

Scientific Research Training for Prospective Mathematics Teachers in China

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This paper presents a summary of the Scientific Research Training (SRT) in Liaocheng Teachers University, which is a representative of normal universities and teachers' colleges in China. There is a survey on the situation of SRT in China systematically and historically.

Tentative ideas about SRT in future are targeted at the teachers and heads of departments in China.

1. GENERAL SITUATION

Now there is in general a special training for the fourth year students majoring in mathematics education and learning in a department of mathematics education in a normal university (or teachers' college) in China. It trains students to do research on mathematics or mathematics education. The training is called Scientific Research Training, briefly SRT, which formally began in 1981 in China. At that time, there was in general no experience to guide students to do research on mathematics or mathematics education in normal university (or teachers' college). Even among teachers only a very few had published a small number of research papers. Since then, through seventeen year practice, a stable frame of organization has been set up. At present, the basic steps for SRT in China are regularly as following:

- The head of the department does collect and print reference subjects, which were compiled or checked by guide teachers of the department for students during the seventh semester.

- After printing, he/she gives each senior student a copy of the subjects.

These subjects in general cover the following three aspects:

- 1) Courses on mathematics or mathematics education,
- 2) Teaching practice and investigation of social aspects, and
- 3) Relatively independent part of a research task which is on someone in the department.

At the same time, the head of the department gives students a lecture or talk interpreting the meaning, importance and concrete procedure of SRT. The lecture or talk must point out what the students should do and not do during SRT.

Every senior student must write one paper and hand it in to the department. Its subject may be any one from the reference subjects or a subject sought out or made up by the student author himself. The head of the department appoints a guide teacher for each student according to his or her subject and instructs the student to consult initially his or her guide teacher on research. Each guide teacher may direct a number of students. Each student collects references or material and writes his/her own manuscript following his or her guide teacher.

Through reviewing by the guide teacher and repeated modifying or correcting by the student author himself, the student author hands his final version of SRT paper in to his or her guide teacher to assess it.

A symposium for exchanging and assessing student SRT papers is held in the department after the SRT papers have been judged by guide teachers. Its author in the symposium reports the most excellent paper in a field. All of the senior students and their guide teachers are expected to attend the symposium.

Finally, the head of the department presides over a guide teacher meeting to decide the scores for students in SRT. All guide teachers must attend the meeting. There are five grades: A, B, C, D and E. Grades A to D mean passing but the E means failing completely. Scores are decided depending only on the opinion of the guide teachers. The opinion is formed according to originality, difficulty, and quantity of calculating and reasoning, exactness of expression, mistakes etc. The best one or two SRT papers are recommended for publication in the Selection of Excellent Graduation Theses of Liaocheng Teachers University in that year.

2. EFFECT

2.1. Effect on Teaching

To make up reference subjects for training students to do research, guide teachers must discover and integrate problems or questions. This requires teachers to study text- books deeply, trying to reform the traditional content with great concentration and offering new opinions often.

To pass SRT smoothly, students pay attention to the subject option, to collecting references and to doing exercises carefully, and have some discussion on academic topics with each other when they learn courses for mathematics or mathematics education. SRT diminishes the negative effects of inertia of thinking preparation of students, strengthens the ability of their thinking divergence, and encourages students to cultivate a habit of original thinking. SRT benefits teachers and students alike, as the boat goes up when the river rises, that is, particular things improve with the improvement of the general situation.

Each year, senior students write papers for SRT. Teachers and students hand in a number of new questions for SRT, and produce a number of results. It no longer limits teaching to just passing on knowledge. Furthermore, fostering the students' ability to read, to create and to write become important points in teaching mathematics.

2.2. Effect on Scientific Research in University

SRT has prompted teachers with stronger and stronger motivation to do research on mathematics or mathematics education and has broadened their thinking as well. Their references have become more and more integrated. The achievement is increasing every year. At present, the majority of teachers have published their own papers. Some of them have published texts or reference books for teaching. A few teachers have published monographs.

2.3. Effect on Teaching Mathematics in Middle School

The majority of students majoring in mathematics education teach mathematics at the middle level school after graduation. That every new teacher has ability and experience in doing scientific research forms an atmosphere conducive to researching mathematics and mathematics education among teachers of middle school. So the number of teachers doing research on mathematics or mathematics education is larger and larger every year and the number of periodicals are increasing more and more.

2.4. Effect on Quality of Graduate Students in Key University

The education of Chinese graduate students of China began in 1978 after the chaotic Cultural Revolution in China. During the 1980's, each university of China enrolled fresh graduate students depending only on the entrance examination. However, the situation changed in the 1990's. The enrollment of graduate students depends not only on the

entrance examination but also on the SRT score and the SRT paper of the students concerned. Every university now takes students who are excellent both in choice of subjects and in quality of writing.

3. MISHAPS

Though the dean, teachers and the students who want to become graduate students attach great importance to SRT, the employers of graduated students majoring in mathematics education almost never pay attention to SRT. When interviewing graduated students, almost no employers want to see the students' SRT paper or score. When key middle schools enroll fresh staff, the president pays attention only to the lecture given by graduated students.

This creates an environment in which the majority of students and some guide teachers do not attach importance to SRT. During SRT, some students neither look for references carefully nor think deeply. They take parts out of some textbooks and summarize them briefly, or have a copy of conclusion from a book, muddle through their SRT paper, and just go through the motions. They hand their SRT papers in, and every thing goes well. Worse, some students even plagiarize others' papers.

Some guide teachers also do not attach importance to SRT. Most of them publish nothing, and never even offer a reference subject for SRT. They do not strictly implement the criterion for assessing SRT scores of students. Though there are five grades of score for assessing students in SRT, they marked the student paper only in three grades, that is, A, B and C, or even two: A and B, so there is no marked difference between good and bad. Some students get by under false pretenses. This phenomenon is like the false, imitation, and inferior commodities filling market.

At present, it is spreading and there is a possibility that it will become a main current. If this kind of action continues to happen, SRT will exist in name only and be worthless action.

4. PERSONAL OBSERVATIONS

4.1. Performing SRT is an essential reform of mathematics education in China

1. Change the guide ideology for mathematics education

The traditional guide ideology for mathematics education in China was mainly passing on knowledge and skill etc. That means what students must be able to do what the teacher has taught, i.e. propagating doctrines of the ancient sages. Now the situation has changed. The purpose of teaching mathematics is fostering the student's ability to apply

mathematical knowledge in practice, to create new concepts of mathematics, to make new methods of mathematics and to construct new theories of mathematics. The process of teaching mathematics helps students without the above abilities. SRT will help to achieve the purpose.

2. Strengthen the foundation of research on mathematics in China

A large number of mathematics researchers now in China were mathematics educators or students majoring in mathematics education. This is an evidence that SRT helped the increasing of number of mathematics researchers. We can expect that SRT will help the increase in future.

3. Accelerate combination of mathematics with economic development

Only if mathematics teachers living throughout the country, from big cities to small villages, possess the interest and ability to apply mathematics into practice, will they use mathematics knowledge to solve the problem around them. This accelerates combination of mathematics and economic development.

4.2. Nature of SRT

SRT teaches students to learn how should they do research on mathematics or mathematics education so learning itself is more important than research results. For each student, the nature of SRT is only a process of learning. For teachers, SRT is part of the whole process of teaching, or an important profile in which teachers help students to know the basic methods of mathematics or mathematics education research. Graduated students who have experienced SRT are aware of diverse and creative ways of thinking, and are used to observe carefully, consider deeply, and write articles often. Through SRT, students have gotten rid of their own sense of inferiority and enhanced their confidence, independence and strength by themselves. They exercise their psychology in SRT. Their psychological quality has been improved.

The purpose of SRT is to help students to have the ability to raise questions beyond the text book, to develop their ability to reason mathematically, calculate, imagine intuitively, abstract mathematical questions from the real life, draw out questions of mathematics education from education or teaching practice, to suppose or guess some results of mathematics, and to express themselves in words. These abilities of graduated students must play an important role when the students work at their posts respectively in the future.

5. TENTATIVE IDEAS ABOUT SRT IN FUTURE

5.1. *Make SRT to be concerned by entire society*

Now most of the executive branches of education at all levels take the proportion of students entering schools of a higher grade as the unique standard to assess middle school. This situation should be changed. The executive branches of education at all levels should take the results of research on education, teaching method and pure science as another standard to assess middle school.

Mathematics teachers of middle school should be adept at teaching mathematics. Outstanding teachers must be specialists in teaching mathematics. However, the teachers of mathematics in middle level schools should not only be adept at teaching mathematics but also possess the ability to do research on education, teaching or science of mathematics. They should have better results in research on education, teaching or science of mathematics. China can develop creative ability in the majority of youngsters only if the nation possesses a large number of excellent mathematicians.

When a key middle school enrolls fresh staff, the school should consider not only the ability to give a lecture but also the followings:

- 1) Ability to do research on science of mathematics,
- 2) The selected subject in SRT, and
- 3) The SRT score.

We can imagine that when key middle schools hire new teachers, if the school looks at the ability to do scientific research as one of the standards, the level of teaching mathematics and research on mathematics in China will be raised to a higher step. The need of society is a strong motive force. If society raises its requirements, SRT in China must be adequate in all aspects. All of the students, teachers, even entire society must be concerned with the improvement of SRT.

5.2. *Normalize overall SRT*

1. *Normalize the organization and the process of implementation*

The department should set up a stable organ that is in charge of SRT; that is, the organ should do the following:

- a) Draw a speech outline with clear points and aimed at the students so the head of the department can mobilize the students to take part in SRT regularly,
- b) Collect, preserve, print and hand out the reference subjects,
- c) Keep the files and records of SRT safely, and

- d) Keep the process of implementing SRT basically stable and enhance it every year.

The department should set up regular records for SRT and administrate them by computer. The following information should be included:

- a) Records of reference subjects for SRT and the indication of contents for each reference subject, references for completing the subjects, raiser and the time raised at, field the subjects belong to respectively, situation of accomplishment etc.,
- b) Records of reference subjects chosen by students; Indication of the names of reference subjects for SRT, names of students who chosen the reference subjects and their academic degree, situation of accomplishment etc., and
- c) Papers written by students and the records of unsolved problems.

Using the files or records of SRT, one can avoid repeating choice of subject and copying of others' papers.

2. *Discuss ways to reform or enhance SRT regularly*

The head of the department convenes two meetings to discuss ways of improving SRT in each year. The guide teachers of SRT and the clerk working in the organ for SRT should attend these meetings. The first meeting should be held about one month before the beginning of SRT. The following should be included:

- The basic situation of the students that year should be observed, for example, the number of students, scores in learning courses, how many students enrolled for the entrance examination to study for the masters degree, what are the interests of these students etc.
- Design the plan for SRT.
- Appoints and announces guide teachers to make up or revise the reference subjects for SRT.

The second meeting should be held within two weeks of the end of SRT. The following should be included:

- Summarize concretely the success and failure in SRT that year.
- Seek out the real reasons.
- Put forward proposals for enhancing the quality of SRT as well as revising the reference subjects.

After the second meeting, the organ for SRT or the head of the department summarizes SRT systematically in written form.

3. *Normalize the assessing of SRT*

The head of the department formulates detailed rules and regulations for assessing SRT. The guide teachers follow them to go over the student's manuscript for SRT and to write the comments in which the following should be included:

- Name of author,
- Title of the paper,
- Main result,
- Strictness and accuracy in proof and calculating,
- Clarity of expression,
- Ease and grace in language, and
- Score.

The standard for assessing SRT and its detailed rules and regulations should benefit the development of student creativity and deal with the relationship between the scores of SRT papers, courses and teaching practice. The student who chooses a subject of great difficulty, works out new results, and creates a new method should get an "A" grade. Conversely, the student who chooses a subject of little difficulty, works out few new results, without any new advance in method, but only summarizes the text book, should get a score of D. The student who plagiarized someone else's paper or just summarized known results should get a score of E. A student who does work on a subject raised by himself should get a higher score than someone who does work on a subject raised by others.

4. *Tentative ideas about qualification and duties of the guide teachers*

The guide teachers for SRT should possess plenty of experience of teaching mathematics, published research results and independently devised reference subjects for SRT. The following are duties of guide teachers.

- a) Raise reference subjects for SRT,
- b) Introduce students to the correct way of doing research on mathematics or mathematics education,
- c) Encourage students to raise new subjects and do research on them,
- d) Check and mark SRT papers to encourage the students reason strictly, calculate carefully, imagine intuitive figures boldly and express as well as write seriously,

- e) Assess papers written by students for SRT and give out the scores, and
- f) Raise new plans for SRT.

5. *Tentative ideas on reference subjects for SRT*

It is clear that the reference subjects can inspire students to do research though the majority of students have never chosen their subjects from the reference subjects. The subjects raised by students themselves are often similar or connected to one of the reference subjects. The reference subjects chosen by students are those which have been mentioned by teachers in class, or which are related to many points of knowledge in text books, or which interest them.

We gave the following three reference subjects:

Subject 1: List and prove the equivalent conditions for developable surface.

Subject 2: Prove that there is no solution for the following ancient problem of mathematics.

In ancient times, there were three houses and three wells in a small village. Construct one path from each house to each well. How should one construct the nine paths so that the nine paths do not intersect each other?

Subject 3: Let n be a positive integer. Calculate the number of topologies on a set of n elements.

Since Subject 1 relates closely to the differential geometry text book, it is often mentioned in class, so the students interested in differential geometry think that they know how to do research on the subject, though they may not be able to list all of the equivalent conditions. A few students who chose the subject, could list more than 40 equivalent conditions for developable surface, and proved the equivalence among them in detail, in a short time.

Though Subject 2 is a strange problem, it is interesting for some students of mathematics. The first author of this paper mentioned it in algebraic topology classes. When the reference subject was made available to students, there was a student who chose it at once. She found three ways of proof with her knowledge learned during undergraduate period.

Subject 3 is a famous and old but still challenging open problem. For many years nobody chose it. When the first author of this paper changed it to calculating the number of topologies on a set of four elements and worked out the number 355 with a computer and the remainder only checking the calculating result of the microcomputer, one student chose it.

From the above, the authors have the following tentative ideas about reference subjects:

- 1) The subjects related to courses of mathematics should be open, higher than the text book, not separate themselves from the text book and more difficult than usual exercises and examination problems. They should also be interesting and solvable with the knowledge learned during undergraduate period. If the subject is too difficult, the student will not or can not deal with the subject; too easy, and it will be useless for SRT.
- 2) Reference subjects related to teaching practice or investigations of society should reflect applied psychology, pedagogy, teaching theory, and theory of probability or of statistics etc.
- 3) Every subject should be original but only summarized.

From the above, we drew the following reference subjects in the field of differential geometry:

- 1) Which properties does a surface with linear Gauss curvature possess?
- 2) Is there any curve with zero torsion on a hyperbolic paraboloid?
- 3) Find the family of osculating cylinders along a given curve with nonzero torsion.
- 4) How can we determine whether a curve with nonzero torsion is on a cylinder (or sphere)?
- 5) Is there a circle on a hyperbolic paraboloid? Is there a parabolic on a helicoid?
- 6) What is the necessary and sufficient condition for a circle (parabolic, hyperbolic, helix) on a surface?
- 7) What is the necessary and sufficient condition for all geodesics of a surface being circles?
- 8) What is the necessary and sufficient condition for all curvature lines on a surface being circles?
- 9) What is the equivalent condition for a surface being ruled?
- 10) Which properties does a surface with linear (nonzero) mean curvature possess?
- 11) How can one find the principal normal (tangent, binormal) line through a given point out of the curve?
- 12) What is the curve on which the curvature κ and torsion τ satisfying $\kappa + \tau = 1$?
- 13) Assume that there is a net of asymptotic lines on a surface. Find the locus of curvature center of the asymptotic lines of the surface if the asymptotic lines are not straight lines.
- 14) Find the locus of curvature centers of curvature lines on a surface.
- 15) Classify the minimal surfaces.

- 16) How many osculants on a curve with nonzero torsion should be deserved to discuss? Which properties do the osculants possess respectively?
- 17) Find the locus of focuses of parabolics osculating to a helix.
- 18) Take oriented segments whose lengths are same a constant, start points on a given curve and directions pointing to the positive direction on the principal normal (tangent, binormal). Find the locus of the middle points.
- 19) Suppose that a straight line through the focus of osculating parabolic of a curve with nonzero torsion and parallel to the binormal at the same point of the curve moves along the curve. What is the surface that the straight line forms?
- 20) Which properties does a curve with non-constant curvature (or torsion) possess?
- 21) What is the surface formed by tangent (principal normal, binormal) line of a curve with constant curvature?
- 22) Suppose that a surface is formed by the principal normal (or binormal) of a curve with constant curvature. What is the sufficient and necessary condition for the surface being a helicoid?
- 23) What is the surface formed by the principal normal (binormal, tangent) of a curve with constant torsion? What is the necessary and sufficient condition for the surface being a helicoid?
- 24) Suppose that a surface is formed by the principal normal (binormal, tangent) of a curve with constant curvature. What is the net of curvature (or asymptotic) lines?
- 25) What is the necessary and sufficient condition for existence of an envelope of a family of surfaces?
- 26) What is the locus of osculating circles to a curve with constant curvature (or torsion)?
- 27) What is the locus of osculating parabolics to a curve with nonzero torsion?
- 28) What is the second (or third) fundamental form of the Gaussian mapping of helicoid?

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