing strength of glulam

Holes were digged to submerge the half diameter of bolt as Figure 6. The bolt diameters were 12.7, 15.9, and 19.1 mm. Tests were carried out vertically to the lamination. Bolts were positioned at a semicircular hole, and the load and deformation under compressive loading were measured continuously. After the uniform deformation, they were completely submerged into the lamination. The test was stopped immediately before starting the compression test of lamination itself. Tests were repeated three times.

2.2.1.4 Dowel bearing strength of lumber

Dowel bearing strength of lumber, F_e , was calculated using equations (2) and (3) from NDS (1997).

Parallel-to-grain loading: $F_{e\parallel} = 11,200(SG)$
 (2)

Perpendicular-to-grain: $F_{e\perp} = 6100(SG^{1.45})D^{-0.5}$
 (3)

At an angle of load to grain θ , Hankinson formula used $F_{e\theta} = \frac{F_{e\parallel}F_{e\perp}}{F_{e\parallel}\sin^2\theta + F_{e\perp}\cos^2\theta}$

where,

F_e : dowel bearing strength, SG : specific gravity of member, and D : diameter of dowel type fasteners.

2.3 Experimental methods

2.3.1 Shear test of bolted connection

All tests were made using seasoned wood with controlled temperature of $20 \pm 3^\circ\text{C}$ and relative humidity of $65 \pm 3\%$ for a sufficiently long period to approximately achieve the equilibrium moisture content of 12%.

Deformations were measured at successive load increments for testing joints during compressive loading. In a connector joint, the load associated with the first relaxation of load is generally related to the shear of the core within the connector, therefore, the test was continued until the ultimate load or a total deformation of 15 mm was achieved.

Test was constructed such that the maximum load was reached between 5-20 min, which requires a rate of motion of the movable crosshead of $1.0 \text{ mm/min} \pm 50\%$. The specific gravity and moisture content of each wood member of each joint were tested using ASTM D 2395 and D 4442 (ASTM D 2395, 1995; ASTM D 4442, 1995). Yield load and proportional limit load were calculated. The connection yield load was determined by fitting a straight line to the initial linear portion of the load-deformation curve. This line was then offset by a deformation equal to 5% of the bolt diameter, and the load at which the offset line intersected the load-deformation curve was selected. In cases where the offset line did not intersect the load-deformation curve, the maximum load was used as the yield

load. The proportional limit load is the load at which the load-deformation curve deviates from a straight line fitted to the initial linear portion of the load-deformation curve. Tests were repeated three times.

To verify the appropriateness of the developed model, two types of bolted connections as Figure 2 were manufactured and tested. For wood-to-steel connection, double shear test was carried out with either parallel and perpendicular loading to the lamination. For wood-to-wood connection, double shear tests were carried with parallel and perpendicular loadings to the grain (ASTM D 4442, 1995; Bohnhoff *et al.*, 1987; Bouchair & Vergne, 1995).

3. RESULTS and DISCUSSION

Basic results for the behavior analysis obtained through actual experiments and an empirical formula are as follows.

3.1 Dowel bearing strength of glulam

As the behaviors of connections are greatly influenced by loading direction to the grain of lumber, two types of tests were carried out with parallel and perpendicular loadings to the lamination, as like grain of lumber in case of structural lumber.

The dowel bearing strengths of glulam for the three bolts used in this study are listed in Table 3. Results of our study are 10% higher than the values of NDS (1997). As the table values of NDS are obtained using an empirical formula without actual experiments, minor difference between the two results can be ignored.

3.2 Dowel bearing strength of lumber

According to the previous studies (McLain, 1975; McLain & Thangjitham, 1983; Smith *et al.*, 1988; Wilkinson, 1992), dowel bearing strength for parallel loading to the grain depends only on the specific gravity of lumber. Dowel bearing strength for perpendicular loading to the grain, however, is reported as a function of specific gravity and bolt diameter.

In this study, equations (2) and (3) were used to calculate the dowel bearing strength of lumbers. The results are Table 4.

3.2.1 Optimum of analysis

In applying the semi-rigid theory to analyze the connection behaviors, results are greatly influenced by determining variables A_{\parallel} , B_{\parallel} , A_{\perp} , and B_{\perp} . Therefore, if optimum values for each connection type (wood-to-wood or wood-to-steel) and number of shear planes (single shear or double shear) are provided, it is possible the equivalent performance analysis to the experi-

Table 3. Dowel bearing strength of glulam with different bolt diameters.

	Sp. Gr.	Dowel bearing strength			
		F_{\parallel} (kgf)	F_{\perp} (kgf)		
			Bolt diameter (mm)		
			12.7	15.9	19.1
Japanese Larch (Our study)	0.45	380(21)*1	210(22)	201(18)	180(20)
Douglas Fir-Larch*2	0.50	392	221	196	182

*1 (): Standard deviation

*2: NDS table values

Table 4. Dowel bearing strength of lumbers with different bolt diameters.

Diameter (mm)	Dowel bearing strength			
	F_{\perp} (kgf)	12.7	15.9	19.1
Japanese Larch*1	353	190	170	155

*1 Specific gravity of Japanese larch is 0.45.

Table 5. Optimum A and B values for joint in single bolted connections.

Angle(°)	Wood-to-steel		Wood-to-wood	
	A*1	B*2	A	B
0	4658.0	0.623	1446.0	1.021
90	-4125.0	-0.807	-4116.0	-0.602

*1 A: Optimum values for capacity analysis of bolted joint

*2 B: Optimum values for capacity analysis of bolted joint

mental methods, through using the analytical methods and the optimum values. Optimum solution was worked out using the least square method.

The main equation for describing the connection behavior is equation (4). However, equation (5), which includes the error term, is more convenient for inducing the optimum values.

$$N = A(10^{Bd} - 1) + e \tag{4}$$

Equation (5) is obtained by arranging the error term in equation (4).

$$e = N - A(10^{Bd} - 1) \tag{5}$$

Optimum values A and B obtained through least square method are shown in Table 5 (More information on this processing is given reference Kim, 1999).

3.2.2 Behavior analysis and discussions

Results of behavior analysis with optimum values for the five types of bolted connections were compared as shown Figure 3. Behaviors of connections

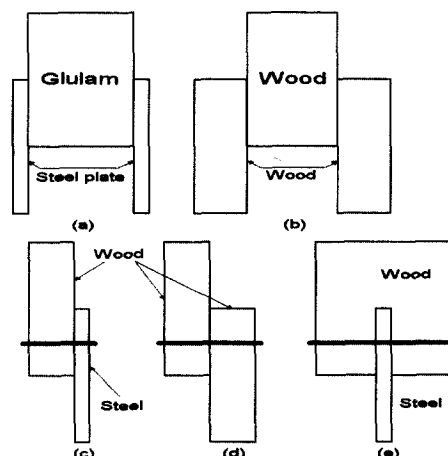


Fig. 3. Configuration of single-bolted connections.

were analyzed and discussed under three categories.

3.2.2.1 Comparison with published results and our theoretical results

Figure 4 is the case type (b) of Figure 3 and compared with previous study results. Results of Smith *et al.* (1988) were composed of actual test results and theoretical results obtained through the fracture mechanics approaching method. Our results were controlled and compared to the above

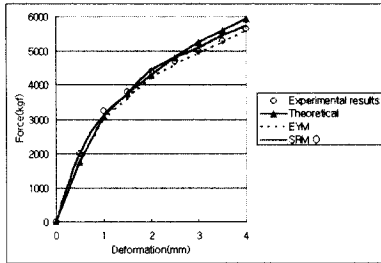


Fig. 4. Comparison of force-deformation results for bolted joints with members loaded parallel to the grain in double shear and single-bolted connections (Ian Smith, 1988. 19. 1 mm bolted, wood-to-wood connections).

Legend : 1. SRM : Semi-rigid method with optimum variables
 2. EYM : European yield model values

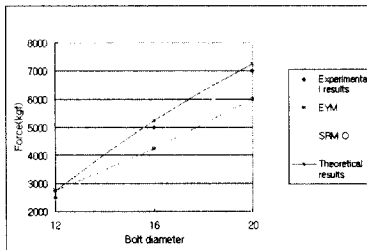


Fig. 5. Influences of bolt diameters; parallel $e1/d=7$, $e2/d=4$. Comparison of force-deformation results for bolted joints with members loaded parallel to the grain in single shear and single-bolted connections (Luc, 1996. 12.9mm bolted, wood-to-steel connections, $e1=$ end distance, $e2=$ edge distance).

results. The yield model, semi-rigid model, and fracture mechanics method also showed similar results. Results of yield model were the lowest as expected, and the differences between the results of semi-rigid model and actual test were insignificant. Fracture mechanics-approaching method was reported as a suitable method for single-bolted connection, but not appropriate for mul-

iple-bolted connection (Peter, 1997).

Figure 5 shows the behaviors of single-bolted with single shear plane bolted connections. The main member was made of wood and side member steel plate. Fracture mechanics was also used to obtain the theoretical results in Luc *et al.* study (Luc & Yasumura, 1996) as like Smith *et al.* (1988). Only the effect a bolt diameter was considered in the study, similar to our results.

3.2.2.2 Comparison with our experimental results and our theoretical results

Figure 6 shows a bolted connection composed of main member, glulam, two side members, 9 mm steel plate, and 15.9 mm bolt, which is equivalent to type (a) of Figure 3. Loading condition was parallel to the lamination, and results of the actual test, semi-rigid model, and yield model were compared. Results of yield model was lower than the actual test results. Though slight difference was observed between the results of the semi-rigid model and the actual test at low level loading, both results were almost identical at the range that the final ultimate load was calculated. The semi-rigid theory can therefore be considered as an appropriate behavior-estimating tool for the bolted connection.

As shown Figure 7, in the case of perpendicular loading to the lamination, the semi-rigid model also showed the best results among all tested methods, though different tendency is appeared to parallel loading condition.

Figures 8 and 9 show the results of 19.1 mm bolted connection under the same conditions of Figures 6 and 7. The smaller load capacity for perpendicular loading to the lamination than the parallel loading condition may be caused by anisotropy properties that the stronger specific strength along the grain and very weak strength perpendicular to the grain for wood. In particular, tensile and shear strengths perpendicular to the grain are smaller as one unit than other

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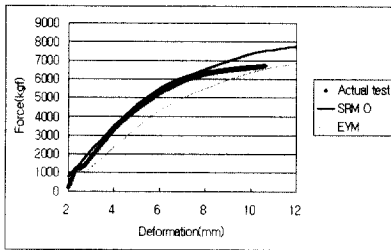


Fig. 6. Comparison of force-deformation results for bolted joints with members loaded parallel to the grain in double shear and single-bolted connections (15.9 mm bolted, wood-to-steel connections).

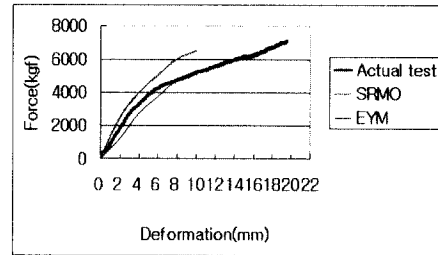


Fig. 9. Comparison of force-deformation results for bolted joints with members loaded perpendicular to the grain in double shear and single-bolted connections (15.9 mm bolted, wood-to-steel connections).

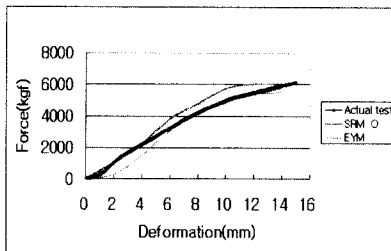


Fig. 7. Comparison of force-deformation results for bolted joints with members loaded perpendicular to the grain in double shear and single-bolted connections (15.9 mm bolted, wood-to-steel connections).

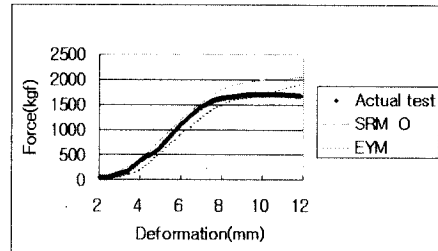


Fig. 10. Comparison of force-deformation results for bolted joints with members loaded parallel to the grain in double shear and single-bolted connections (15.9 mm bolted, wood-to-wood connections).

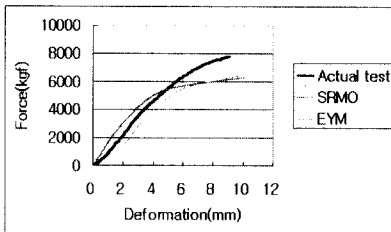


Fig. 8. Comparison of force-deformation results for bolted joints with members loaded parallel to the grain in double shear and single-bolted connections (19.1 mm bolted, wood-to-steel connections).

strengths (Luc & Yasumura, 1996). Therefore, in considering the load transfer between the structural building elements, caution must be taken to avoid splitting or shear failure within the bolted connection by keeping the main shear force away from the perpendicular to the grain.

Both the main and side members of the bolted connection used lumber as shown Figure 10. The bolt diameter was 15.9 mm and type (b) of Figure 3. Results of the yield model were also lower than the actual test results, which those of the semi-rigid model were higher.

3.2.2.3 Comparison results for imaginary connection type

Figure 11 shows an imaginary bolted connection of the type (d) of Figure 3. Main and side members of the connection were both made of wood, and single-bolted with single shear plane connection. At the initial step, load-slip curve showed an analogous slope, but ultimate values were significantly different. Nevertheless, large differences are not found supposing that actual yield load are considered as criterion.

Figure 12 is the case (e) of Figure 3 that is the recently many used bolted connection type. Since the yield model requires the size of the main member be larger than the of side member, type (e) of Figure 3 cannot be analyzed through this model. However, to compare the empirical results of Japan and our results, forcibly results of yield model also were presented.

Double shear plane connection with steel plated core can be analyzed by considering the symmetric connection of a single shear plane connection in which the steel plate is the main member and wood is the side member. However, behaviors of that type connection shall be different to double value for behavior of single bolted connection. The added Japan results are calculated values using design criterion of Japan

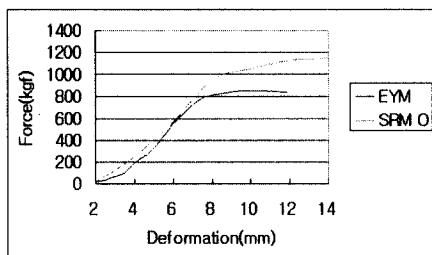


Fig. 11. Comparison of force-deformation results for bolted joints with members loaded parallel to the grain in double shear and single-bolted connections (imaginary 15.9 mm bolted, wood-to-steel connections).

(The society of construction in Japan, 1988).

On the whole, the differences between results of yield model and analytical results of semi-rigid model did not exceed 10% for double shear plane (wood-to-steel), which was mainly caused by the presence of the steel plate. Since the ultimate tensile strength is generally used as the yield strength, due to the fact steel plate has a higher yield strength than wood, plastic effect within the main member could be larger than the connection that uses wood as the side member. That is, since the side member could endure heavier load than the expected load, contact areas between the bolt and bolt-hole increased. Connection together with steel plate can support the additional load after the elastic domain has passed, in consequence of the increased effect of plastic property within wood itself is possible.

From existed information (Bouchair & Vergne, 1995; Kim, 1998), a gap between bolt and bolt-hole mainly affect the revelation of plasticity, and plasticity starts at a loading angle of 30° . Practical plasticity are developed all around the hole from that angle, since angle zero plasticity are only developed beneath the bolt, which could help explain our results.

Inaccurate characteristic values of steel plate and bolt may have led to poorer results in the yield

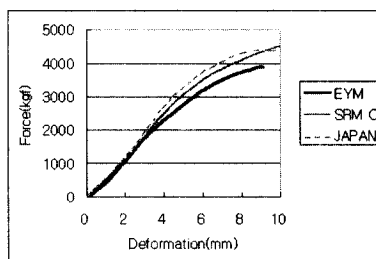


Fig.12. Comparison of analysis with experimental and optimum variables in double shear and single-bolted connection (wood-to-steel, main member-steel, side member: wood).

model compared to the other models. Therefore, if the strength values of steel plate and bolt are directly tested, more reliable results could be obtained.

On the other hand, owing to the difficulty in obtaining wood with straight grains, wood-to-wood bolted connection was manufactured using materials with a thickness difference between the main member and the side member of 5 mm; thus the development of the plasticity effect was reduced. The wooden member also resulted in a similar effect. Therefore, the reliability of yield model results for the wood-to-wood connection decreased, compared to the wood-to-steel connection. Another reason for the difference between wood-to-wood and wood-to-steel connections may be the non-uniformity of materials around the bolt-holes.

Considering that the nonlinear property within the material is developed due to non-uniformity, resulting in the reduction of approximately 20% in stress of the material (Rahman *et al.*, 1988), and local properties of material greatly affect the behavior of the connection (Bodig *et al.*, 1991), further studies should be done to develop a more accurate analytical method that can reflect these peculiarities.

4. CONCLUSION

In the case of a single-bolted connection, if accurate characteristic properties of material could be guaranteed, it would be more convenient and economical to perform the behavior analysis with a model based on the semi-rigid theory, instead of the existing yield model which is complex and hard, and the empirical formula which produces errors, giving different results from the actual ones. For the multiple-bolted connection, however, it was difficult to develop an analytical model applying the elasto-plastic theory. In particular, the difficulty in modeling that precisely reflected the interaction among

bolts and mixed-action between the member and bolts increased as the sizes of bolts increased. Analytically approaching method that can consider all the possible conditions in fact, does not exist.

However, in the case of a single-bolted connection, if variables of equation determining the load and deformation can be appropriately controlled, the analytical method in conjunction with a semi-rigid theory can be effectively applied to obtain the desired predicted value, considering that the right solution could be derived through a simpler equation using a less difficult method compared to the existing yield model.

It can therefore be concluded that the analytical method using a semi-rigid theory can be used in the behavior analysis of bolted connection since this method showed an excellent analysis ability of behavior until number of bolt is two.

Although analytical method has a disadvantage in that the number of bolt is limited to two, it is still more advantageous than the numerical method which is complicated and time-consuming.

The optimum values were obtained to analyze the behavior of a single-bolted connection.

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