J. KSIAM Vol.3, No. 2, 121-135, 1999

Enhanced Algorithms for Reliability Calculation of Complex System[1)]

Seong Cheol Lee

Abstract

This paper studies the problem of inverting minimal path sets to obtain minimal cut sets for complex system. We describe efficiency of inversion algorithm by the use of boolean algebra and we develop inclusion-exclusion algorithm and pivotal decomposition algorithm for reliability calculation of complex system. Several examples are illustrated and the computation speeds between the two algorithms are undertaken.

1. Introduction

The reliability literature of the past 10 years contains many papers with reliability calculation of coherent structure. This paper extends [10] and discussed the some algorithms for reliability calculation. Problems related to the coherent structure of the system are based on [6]. The path set and cut set method for determining system reliability [1] is used. In section 2, we faintly survey an inversion algorithm by Heidtmann to the case of inverting paths and cuts of 2-state systems. The method in section 2 is based on symbolic manipulation of boolean functions by applying two de Morgan's laws. To invert paths from cuts and vice versa the boolean structure function must be derived, inverted, and reduced. Then the sets can be deduced. In section 3 and 4, we propose an inclusion-exclusion algorithm and a pivotal decomposition algorithm by the use of inclusion-exclusion formula and the decomposition rule. These algorithms are easy to program and expedient for automated computation.

Notation

N set of integers from 1 to n

This paper was support by Research Fund, Namseoul University 1998

Key word : inversion algorithm, inclusion-exclusion algorithm, pivotal decomposition algorithm

I	subset of N
2^N	power set of N
$N\{A\}$	number of element of A
A'	inverse of A
A^*	star function of A
x_i	boolean variable of i-th component
$\bar{x}_i$	complement boolean variable of i-th component
S	success expression of the system
R	reliability expression of the system
$\psi(x)$	system structure function
$h(p)$	system reliability function

2. Inversion Algorithm and Algebraic Technique

K. D. Heidtmann suggested an inversion algorithm in his paper [3]. Now we will try to describe his algorithm in simpler terms. Each component of a system of n component is uniquely represented by its index $i \in N$, and any assembly of components by a subset I of N. So any path or cut is a subset of N, and set of all paths, or the set of all cuts, is a subset of the power set 2^N. The concept of inverse combines two complements to invert paths and cuts as subsets A and A' of 2^N; that is, if A is set of paths, then A' is set of cuts; and vice versa.

Let $A \subset 2^N$, $I \subset N$. From the complement $\bar{I}$ of every I for $I \in A$. Eliminate from 2^N all such $\bar{I}$. The result is A', the inverse of A. More formally, let $A \subset 2^N$, $A' \subset 2^N$, $I \subset N$. Then A' is the inverse of A if and only if for every possible I, either $I \in A$ or $\bar{I} \in A'$. The inverse property is reciprocal: $(A')' = A$. The number of elements of A, plus the number of elements in A' is 2^n ; $n\{A\} + n\{A'\} = 2^n$. There exists one and only one inverse of A.

Let $A \subset 2^N$, $I \subset N$. From the complement $\bar{I}$ of every for $I \in A$. A^* is the set of all those $\bar{I}$. A and $(A')^*$ are a partition of 2^N. $A' = (A^{c*}) = (A^c)^*$. These inverse and star relationships were derived and proved in Lee(1996)[8].

In applying the inverse concept by hand calculation, many sets must be looked at because 2^N contains 2^n elements. Thus it is helpful to verify the correctness of a computed A' by the following test which is based on the facts that A and $(A')^*$ form a partition of 2^N, and $n\{2^N\} = 2^n$.

Test : Let A' be the inverse of A, $A \subset 2^N$. Then A and A' satisfy
$n\{A\} + n\{A'\} = 2^n$.

For automated calculation, the inversion algorithm can be used. After execution, A' contains all cuts if A contains all paths, and vice versa.

Inversion Algorithm

Input : A, N

step 1. Compute A^*. The I are the elements of 2^N.

step 2. Set $I \leftarrow \emptyset$ and $A' \leftarrow \emptyset$.

step 3. If $I \notin A^*$ *then* $A' \leftarrow A' \cup \{ I \}$.

step 4. If $I \neq N$ *then* replace I by its successor and go to 3.

step 5. Stop(A' is inverse of A).

The test yields n{ A}+ n{ A'} = 3+ 5 = 2^3.

Example 1. The bridge structure is shown in the following diagram

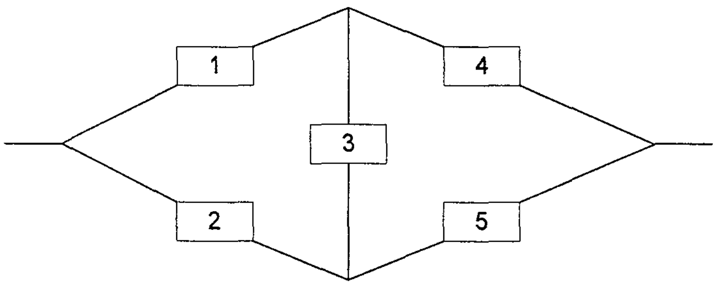

Fig.1 5-component bridge system

There are five components; $n=5$, $N=\{1, 2, 3, 4, 5\}$

$2^N = \{\emptyset, \{1\}, \cdots, \{5\}, \{1,2\}, \cdots, \{4,5\}, \{1,2,3\}, \cdots, \{3,4,5\}, \{1,2,3,4\}, \cdots, \{2,3,4,5\}, \{1,2,3,4,5\}\}$

$n\{2^N\} = 2^5$.

This system has 16 paths which conclude 4 minimal path sets.

$$A = \{I_1, I_2, \cdots, I_{16}\} = \{\{1,4\}, \{2,5\}, \cdots, \{1,2,3,4,5\}\}.$$

$$\overline{I_1} = \{\overline{1,4}\} = \{2,3,5\}, \overline{I_2} = \{\overline{2,5}\} = \{1,3,4\}, \cdots, \overline{I_{16}} = \{\overline{1,2,3,4,5}\} = \emptyset.$$

Thus $A^* = \{\{2,3,5\}, \{1,3,4\}, \cdots, \emptyset\}$.

By remove the elements of A^* from 2^N.

$$A'=\{\{1,2,3\},\{4,5\},\cdots,\{1,2,3,4,5\}\}.$$

The test yields : $n\{A\}+n\{A'\}=16+16=2^5$.

According to the above example, the inversion algorithm is useful in finding inverting paths and cuts of 2-states systems, but it depends upon set theory. Therefore, we need another algorithm in order to easily calculate system reliability.

Now, we present an algebraic technique computing system reliability. The system success expression S is given by the union of all the system minimal path sets: that is

$$S=\bigcup_{i=1}^{m} P_i$$
$$=P_1+\overline{P_1}P_2+\overline{P_1}\,\overline{P_2}P_3+\ldots+\overline{P_1}\,\overline{P_2}\ldots\overline{P_{m-1}}P_m \tag{1}$$

Each P_i in (1) is a product term of the form

$$P_i=\prod_{j=1}^{n_i} x_{ij}, \qquad i=1,2,\ldots,m \tag{2}$$

When (2) is substituted in (1), and all P_i are expanded using DeMorgan's Law, the product terms in the resulting sum of product expression lose their disjointedness. By above two formulas, we reduce following algorithm which gives a minimum reliability expression.

Boolean Algorithm

Step 1. Enumerate all the m paths of the system and arrange these in a sequence

$$P_1, P_2, \ldots, P_m.$$

Step 2. Write S in a more convenient from given by (1)

$$S=P_1+\overline{P_1}(P_2+\overline{P_2}(P_3+\overline{P_3}(\ldots(P_{m-1}+\overline{P_{m-1}}(P_m))\ldots))). \tag{3}$$

Step 3. Substitute expression of P_m in (3); let $j=m-1$.

Step 4. In the resulting expression substitute expression of P_i ; let $j=j-1$.

Step 5. Repeat step 4 if $j\geq 1$.

Step 6. Replace logical variables by their reliabilities to get the required reliability expression.

For illustration, consider the 5-component bridge system shown in Fig. 1 which is also solved in [8].

Step 1. Paths are $P_1=\{1,4\}$, $P_2=\{1,3,5\}$, $P_3=\{2,5\}$, $P_4=\{2,3,4\}$.

Step 2. $S=P_1+\overline{P_1}(P_2+\overline{P_2}(P_3+\overline{P_3}(P_4)))$

Step 3, 4, 5.

$$S=x_1x_4+\overline{x_1x_4}(x_1x_3x_5+\overline{x_1x_3x_5}(x_2x_5+\overline{x_2x_5}(x_2x_3x_4)))$$
$$=x_1x_4+\overline{x_1x_4}x_1x_3x_5+\overline{x_1x_4x_1x_3x_5}x_2x_5+\overline{x_1x_4x_1x_3x_5x_2x_5}x_2x_3x_4$$
$$=x_1x_4+x_1x_3\overline{x_4}x_5+\overline{x_1}x_2\overline{x_3x_4}x_5+\overline{x_1}x_2x_3x_4\overline{x_5}$$

Step 6. $R=p_1p_4+p_1p_3(1-p_4)p_5+(1-p_1)p_2(1-p_3)(1-p_4)p_5+(1-p_1)p_2p_3p_4(1-p_5)$

Any assignment of 0-1 values to the x_i's that makes the cut set polynomial equal to 1 corresponds to system failure; ie, no path of good arcs exists in the bridge system. Now we propose some algorithms for reliability calculation of simple and complex system.

3. Inclusion-Exclusion Algorithm

The inclusion-exclusion rule came from the additive law of probability. So the inclusion-exclusion method provides successive upper and lower bounds on system reliability which converge to the exact system reliability. In system reliability calculations by the inclusion-exclusion method, large numbers of pairs of identical terms with opposite signs cancel. For any system with n minimal path sets the number of terms generated in step i of the method is $\binom{n}{i}$, so that the Poincaré formula consist of $\sum_{i=1}^{n}\binom{n}{i}=2^n-1$ terms.

Two of the terms cancel if a union of i minimal path sets contains exactly the same components as a union of j minimal path sets ($1\le i$, $j\le n$, $|i-j|=1$) Therefore the reliability analysis of all systems having pairwise disjoint minimal path sets, i.e. which have redundant component, is affected by this cancelling terms. For nearly all large complex systems the number of cancelling terms is enormous, so that avoiding these terms affords an important computational advantage.

Reliability analysis by the original method of inclusion-exclusion assumes the knowledge of all minimal path or cut sets[1].

Let $E_i(\overline{E}_i)$ be the event that i-th component x_i is functioning (failed) with probability $p_i(1-p_i)$.

Let $A_r(\overline{A_r})$ be the event that all components in r-th minimal path set P_r is functioning (failed).

i.e.

$$A_r \equiv \bigcap_{x_i \subset P_r} E_i, \quad \overline{A_r} \equiv \bigcap_{x_i \subset K_r} \overline{E_i} \tag{4}$$

where K_r is r-th minimal cut set. Then

$$P(A_r) = P\,(\bigcap_{x_i \subset P_r} E_i), \quad P(\overline{A_r}) = P\,(\bigcap_{x_i \subset K_r} E_i) \tag{5}$$

System success corresponds to event $U_{r=1}^{n} A_r$ if the system has n minimal path sets.

The the system reliability function

$$h(p) = P[\bigcup_{r=1}^{n} A_r] \tag{6}$$

Let

$$S_k = \sum_{1 \leq i_1 < i_2, \cdots i_k \leq n} P[Ai_1 \cap Ai_2 \cap \cdots \cap Ai_k]$$

by the inclusion-exclusion principal[2]

$$h(p) = \sum_{k=1}^{n} (-1)^{k-1} S_k \tag{7}$$

and

$$h(p) \leq S_1$$
$$h(p) \geq S_1 - S_2 \tag{8}$$
$$h(p) \leq S_1 - S_2 + S_3$$

and so on.

Now we define the approximation to system reliability function $h(p)$ of step m by

$$h^{(m)}(p) \equiv \sum_{k=1}^{m} (-1)^{k-1} S_k \tag{9}$$

For $m > 1$

$$h^{(m)}(p) = h^{(m-1)}(p) + (-1)^{m-1} S_m \tag{10}$$

$$h^{(n)}(p) = h(p) \tag{11}$$

Although it is not true in general that the upper bounds decrease and the lower bounds increase, in practice it may be necessary to calculate only a few S_k's to obtain a close approximation. Of course similar formulas for computing system unreliability $\overline{h}(p)$ in terms of minimal cut sets and component unreliabilities $1 - p_i$ can be given. Now we state the algorithm in detail.

Inclusion- Exclusion Algorithm

Input : $n,\ p_1,\ p_2,\ \cdots,\ p_n,\ N_0$ or ε

Output : $h(p)$

step 1. set $i=1$

step 2. while $i \le N_0$ do step 3~5

step 3. set $S_i = \sum_{1 \le r_1 < \cdots < r_i \le n} P_{r_1} \cdots P_{r_i}$ (*compute* S_i)

$h(p) = \sum_{i=1}^{n} (-1)^{i-1} S_i$

step 4. if $|S_i - S_{i-1}| < \varepsilon$

output $h(p)$

stop

step 5. set $i = i+1$

$S_i = S_{i+1}$

step 6. output (method failed after N_0 iterations, N_0 or $h(p)$)

4. Pivotal decomposition algorithm

This section, gives the algorithm for pivotal decomposition rule. The following identity holds for any structure function ϕ of order n:

$$\phi(x) = x_i \cdot \phi(1_i, x) + (1-x_i) \cdot \phi(0_i, x) \tag{12}$$

We immediately obtain the corresponding pivotal decomposition of the reliability function.

$$h(p) = E[\phi(X)]$$

$$= p_i \cdot h(1_i, p) + (1-p_i) \cdot h(0_i, p) \quad i = 1, \cdots, n \tag{13}$$

Now we proposed the following algorithm.

Algorithm for series system

Input : $n,\ p_1,\ p_2,\ \cdots,\ p_n$

Output : $h(p)$

step 1. $i=1$

step 2. while $i \le n$ do step 3~4

step 3. set $h(0_i, p) = 0$

$h(1_i, p) = p_{i+1} \cdot h(1_{i+1}, p) + (1-p_{i+1}) \cdot h(0_{i+1}, p)$

$h(p) = p_i \cdot h(1_i, p) + (1-p_i) \cdot h(0_i, p)$ (compute $h(p)$)

step 4. $i = i + 1$

step 5. output $h(p)$

Algorithm for parallel system

Input : $n, p_1, p_2, \cdots, p_n$

Output : $h(p)$

step 1. $i = 1$

step 2. while $i \le n$ do step 3~4

step 3. set $h(1_i, p) = 1$

$h(0_i, p) = p_{i+1} h(1_{i+1}, p) + (1-p_{i+1}) h(0_{i+1}, p)$

$h(p) = p_i \cdot h(1_i, p) + (1-p_i) \cdot h(0_i, p)$ (compute $h(p)$)

step 4. $i = i + 1$

step 5. output $h(p)$

5. Numerical Examples

Now we present several examples. We applied the inclusion-exclusion method in actual practice which concerned airplane operation systems (Lee 1991, 1993)[5],[6]. Suppose that an airplane engine will operate, when in flight, with probability p_i independently from engine to engine; Suppose that the airplane will make a successful flight if at least 50% of its engines remain operative.

Example 2. (1-out-of-2:G system) We consider 2-engine plane. From (Lee 1993-1)[6], it has two minimal path sets

$$P_1 = \{1\}, \quad P_2 = \{2\}$$

Thus the first bound on the reliability $h^{(1)}(p)$ is

$$h^{(1)}(p) = S_1 = \sum_{i=1}^{2} P(Ai)$$

$$= p_1 + p_2$$

And the second bound is

$$h^{(2)}(p) = h^{(1)}(p) - S_2$$

$$= h^{(1)}(p) - P(A_1 \cap A_2)$$

$$= p_1 + p_2 - p_1 p_2$$

Hence by (11), system reliability

$$h(p) = h^{(2)}(p)$$

The 1-out-of-2 : G system is also a 2-out of-2 : F system with only one minimal cut set

$$K_1 = \{1, 2\}$$

Hence unreliability

$$\bar{h}(p) = P(\overline{A_1})$$
$$= (1-p_1)(1-p_2)$$

Next we consider 4-engine plane.

Example 3.(2-out-of-4: G system) From[5] it has six minimal path sets;

$$P_1 = \{1, 2\}, \quad P_2 = \{1, 3\}, \quad P_3 = \{1, 4\}$$
$$P_4 = \{2, 3\}, \quad P_5 = \{2, 4\}, \quad P_6 = \{3, 4\}$$

Thus the first bound on the reliability is

$$h^{(1)}(p) = S_1 = \sum_{i=1}^{6} P(Ai)$$

$$= p_1 p_2 + p_1 p_3 + p_1 p_4 + p_2 p_3 + p_2 p_2 + p_3 p_4$$

Since

$$P(A_1 \cap A_2) = p_1 p_2 p_3 , \quad P(A_1 \cap A_6) = p_1 p_2 p_3 p_4$$

Hence the second bound is

$$h^{(2)}(p) = h^{(1)}(p) - S_2$$

$$= h^{(1)}(p) - \sum_{1 \le i_1 < i_2 \le 6} P(Ai_1 \cap Ai_2)$$

$$= h^{(1)}(p) - 3(p_1 p_2 p_3 + p_1 p_2 p_4 + p_1 p_3 p_4 + p_2 p_3 p_4 + p_1 p_2 p_3 p_4)$$

Similarly

$$h^{(3)}(p) = h^{(2)}(p) + S_3$$

$$= h^{(2)}(p) - \sum_{1 \le i_1 \le i_2 \le i_3 \le 6} P(Ai_1 \cap Ai_2 \cap Ai_3)$$

$$= h^{(2)}(p) + p_1 p_2 p_3 + p_1 p_2 p_4 + p_1 p_3 p_4 + p_2 p_3 p_4 + 16 p_1 p_2 p_3 p_4$$

The 4 terms of $h^{(3)}(p)$ being products of 3 factors cancel 4 terms of $h^{(2)}(p)$, and the 3 terms $p_1p_2p_3p_4$ of $h^{(2)}(p)$ cancel 3 of the 16 terms of $h^{(3)}(p)$.

This results in

$$h^{(4)}(p) = h^{(3)}(p) - 15p_1 p_2 p_3 p_4$$

$$h(p) = h^{(6)}(p) = h^{(5)}(p) - p_1 p_2 p_3 p_4$$

$$h^{(5)}(p) = h^{(4)}(p) + 6p_1 p_2 p_3 p_4$$

with 46 cancelling terms.

The 2-out-of-4 : G system is also a 3-out-of-4 : F system with four minimal cut sets

$$K_1 = \{1, 2, 3\} \qquad K_2 = \{1, 2, 4\} \qquad K_3 = \{1, 3, 4\} \qquad K_4 = \{2, 3, 4\}$$

$$\overline{h^{(1)}}(p) = \sum_{i=1}^{4} P(\overline{A_i})$$

$$= (1-p_1)(1-p_2)(1-p_3) + (1-p_1)(1-p_2)(1-p_4) + (1-p_1)(1-p_3)(1-p_4) + (1-p_2)(1-p_3)(1-p_4)$$

$$\overline{h^{(2)}}(p) = \overline{h^{(1)}}(p) - 6(1-p_1)(1-p_2)(1-p_3)(1-p_4)$$

$$\overline{h^{(3)}}(p) = \overline{h^{(2)}}(p) + 4(1-p_1)(1-p_2)(1-p_3)(1-p_4)$$

$$\overline{h}(p) = \overline{h^{(4)}}(p) = \overline{h^{(3)}}(p) - (1-p_1)(1-p_2)(1-p_3)(1-p_4)$$

There are 8 cancelling terms instead of 46 and much less computation because $2^6 - 2^4 = 48$ fewer terms.

Example 4. We consider a 3-component series system and a 3-component parallel system with success probabilities $p_1 = 0.6$, $p_2 = 0.7$, $p_3 = 0.8$ in Fig.2 and Fig.3. By pivotal decomposition algorithm, series system reliability is

$$\begin{aligned} h(p) &= p_1 \cdot h(1_1, p) + (1-p_1) \cdot h(0_1, p) \\ &= p_1\{p_2 h(1_2, p) + (1-p_2) h(0_2, p)\} + (1-p_1) h(0_1, p) \\ &= p_1\{p_2 p_3 + (1-p_2) \cdot 0\} + (1-p_1) \cdot 0 \\ &= p_1 p_2 p_3 \\ &= 0.336 \end{aligned}$$

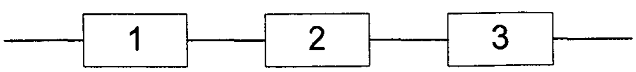

Fig.2 3-component series system

parallel system reliability is

$$\begin{aligned} h(p) &= p_1 \cdot h(1_1,p)+(1-p_1)\cdot h(0_1,p) \\ &= p_1\cdot 1 + (1-p_1)\{p_2 h(1_2,p) + (1-p_2)\,h(0_2,p)\} \\ &= p_1 + (1-p_1)\{p_2+(1-p_2)p_3\} \\ &= 0.6+0.4(0.7+0.3\times 0.8) \\ &= 0.976 \end{aligned}$$

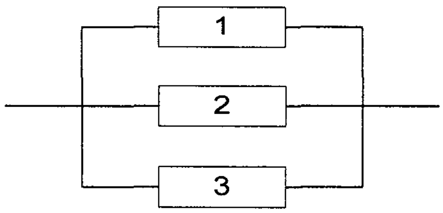

Fig.3 3-component parallel system

For a general non-series parallel system (having unequal probabilities of success), the only known practical method of exact analysis is the path and cut-set method.

Example 5.(7-component bridge system) Consider bridge system in fig.4.

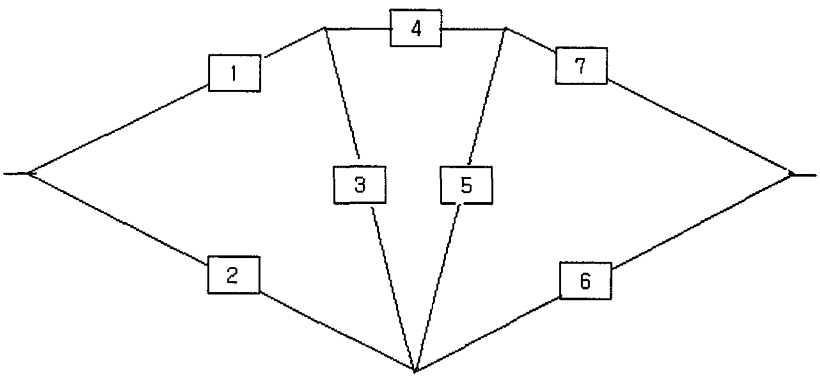

Fig.4 7-component bridge system

There are 7 minimal path set;

$$P_1=\{1,4,7\},\ P_2=\{1,3,6\},\ P_3=\{1,3,5,7\},\ P_4=\{1,4,5,6\},$$

$$P_5=\{2,6\},\ P_6=\{2,3,4,7\},\ P_7=\{2,5,7\}$$

System reliability is

$$\begin{aligned} h(p) = {} & p_1p_4p_7 + (1-p_1p_4p_7)\,[p_1p_3p_6+(1-p_1p_3p_6)\{p_1p_3p_5p_7+(1-p_1p_3p_5p_7) \\ & \cdot\{p_1p_4p_5p_6+(1-p_1p_4p_5p_6)\{p_2p_6+(1-p_2p_6)\{p_2p_3p_4p_7+(1-p_2p_3p_4p_7)p_2p_5p_7\}\}\}\}] \end{aligned}$$

Example 6. (8-component complex system) Consider complex system in fig.5.

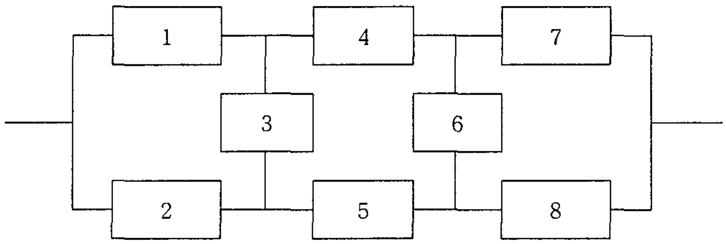

Fig.5 8-component complex system

There are 8 minimal path sets;

$$P_1=\{1,4,7\},\ P_2=\{2,5,8\},\ P_3=\{1,3,5,8\},\ P_4=\{1,4,6,8\},\ P_5=\{2,3,4,7\},$$
$$P_6=\{2,5,6,7\},\ P_7=\{1,3,5,6,7\},\ P_8=\{2,3,4,6,8\}$$

System reliability is

$$\begin{aligned} h(p) = {} & p_1p_4p_7 + (1-p_1p_4p_7)\,[p_2p_5p_8+(1-p_2p_5p_8)\{p_1p_3p_5p_8+(1-p_1p_3p_5p_8) \\ & \cdot\{p_1p_4p_6p_8+(1-p_1p_4p_6p_8)\{p_2p_3p_4p_7)+(1-p_2p_3p_4p_7)\{p_2p_5p_6p_7+(1-p_2p_5p_6p_7) \\ & \cdot\{p_1p_3p_5p_6p_7+(1-p_1p_3p_5p_6p_7)p_2p_3p_4p_6p_8\}\}\}\}\}] \end{aligned}$$

Now, we consider the following example in order to compare two algorithms in computational complexity.

Example 7. In example 1, there are 4 minimal path sets

$$P_1=\{1,4\},\ P_2=\{1,3,5\},\ P_3=\{2,5\},\ P_4=\{2,3,4\}$$

By inclusion-exclusion algorithm, reliability function is

$$\begin{aligned} h(p) = {} & p_1p_4+p_1p_3p_5+p_2p_5+p_2p_3p_4-(p_1p_3p_4p_5+p_1p_2p_4p_5+p_1p_2p_3p_4+p_1p_2p_3p_5+p_1p_2p_3p_4p_5 \\ & +p_2p_3p_4p_5)+(p_1p_2p_3p_4p_5+p_1p_2p_3p_4p_5+p_1p_2p_3p_4p_5+p_1p_2p_3p_4p_5)-p_1p_2p_3p_4p_5 \end{aligned}$$

$$=p_1p_4+p_1p_3p_5+p_2p_5+p_2p_3p_4-(p_1p_3p_4p_5+p_1p_2p_4p_5+p_1p_2p_3p_4+p_2p_3p_4p_5)+4p_1p_2p_3p_4p_5$$

By pivotal decomposition algorithm, reliability function is

$$h(p)=p_1p_4+(1-p_1p_4)[p_1p_3p_5+(1-p_1p_3p_5)\{p_2p_5+(1-p_2p_5)p_2p_3p_4\}].$$

Thus pivotal decomposition algorithm is more useful for reliability calculation in 5-bridge structure.

6. Computational Results

This section discusses the results obtained when algorithms inclusion-exclusion and pivotal decomposition were programed and run on a variety of complex system reliability calculation problems.

Following tables represent reliability calculation of n-component structure used two algorithms.

table 1. series structure

	inclusion - exclusion	pivotal decomposition
n=2	p_1p_2	$p_1p_2+(1-p_1)0$
n=3	$p_1p_2p_3$	$p_1\{p_2p_3+(1-p_2)0\}+(1-p_1)0$
n=4	$p_1p_2p_3p_4$	$p_1[p_2\{p_3p_4+(1-p_3)0+(1-p_2)0\}]+(1-p_1)0$

table 2. parallel structure

	inclusion - exclusion	pivotal decomposition
n=2	$p_1+p_2-p_1p_2$	$p_1+(1-p_1)p_2$
n=3	$p_1+p_2+p_3-(p_1p_2+p_1p_3+p_2p_3)+p_1p_2p_3$	$p_1+(1-p_1)\{p_2+(1-p_2)p_3\}$
n=4	$p_1+p_2+p_3+p_4-(p_1p_2+p_1p_3+p_1p_4+p_2p_3$ $+p_2p_4+p_3p_4)+(p_1p_2p_3+p_1p_2p_4+p_1p_3p_4$ $+p_2p_3p_4)-p_1p_2p_3p_4$	$p_1+(1-p_1)[p_2+(1-p_2)\{p_3+(1-p_3)p_4\}]$

By above tables, we show that in case of series structure, the inclusion-exclusion algorithm is proper in computational complexity, but pivotal decomposition algorithm is proper in computational complexity in case of parallel structure. Now we compare the CPU time between two algorithms in complex system. Table 3 displays elapsed computation time in CPU seconds for each of the two algorithms.

table 3. CPU Time for the algorithms

system(Algorithm)	# Min Paths	1	2	3	4	5	mean	CPU sec
6-component parallel (inclusion-exclusion)	6	53.45s	53.63s	53.55s	53.27s	53.59s	53.50s	5.35ms
4-component parallel (pivotal decomposition)	4	3.69s	3.56s	3.62s	3.56s	3.66s	3.62s	0.36ms
2-out-of-4:G system (inclusion-exclusion)	6	59.54s	59.49s	59.05s	59.15s	59.19s	59.28s	5.93ms
5-component bridge system (inclusion-exclusion)	4	30.63s	30.67s	30.36s	30.54s	30.41s	30.52s	3.05ms
5-component bridge system (pivotal decomposition)	4	4.29s	3.99s	4.07s	4.12s	3.94s	4.08s	0.41ms
7-component bridge system (inclusion-exclusion)	7	59.25s	59.60s	59.53s	59.69s	59.61s	59.54s	5.95ms
7-component bridge system (pivotal decomposition)	7	4.30s	4.13s	4.23s	4.20s	4.25s	4.22s	0.42ms
8-component bridge system (inclusion-exclusion)	8	67.02s	66.56s	66.99s	66.80s	66.88s	66.85s	6.69ms
8-component bridge system (pivotal decomposition)	8	3.90s	3.78s	3.94s	3.98s	3.86s	3.89s	0.39ms

* Operation Environment
PC - IBM 586
CPU - Pentium II 350MHz
RAM - 128MB (Synchronous DRAM 100MHz)
OS - Windows 98
Compiler - Microsoft Visual C++ 6.0
0.08s/10000(iteration) = 8μ s = 0.008ms

By above table, we suggest that pivotal decomposition algorithm provides an efficient method for complex system. We expect that our method, applied in this paper, is further extended to the case when components of the system are given multi-states.

References

[1] R. E. Barlow, F. Proschan, *Statistical Theory of Reliability and Life testing*, Holt, Reinhardt & Winston Inc., 1975.

[2] W. Feller, *An Introduction to Probability Theory and Its Applications* I. II.

John Wiley & Sons Inc., New York, 1968.

[3] K. D. Heidtmann, "Inverting Paths and Cuts of 2-State Systems", *IEEE Transactions on Reliability*, Vol. R-32 No. 5, pp.469-471, 1983.

[4] R. L. Kenyon, R. J. Newell, "Steady-State Availability of k-out of-n: G system with single Repair", *IEEE Transactions on Reliability*, Vol. R-32 No. 2, 1983.

[5] S. C. Lee, "System Diagrams and Event Trees for Coherent Structure", *Bulletin of the Institute for basic science,* Vol. 12, Inha Univ., 1991.

[6] S. C. Lee, "Probability of System Failure of Coherent Structure", *Bulletin of the Institute for Applied science,* Korea Aviation Univ., 1993.

[7] S. C. Lee, "Determination of Probability of Component or Subsystem Failure", *Journal of Korean Society for Quality Control,* Vol. 2, No. 2, pp.121-130, 1993.

[8] S. C. Lee, "Reliability Expression for Complex System", *Journal of Korean Commun- ications in Statistics,* Vol. 3, No. 3, pp.125-133, 1996.

[9] S. C. Lee, "Algorithm for System Reliability Calculation", *Journal of Namseoul Univ.,* Vol. 3, pp.75-87, 1997.

[10] S. C. Lee, "Development and analysis of Algorithms for Reliability Calculation of Coherent structure", *Journal of the Society of Korea Industrial and Systems Engineering.,* Vol. 21, pp.153-164, 1998.

[11] 三根久, 河合一, *信頼性 保全性の基礎數理*, 日科技連, 1984.

Department of General Education,
Namseoul University, Cheonan 330-800, Korea
sclee@nsu.ac.kr