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AN ANALYSIS OF $MMPP/D_1, D_2/1/B$ QUEUE FOR TRAFFIC SHAPING OF VOICE IN ATM NETWORK

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Abstract. Recently in telecommunication, BISDN (Broadband Integrated Service Digital Network) has received considerable attention for its capability of providing a common interface for future communication needs including voice, data and video. Since all information in BISDN are statistically multiplexed and are transported in high speed by means of discrete units of 53-octet ATM (asynchronous Transfer Mode) cells, appropriate traffic control needs. For traffic shaping of voice, the output cell discarding scheme has been proposed. We analyze the scheme with a $MMPP/D_1, D_2/1/B$ queueing system to obtain performance measures such as loss probability and waiting time distribution.

1. Introduction

The Asynchronous Transfer Mode (ATM) has been selected as a mode of transmission and switching in the BISDN (Broadband Integrated Service Digital Networks), because of its efficiency and flexibility. The ATM is based on asynchronous time division multiplexing and fast packet switching technology. In ATM networks, all information are transmitted in a fixed-size packet called cell which has a 48-octet information field and 5-octet header. The header contains various information required to transfer the information field across the network.

The ATM networks support diverse services which require the different Quality of Service (QoS) such as voice, data and video. Since user terminals in BISDN generate cells only when they have information to transmit and these cells are statistically multiplexed, the traffic stream fluctuates uncertainly. Therefore, traffics such as voice and video have properties of time-correlation and burstiness. This characteristics of traffic may cause to congestion of network, so appropriate traffic control needs.

Voice traffic has delay-sensitive but loss-insensitive characteristic. An effective method to support voice traffic in ATM networks is use of output cell discarding (CD) scheme. The output CD scheme operates as follows: Voice information is stored in pair of cells to separate the more significant and less significant bits. The cell containing the more significant bits is identified as high priority cell (i.e. nondiscardable in network) and the cell containing the less significant bits is identified as low priority cell (i.e. discardable in network). The low-priority cells may be discarded during congestion of network. This output CD scheme results in significant transmission bandwidth saving

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and resiliency of the network during congestion. Therefore, the spare bandwidth obtained by CD scheme can be used to support different traffic such as data and video. Also, this smoothing effect of voice helps in avoiding buffer overflow[2,3].

To model the bursty voice traffic, we use a Markov-modulated Poisson process(MMPP) in pair of cells. We put a threshold on buffer considering congestion of network. If the buffer occupancy at transmission epoch is less than or equal to the threshold, the service time is D_1 (the transmission time of cell pair). Otherwise, the service time is $D_2 (= D_1/2$, because low-priority cell is discarded). We assume a finite capacity (B) queue for practical applications. Then, the output CD scheme is modeled by the queueing system $MMPP/D_1, D_2/1/B$ with one threshold. In following section, we analyze the queueing model by using the embedded Markov chain and the supplementary variable method.

2. Description of model and MMPP

A Markov-modulated Poisson process(MMPP) has been used to model the video and the packetized voice traffic. The MMPP can be constructed by a Poisson process with a rate that varies according to an N-state irreducible continuous-time Markov process $\{J(t), t \geq 0\}$ (called the underlying Markov process). When the underlying Markov process is in state i at time t, arrivals occur according to a Poisson process of rate λ_i. The sojourn time of the state i follows exponential distribution with mean $\frac{1}{\sigma_i}$. Then, the MMPP is characterized by the Markov process $\{J(t), t \geq 0\}$ with the transition rate matrix Q and the arrival rate matrix $\Lambda \triangleq$ diag $(\lambda_1, \lambda_2, \cdots, \lambda_N)$. The transition rate matrix Q is as follows:

$$Q = \begin{vmatrix} -\sigma_1 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & -\sigma_2 & \dots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \dots & -\sigma_N \end{vmatrix}.$$

The steady-state probability vector Π of the underlying Markov process $\{J(t), t \geq 0\}$ is given by solving the following equations

$$\Pi Q = 0, \quad \Pi e = 1, \qquad e = (1, 1, \cdots, 1)^T.$$

The arriving cells in pair are first queued in a buffer of finite capacity B in unit of pair of cells. Cells arriving when the buffer is full are lost, and cell pairs in buffer are served on the first-come first-service basis.
Introduce the notations

$$M(t) = \text{the number of cell pairs arriving during the interval } (0, t],$$
$$J(t) = \text{the state of the underlying Markov process at time } t.$$

Now we define the conditional probabilities

$$p_{i,j}(n, t) = P\{M(t) = n, J(t) = j | M(0) = 0, J(0) = i\}, \quad n \geq 0, \quad 1 \leq j \leq N.$$

Then, it is easily shown that the $N\times N$ matrix of probabilities $P(n,t)=(p_{i,j}(n,t))_{1\le i,j\le N}$, has the probability generating function

$$\begin{aligned}\bar{P}(z,t)&=\sum_{n=0}^{\infty}P(n,t)z^n, \quad |z|\le 1,\\ &=e^{R(z)t},\end{aligned}$$

where $R(z)=Q+(z-1)\Lambda$.

3. Analysis of queue length distribution

3.1 The queue length distribution at transmission epochs

Introduce the notations

$$\begin{aligned}\tau_n &= \text{ the } n\text{-th service completion epoch}, \quad n\ge 1, \quad \tau_0\triangleq 0,\\ N_n &= \text{ the queue length at time } \tau_n+,\\ J_n &= \text{ the state of the underlying Markov process at time } \tau_n+.\end{aligned}$$

Then, the process $\{(N_n,J_n),n\ge 0\}$ forms a Markov chain with finite state space $\{0,1,\cdots,B-1\}\times\{1,2,\cdots,N\}$.

Define the limiting probabilities $x_{k,i}$ and its probability vectors as

$$\begin{aligned}x_{k,i}&\triangleq\lim_{n\to\infty}P\{N_n=k,J_n=i\},\\ x&\triangleq(x_0,x_1,\cdots,x_{B-1}) \text{ with } x_k\triangleq(x_{k,1},x_{k,2},\cdots,x_{k,N}).\end{aligned}$$

The transition probability matrix $\overline{Q}_1$ of the Markov chain $\{(N_n,J_n),n\ge 0\}$ is given by

$$\overline{Q}_1=\left|\begin{array}{cccccccccc}
A_0^{'} & A_1^{'} & A_2^{'} & \dots & A_{L_1-1}^{'} & A_{L_1}^{'} & A_{L_1+1}^{'} & \dots & A_{B-2}^{'} & \overline{A}_{B-1}^{'}\\
A_0 & A_1 & A_2 & \dots & A_{L_1-1} & A_{L_1} & A_{L_1+1} & \dots & A_{B-2} & \overline{A}_{B-1}\\
0 & A_0 & A_1 & \dots & A_{L_1-2} & A_{L_1-1} & A_{L_1} & \dots & A_{B-3} & \overline{A}_{B-2}\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & 0 & \dots & A_1 & A_2 & A_3 & \dots & A_{B-L_1} & \overline{A}_{B-L_1+1}\\
0 & 0 & 0 & \dots & A_0 & A_1 & A_2 & \dots & A_{B-L_1-1} & \overline{A}_{B-L_1}\\
0 & 0 & 0 & \dots & 0 & B_0 & B_1 & \dots & B_{B-L_1-2} & \overline{B}_{B-L_1-1}\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & B_1 & \overline{B}_2\\
0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & B_0 & \overline{B}_1
\end{array}\right|$$

where the blocks $A_k, B_k, A_k^{'}, \overline{A}_k, \overline{B}_k$, and $\overline{A}_k^{'}$ are as following:

$$\begin{aligned}&A_k=P(k,D_1), \quad B_k=P(k,D_2), \quad \overline{A}_k=\sum_{n=k}^{\infty}A_n, \quad \overline{B}_k=\sum_{n=k}^{\infty}B_n,\\ &A_k^{'}=\int_0^{\infty}P(0,t)\Lambda dt A_k=(\Lambda-Q)^{-1}\Lambda A_k, \qquad \overline{A}_k^{'}=\sum_{n=k}^{\infty}A_n^{'}.\end{aligned}$$

The steady-state probability vector x of the Markov chain $\{(N_n, J_n), n \geq 0\}$ is obtained from the equations

$$x\, \overline{Q}_1 \;=\; x, \qquad x\, e \;=\; 1.$$

3.2 The queue length distribution at an arbitrary time

In this subsection we derive the queue length distribution at an arbitrary time. Let $N(t)$ be the queue length (including the cell in service) at time t.

$$R(t) = \begin{cases} 1 & \text{if the service time of the cell is by } D_1 \text{ at time } t, \\ 2 & \text{if the service time of the cell is by } D_2 \text{ at time } t. \end{cases}$$

and

$$\xi = \begin{cases} 0 & \text{if the server is idle }, \\ 1 & \text{if the server is busy}. \end{cases}$$

Define the limiting probabilities

$$\begin{aligned} y_0 &= \lim_{t\to\infty} P\{N(t) = 0, \xi = 0\}, \\ y_n &= \lim_{t\to\infty} P\{N(t) = n, \xi = 1\}, \qquad n \geq 1. \end{aligned}$$

First we compute the vector y_0 that the system is idle. Analogously to Choi[1], we have

$$y_0 = \frac{1}{C_1} x_0 (\Lambda - Q)^{-1}. \tag{1}$$

where $C_1 = x_0(\Lambda - Q)^{-1}e + D_2 + (D_1 - D_2)\sum_{n=0}^{L_1} x_n e$. Let $\hat{T}$ and $\tilde{T}$ are the respective remaining and elapsed service time for the cell in service. In order to obtain the queue length distribution $y_n (n \geq 1)$ at arbitrary time, we define the joint probability distribution of the queue length and the remaining service time at arbitrary time τ.

$$\alpha_r(n, j, t)dt = P\{N(\tau) = n, J(\tau) = j, R(\tau) = r, t < \hat{T} \leq t + dt, \xi = 1\},$$

and its Laplace transform and the vectors

$$\begin{aligned} \alpha_r^*(n, j, s) &= \int_0^\infty e^{-st} \alpha_r(n, j, t)dt, \\ \alpha_r^*(n, s) &= (\alpha_r^*(n, 1, s), \cdots, \alpha_r^*(n, N, s)), \qquad r = 1, 2, \\ \alpha^*(n, s) &= \alpha_1^*(n, s) + \alpha_2^*(n, s). \end{aligned}$$

We furthermore define the conditional probability $\beta_r(n, j_1, j_2, t)\ dt (r = 1, 2)$ and its Laplace transform

$$\begin{aligned} \beta_r(n, j_1, j_2, t)dt = P\{&\chi(\tilde{T}) = n, J(\bar{\tau} + \tilde{T}) = j_2, R(\bar{\tau} + \tilde{T}) = r, \\ &t < \hat{T} \leq t + dt, \xi = 1 | J_{\bar{\tau}} = j_1\}, \\ \beta_r^*(n, j_1, j_2, s) &= \int_0^\infty e^{-st} \beta_r(n, j_1, j_2, t)dt, \\ \beta_r^*(n, s) &= (\beta_r^*(n, j_1, j_2, s))_{1 \leq j_1, j_2 \leq N}, \end{aligned}$$

where $\chi(T)$ is the number of cells arriving during the time T. Then, the vectors $\alpha_r^*(n,s)$ can be represented as the following equations:

(2)

$$\alpha_1^*(n,s) = \frac{D_1}{C_1}[x_0(\Lambda - Q)^{-1}\Lambda\beta_1^*(n-1,s) + \sum_{k=1}^{\min(n,L_1)} x_k\beta_1^*(n-k,s)],$$

(3)

$$\alpha_1^*(B,s) = \frac{D_1}{C_1}[x_0(\Lambda - Q)^{-1}\Lambda\{\sum_{m=B-1}^{\infty}\beta_1^*(m,s)\} + \sum_{k=1}^{L_1} x_k\{\sum_{m=B-k}^{\infty}\beta_1^*(n-k,s)\}],$$

(4) $$\alpha_2^*(n,s) = 0, \qquad 1 \le n \le L_1,$$

(5) $$\alpha_2^*(n,s) = \frac{D_2}{C_1}\sum_{k=L_1+1}^{n} x_k\beta_2^*(n-k,s), \qquad L_1+1 \le n \le B-1,$$

(6) $$\alpha_2^*(B,s) = \frac{D_2}{C_1}\sum_{k=L_1+1}^{B-1} x_k[\sum_{m=B-k}^{\infty}\beta_2^*(m,s)].$$

We finally obtain that

(7)

$$\alpha^*(n,s) = \alpha_1^*(n,s) + \alpha_2^*(n,s),$$

$$= \begin{cases} \frac{D_1}{C_1}[x_0(\Lambda - Q)^{-1}\Lambda\beta_1^*(n-1,s) + \sum_{k=1}^{n} x_k\beta_1^*(n-k,s)], & 1 \le n \le L_1, \\ \frac{D_1}{C_1}[x_0(\Lambda - Q)^{-1}\Lambda\beta_1^*(n-1,s) + \sum_{k=1}^{L_1} x_k\beta_1^*(n-k,s)] & \\ \quad + \frac{D_2}{C_1}\sum_{k=L_1+1}^{n} x_k\beta_2^*(n-k,s), & L_1+1 \le n \le B-1, \\ \frac{D_1}{C_1}[x_0(\Lambda - Q)^{-1}\Lambda\{\sum_{m=B-1}^{\infty}\beta_1^*(m,s)\} + \sum_{k=1}^{L_1} x_k\{\sum_{m=B-k}^{\infty}\beta_1^*(m,s)\}] & \\ \quad + \frac{D_2}{C_1}\sum_{k=L_1+1}^{B-1} x_k\{\sum_{m=B-k}^{\infty}\beta_2^*(m,s)\}, & n = B. \end{cases}$$

In order to obtain $\beta_r^*(n,s)(r=1,2)$, we consider the following equation

(8) $$\sum_{n=0}^{\infty}\beta_1^*(n,s)z^n = E[e^{-s\hat{T}}e^{R(z)\tilde{T}}] = e^{R(z)D_1}E[e^{-(sI+R(z))\hat{T}}],$$

where $D_1 = \tilde{T} + \hat{T}$. Since $E[e^{-s\hat{T}}] = \int_0^{D_1} e^{-st}\frac{1}{D_1}dt = \frac{1-e^{-sD_1}}{sD_1}$,

$$\sum_{n=0}^{\infty}\beta_1^*(n,s)z^n = e^{R(z)D_1}[I-e^{-(sI+R(z))D_1}][(sI+R(z))D_1]^{-1}$$

$$(9) \qquad = \frac{1}{D_1}[e^{R(z)D_1}-e^{-sD_1}I](sI+R(z))^{-1}.$$

It is known that

$$\sum_{n=0}^{\infty}A_n z^n = \sum_{n=0}^{\infty}P(n,D_1)z^n = e^{R(z)D_1}.$$

Substituting above equation to (9), we obtain

$$\begin{aligned}\sum_{n=0}^{\infty}\beta_1^*(n,s)z^n &= \frac{1}{D_1}[\sum_{n=0}^{\infty}A_n z^n - e^{-sD_1}I](sI+R(z))^{-1}\\ &= \frac{1}{D_1}[\sum_{n=0}^{\infty}A_n z^n - e^{-sD_1}I][\sum_{n=0}^{\infty}R_n(s)z^n]\\ &= \frac{1}{D_1}[\sum_{n=0}^{\infty}\sum_{k=0}^{n}A_k R_{n-k}(s) - \sum_{n=0}^{\infty}e^{-sD_1}R_n(s)]z^n\end{aligned}$$

where $R_n(s)=(sI-\Lambda+Q)^{-1}[\Lambda(\Lambda-sI-Q)^{-1}]^n$. Thus, $\beta_1^*(n,s)$ is given by

$$\beta_1^*(n,s) = \frac{1}{D_1}[\sum_{m=0}^{n}A_m R_{n-m}(s) - e^{-sD_1}R_n(s)].$$

Similarly, we can obtain $\beta_2^*(n,s)$ as following:

$$\beta_2^*(n,s) = \frac{1}{D_2}[\sum_{m=0}^{n}B_m R_{n-m}(s) - e^{-sD_2}R_n(s)].$$

Substituting $\beta_1^*(n,s)$ and $\beta_2^*(n,s)$ to $\alpha^*(n,s)$, we obtain

(10)

$$\alpha^*(n,s)$$

$$= \begin{cases} \frac{1}{C_1}[x_0(\Lambda-Q)^{-1}\Lambda\sum_{m=0}^{n-1}A_m R_{n-1-m}(s) + \sum_{k=1}^{n}x_k\sum_{m=0}^{n-k}A_m R_{n-k-m}(s) \\ \quad - e^{-sD_1}\{x_0(\Lambda-Q)^{-1}\Lambda R_{n-1}(s) + \sum_{k=1}^{n}x_k R_{n-k}(s)], \qquad 1\le n\le L_1, \\ \frac{1}{C_1}[x_0(\Lambda-Q)^{-1}\Lambda\sum_{m=0}^{n-1}A_m R_{n-1-m}(s) + \sum_{k=1}^{L_1}x_k\sum_{m=0}^{n-k}A_m R_{n-k-m}(s) \\ \quad - e^{-sD_1}\{x_0(\Lambda-Q)^{-1}\Lambda R_{n-1}(s) + \sum_{k=1}^{L_1}x_k R_{n-k}(s) \\ \quad + \sum_{k=L_1+1}^{n}x_k\sum_{m=0}^{n-k}B_m R_{n-k-m}(s) - e^{-sD_2}\sum_{k=L_1+1}^{n}x_k R_{n-k}(s)], \\ \qquad L_1+1\le n\le B-1. \end{cases}$$

Finally, we obtain the queue length probabilities $y_n (n \geq 1)$ at an arbitrary time:
For $1 \leq n \leq L_1$,

$$\begin{aligned} y_n =& \alpha^*(n,0) \\ =& \frac{1}{C_1}[x_0(\Lambda - Q)^{-1}\Lambda \sum_{m=0}^{n-1} A_m(Q-\Lambda)^{-1}\{\Lambda(\Lambda - Q)^{-1}\}^{n-1-m} \\ &+ \sum_{k=1}^{n} x_k \sum_{m=0}^{n-k} A_m (Q-\Lambda)^{-1}\{\Lambda(\Lambda-Q)^{-1}\}^{n-k-m} \\ &- \sum_{k=0}^{n} x_k (Q-\Lambda)^{-1}\{\Lambda(\Lambda-Q)^{-1}\}^{n-k}]. \end{aligned} \tag{11}$$

For $L_1 + 1 \leq n \leq B-1$,

$$\begin{aligned} y_n =& \frac{1}{C_1}[x_0(\Lambda - Q)^{-1}\Lambda \sum_{m=0}^{n-1} A_m(Q-\Lambda)^{-1}\{\Lambda(\Lambda - Q)^{-1}\}^{n-1-m} \\ &+ \sum_{k=1}^{L_1} x_k \sum_{m=0}^{n-k} A_m (Q-\Lambda)^{-1}\{\Lambda(\Lambda-Q)^{-1}\}^{n-k-m} \\ &- \sum_{k=0}^{L_1} x_k (Q-\Lambda)^{-1}\{\Lambda(\Lambda-Q)^{-1}\}^{n-k} \\ &+ \sum_{k=L_1+1}^{n} x_k \sum_{m=0}^{n-k} B_m (Q-\Lambda)^{-1}\{\Lambda(\Lambda-Q)^{-1}\}^{n-k-m} \\ &- \sum_{k=L_1+1}^{n} x_k (Q-\Lambda)^{-1}\{\Lambda(\Lambda-Q)^{-1}\}^{n-k}], \end{aligned}$$

and

$$y_B = \Pi - \sum_{k=0}^{B-1} y_k.$$

Using the probabilities $y_n (n \geq 0)$ obtained above, we obtain performance measures such as loss (P_{loss}) and mean queue length (M_q):

$$P_{\text{loss}} = \frac{y_B \Lambda e}{\sum_{i=0}^{B} y_i \Lambda e} = \frac{y_B \Lambda e}{\Pi \Lambda e}, \qquad M_q = \sum_{i=0}^{B} i y_i e.$$

4. Analysis of waiting time distribution

In order to derive the waiting time distribution of an arbitrary cell pair, let's tag a cell pair arriving at time τ. Suppose that there are $i (1 \leq i \leq B-1)$ cell pairs in the system at time τ. Since the service time may change according to the buffer occupancy at service completion epoch, we need to know the time(U^{i-1}) required to complete transmission of $(i-1)$ cells at service completion epoch of the cell under service present at time τ. We first define the hitting time of the level more than the threshold L_1 from the level less than or equal to the threshold L_1 at service completion epoch and of the threshold L_1 from the level more than the threshold L_1:

$$\begin{aligned}
Y_{k,m}(j_1,j_2) &\triangleq inf\{n \geq 1; (N_n, J_n) = (m, j_2), N_n \in A | (N_0, J_0) = (k, j_1)\}, \\
& \qquad k = 1, \cdots, L_1, \quad m = L_1+1, \cdots, B-1, \\
Z_{k,L_1}(j_1,j_2) &\triangleq inf\{n \geq 1; (N_n, J_n) = (L_1, j_2) | (N_0, J_0) = (k, j_1)\}, \\
& \qquad k = L_1+1, \cdots, B-1, \quad 1 \leq j_1, j_2 \leq N,
\end{aligned}$$

where $A = \{L_1+1, \cdots, B-1\}$. Introduce the matrices $P_1, P_1^{'}, \overline{P}_1$, and $\overline{P}_1^{'}$ of order BN to obtain distribution of $Y_{k,m}(j_1,j_2)$ and $Z_{k,L_1}(j_1,j_2)$:

$$P_1 = \left| \begin{array}{cccccccccc}
A_0^{'} & A_1^{'} & A_2^{'} & \ldots & A_{L_1-1}^{'} & A_{L_1}^{'} & A_{L_1+1}^{'} & \cdots & A_{B-2}^{'} & \overline{A}_{B-1}^{'} \\
A_0 & A_1 & A_2 & \ldots & A_{L_1-1} & A_L & A_{L_1+1} & \cdots & A_{B-2} & \overline{A}_{B-1} \\
0 & A_0 & A_1 & \ldots & A_{L_1-2} & A_{L_1-1} & A_{L_1} & \cdots & A_{B-3} & \overline{A}_{B-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & A_1 & A_2 & A_3 & \cdots & A_{B-L_1} & \overline{A}_{B-L_1+1} \\
0 & 0 & 0 & \ldots & A_0 & A_1 & A_2 & \cdots & A_{B-L_1-1} & \overline{A}_{B-L_1} \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0
\end{array} \right|$$

and the matrix $P_1^{'}$ is the same as the matrix P_1 except that all rows and columns more than L_1 are block 0.

$$\overline{P}_1 = \left| \begin{array}{cccccccc}
0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & \ldots & B_0 & B_1 & B_2 & \ldots & 0 & 0 \\
0 & \ldots & 0 & B_0 & B_1 & \ldots & 0 & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \ldots & 0 & 0 & 0 & \ldots & B_1 & \overline{B}_2 \\
0 & \ldots & 0 & 0 & 0 & \ldots & B_0 & \overline{B}_1
\end{array} \right|$$

and the matrix $\overline{P}_1^{'}$ is the same as the matrix $\overline{P}_1$ except that (L_1+1, L_1)-block B_0 in the matrix $\overline{P}_1$ is replaced by block 0.
For $k = 1, \cdots, L_1, m = L_1+1, \cdots, B-1$, the event $\{Y_{k,m}(j_1,j_2) = l\}$ means that the Markov chain $\{(N_n, J_n), n \geq 0\}$ starting at the state (k, j_1) stays in the level less than

the level L_1+1 during $l-1$ transitions and at the l-th transition the Markov chain hits the state (m, j_2). Therefore, we have

$$\begin{aligned} P\{Y_{k,m}(j_1,j_2)=l\} &= [P_1^{'(l-1)}P_1](k,j_1;m,j_2), \\ &\triangleq f^l_{k,m}(j_1,j_2), \end{aligned}$$

where $[X](k,j_1;m,j_2)$ is the (j_1,j_2)-element of the (k,m)-block of the matrix X. Similarly, we obtain distribution for the random variable $Z_{k,L_1}(j_1,j_2)$

$$\begin{aligned} P\{Z_{k,L_1}(j_1,j_2)=l\} &= [\overline{P}_1^{'(l-1)}\overline{P}_1](k,j_1;L_1,j_2), \\ &\triangleq g^l_{k,L_1}(j_1,j_2), \qquad k=L_1+1,\cdots,B-1. \end{aligned}$$

Then, the Laplace transform of the time(U^{i-1}) required to complete the service of $(i-1)$ cell pairs is given by
For $1\le i,k\le L_1$,

$$\begin{aligned} &E[e^{-sU^{i-1}}|(N_n,J_n)=(k,j)] \\ &= \sum_{m_0=L_1+1}^{B-1}\sum_{j_0}[\sum_{a_0=1}^{i-1}E[e^{-sU^{i-1}}|(N_n,J_n)=(k,j),Y_{k,m_0}(j,j_0)=a_0]P\{Y_{k,m_0}(j,j_0)=a_0\} \\ &+\sum_{a_0=i}^{\infty}E[e^{-sU^{i-1}}|(N_n,J_n)=(k,j),Y_{k,m_0}(j,j_0)=a_0]P\{Y_{k,m_0}(j,j_0)=a_0\}] \\ &= \sum_{m_0=L_1+1}^{B-1}\sum_{j_0}[\sum_{a_0=1}^{i-1}E[e^{-sU^{i-1}}|(N_n,J_n)=(k,j),Y_{k,m_0}(j,j_0)=a_0]f^{a_0}_{k,m_0}(j,j_0) \\ &+e^{-s(i-1)D_1}\sum_{a_0=i}^{\infty}f^{a_0}_{k,m_0}(j,j_0)], \end{aligned}$$

Conditioning $Z_{m,L_1}(j,j^{'})$ and $Y_{L_1,m}(j,j^{'})$ at $E[e^{-sU^{i-1}}|(N_{\tau_n},J_{\tau_n})=(k,j),Y_{k,m_0}(j,j_0)=a_0]$, the summation below is finite:

$$\begin{aligned} &E[e^{-sU^{i-1}}|(N_n,J_n)=(k,j)] \\ &\qquad = \sum_{l=0}(A^1_l(k,j)+B^1_l(k,j)) \\ &\qquad \triangleq W^{i-1}_{k,j}(s), \quad k=1,2,\cdots,L_1, \quad 1\le j\le N, \end{aligned}$$

where

$$A_0^1(k,j) = e^{-s(i-1)D_1} \sum_{m_0=L_1+1}^{B-1} \big[e^T - \sum_{a_0=1}^{i-1} f_{k,m_0}^{a_0} e\big]_j, \quad B_0^1(k,j) \triangleq 0,$$

$$A_l^1(k,j) = \sum_{m_0} \cdots \sum_{m_l} \sum_{a_0=1}^{i-1} \sum_{b_1=1}^{i-1-a_0} \cdots \sum_{b_l=1}^{i-1-\sum_0^{l-1} a_n - \sum_1^{l-1} b_n} e^{-s((i-1)D_1 + (D_2-D_1)\sum_1^l b_n)}$$

$$\big[f_{k,m_0}^{a_0} \prod_{r=1}^{l-1} \{g_{m_{r-1},L_1}^{b_r} f_{L_1,m_r}^{a_r}\} \{e^T - \sum_{a_l=1}^{i-1-\sum_0^{l-1} a_n - \sum_1^{l-1} b_n} f_{L_1,m_l}^{a_l} e\}\big]_j,$$

$$B_l^1(k,j) = \sum_{m_0} \cdots \sum_{m_{l-1}} \sum_{a_0=1}^{i-1} \sum_{b_1=1}^{i-1-a_0} \cdots \sum_{a_{l-1}=1}^{i-1-\sum_0^{l-2} a_n - \sum_1^{l-1} b_n} e^{-s((i-1)D_2 + (D_1-D_2)\sum_0^{l-1} a_n)}$$

$$\big[f_{k,m_0}^{a_0} \prod_{r=1}^{l-1} \{g_{m_{r-1},L_1}^{b_r} f_{L_1,m_r}^{a_r}\} \{e^T - \sum_{b_l=1}^{i-1-\sum_0^{l-1} a_n - \sum_1^{l-1} b_n} g_{m_{l-1},L_1}^{b_l} e\}\big]_j,$$

and $[X]_j = j-$th component of row vector X.

For $L_1 + 1 \le k \le B-1$,

$$\begin{aligned} E[e^{-sU^{i-1}} | (N_n, J_n) = (k,j)] &= \sum_{l=0} (\overline{A}_l^1(k,j) + \overline{B}_l^1(k,j)), \\ &\triangleq W_{k,j}^{i-1}(s), \quad 1 \le j \le N. \end{aligned}$$

where

$$\overline{A}_0^1(k,j) \triangleq 0, \qquad \overline{B}_0^1(k,j) = e^{-s(i-1)D_2} \big[e^T - \sum_{b_0=1}^{i-1} g_{k,L_1}^{b_0} e\big]_j,$$

$$\overline{A}_l^1(k,j) = \sum_{m_1} \cdots \sum_{m_l} \sum_{b_0=1}^{i-1} \sum_{a_1=1}^{i-1-b_0} \cdots \sum_{b_{l-1}=1}^{i-1-\sum_1^{l-1} a_n - \sum_0^{l-2} b_n} e^{-s((i-1)D_1 + (D_2-D_1)\sum_0^{l-1} b_n)}$$

$$\big[g_{k,L_1}^{b_0} \prod_{r=1}^{l-1} \{f_{L_1,m_r}^{a_r} g_{m_r,L_1}^{b_r}\} \{e^T - \sum_{a_l=1}^{i-1-\sum_1^{l-1} a_n - \sum_0^{l-1} b_n} f_{L_1,m_l}^{a_l} e\}\big]_j,$$

$$\overline{B}_l^1(k,j) = \sum_{m_1} \cdots \sum_{m_l} \sum_{b_0=1}^{i-1} \sum_{a_1=1}^{i-1-b_0} \cdots \sum_{a_l=1}^{i-1-\sum_1^{l-1} a_n - \sum_0^{l-1} b_n} e^{-s((i-1)D_2 + (D_1-D_2)\sum_1^{l-1} a_n)}$$

$$\big[g_{k,L_1}^{b_0} \prod_{r=1}^{l-1} \{f_{L_1,m_r}^{a_r} g_{m_r,L_1}^{b_r}\} f_{L_1,m_l}^{a_l} \{e^T - \sum_{b_l=1}^{i-1-\sum_1^{l-1} a_n - \sum_0^{l-1} b_n} g_{m_l,L_1}^{b_l} e\}\big]_j.$$

Since the service time may change according to buffer occupancy, we must know the number of cell pairs arriving from an arbitrary time τ to the service completion epoch. Consider the following joint probabilities:

$$P\{N(\tau)=0,\text{ An arrival is in }(\tau,\tau+d\tau)\}=y_0\Lambda e d\tau.$$

For $1\leq n\leq B-1,\quad n+l<B-1,$

$$\begin{aligned}&P\{N(\tau)=n,N_{\tau_{k+1}}=n+l,J_{\tau_{k+1}}=j,R(\tau)=1,\text{ An arrival is in }(\tau,\tau+d\tau),\\&\qquad\qquad t<\hat{T}\leq t+dt,\xi=1\}\\&\quad=\frac{1}{C_1}[x_0(\Lambda-Q)^{-1}\Lambda P(n-1,D_1-t)\Lambda P(l,t)d\tau dt\\&\quad+\sum_{i=1}^{\min(n,L_1)}x_iP(n-i,D_1-t)\Lambda P(l,t)d\tau dt]_j.\end{aligned}$$

For $1\leq n\leq B-1,$

$$\begin{aligned}&P\{N(\tau)=n,N_{\tau_{k+1}}=B-1,J_{\tau_{k+1}}=j,R(\tau)=1,\text{An arrival is in }(\tau,\tau+d\tau),\\&\qquad\qquad t<\hat{T}\leq t+dt,\xi=1\}\\&\quad=\frac{1}{C_1}[x_0(\Lambda-Q)^{-1}\Lambda P(n-1,D_1-t)\Lambda\overline{P}(B-n-1,t)d\tau dt\\&\quad+\sum_{i=1}^{\min(n,L_1)}x_iP(n-i,D_1-t)\Lambda\overline{P}(B-n-1,t)d\tau dt]_j,\\&\quad\text{where }\overline{P}(k,t)=\sum_{l=k}^{\infty}P(l,t).\end{aligned}$$

For $L_1+1\leq n\leq B-1,\ n+l<B-1,$

$$\begin{aligned}&P\{N(\tau)=n,N_{\tau_{k+1}}=n+l,J_{\tau_{k+1}}=j,R(\tau)=2,\text{ An arrival is in }(\tau,\tau+d\tau),\\&\qquad\qquad t<\hat{T}\leq t+dt,\xi=1\}\\&\quad=\frac{1}{C_1}[\sum_{i=L_1+1}^{n}x_iP(n-i,D_2-t)\Lambda P(l,t)d\tau dt]_j\end{aligned}$$

For $L_1+1\leq n\leq B-1,$

$$\begin{aligned}&P\{N(\tau)=n,N_{\tau_{k+1}}=B-1,J_{\tau_{k+1}}=j,R(\tau)=2,\text{ An arrival is in }(\tau,\tau+d\tau),\\&\qquad\qquad t<\hat{T}\leq t+dt,\xi=1\}\\&\quad=\frac{1}{C_1}[\sum_{i=L_1+1}^{n}x_iP(n-i,D_2-t)\Lambda\overline{P}(B-n-1,t)d\tau dt]_j\end{aligned}$$

By combining above results, we obtain the Laplace transform for the waiting time of a cell pair:

$$
\begin{aligned}
E[e^{-sW}] = & \frac{1}{(1-P_{\text{loss}})\Pi\Lambda e}[y_0\Lambda e \\
& + \frac{1}{C_1}\{\sum_{n=1}^{B-2}\sum_{l=0}^{B-n-2} x_0(\Lambda - Q)^{-1}\Lambda \int_0^{D_1} e^{-st}P(n-1, D_1-t)\Lambda P(l,t)dtW_{n+l}^{n-1}(s) \\
& + \sum_{n=1}^{B-1} x_0(\Lambda - Q)^{-1}\Lambda \int_0^{D_1} e^{-st}P(n-1, D_1-t)\Lambda \overline{P}(B-n-1,t)dtW_{B-1}^{n-1}(s) \\
& + \sum_{i=1}^{L_1}\sum_{n=i}^{B-2}\sum_{l=0}^{B-n-2} \int_0^{D_1} e^{-st}x_iP(n-i, D_1-t)\Lambda P(l,t)dtW_{n+l}^{n-1}(s) \\
& + \sum_{i=1}^{L_1}\sum_{n=i}^{B-1} \int_0^{D_1} e^{-st}x_iP(n-i, D_1-t)\Lambda \overline{P}(B-n-1,t)dtW_{B-1}^{n-1}(s) \\
& + \sum_{i=L_1+1}^{B-2}\sum_{n=i}^{B-2}\sum_{l=0}^{B-n-2} \int_0^{D_2} e^{-st}x_iP(n-i, D_2-t)\Lambda P(l,t)dtW_{n+l}^{n-1}(s) \\
& + \sum_{i=L_1+1}^{B-1}\sum_{n=i}^{B-1} \int_0^{D_2} e^{-st}x_iP(n-i, D_2-t)\Lambda \overline{P}(B-n-1,t)dtW_{B-1}^{n-1}(s)\}].
\end{aligned}
$$

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