A Simple Method for Hairstyle Modeling and Animation
자연스러운 머리카락 모델링 및 애니메이션

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Abstract

본 논문에서는 머리카락의 모델링 및 애니메이션을 다룬다. 자연스러운 머리모양의 모델링을 위한 방법을 제시하고 디자인된 머리카락의 애니메이션을 구현한다. 빠른 구현을 위하여 역동적인 충돌감지법을 제안하고 머리카락을 군으로 나누어서 다룬다.

1. Introduction

Realistic representation and animation of human hair remains intractable. Rosenblum, Carlson and Tripp III [2] presented an approach to model the characteristics of human hair. They used a mass-spring-hinge system to control the position and orientation of a single strand. However they did not consider hairstyle modeling and their dynamic model only supports hair that is originally straight. Anjyo, Usami and Kurihara [1] used a simplified cantilever beam simulation for hairstyle modeling and used one-dimensional projective differential equations of angular momenta for linked rigid sticks to describe the dynamical behavior of a strand. Although they dealt with both hairstyle modeling and hair animation, their hairstyling is missing an important element: style preservation. Under external forces, the hair can change shape. But as the external forces go away, the hair should more or less restore the original style. In Anjyo et al.'s work, however, the original style can never be restored once it is animated into another shape.

We propose a novel method for simultaneous manipulation of hairstyle modeling and hair animation. For hairstyle modeling we use a rod equation as in [1]. However, we use a rod equation for the slope of elastic curve instead of a rod equation for the deflection which was used in [1]. Since the length is fixed during deflections, it is natural to use polar coordinates and thus our approach is more suitable in hairstyle modeling. For hair animation, we employ one-dimensional projective differential equations of angular momenta for linked rigid sticks [1] and mass-spring-hinge system [2]. Unlike the previous work, we give explicit relation between hairstyle modeling and hair animation. This enables us to animate the styled hair and to give the recovery force to preserve the original hairstyle. In both we only consider collision between hair and a head. For hair-hair interaction, we employ dynamic collision detection procedure and impose friction on the collision surfaces to simulate

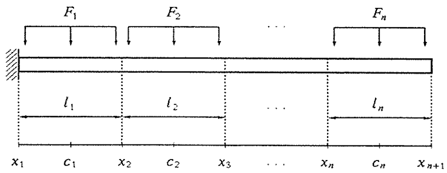


Figure 1: A cantilever rod; the model for hair bending.

the collision effect. We divide the hair into groups to save the cost for the simulation of hair dynamics.

2. Hairstyle Modeling

Modeling the nature of hair bending is an important part of the hairstyle modeling. The bending is simulated using the rod or beam deformation. There are two types of deformation of a rod. One is bending the rod and the other is twisting it. We shall consider only the former case and ignore the torsional deformation due to shearing force. We shall consider the two-dimensional case first, and extend it to three-dimensional case later. In the equilibrium elastic rod deflection due to bending satisfies

$$\chi = \frac{M}{EI} \tag{1}$$

where χ is the curvature of the elastic curve or neutral surface, M is the bending moment, E is Young's modulus and I is the moment of inertia or the second momentum of the cross-sectional area around the neutral axis. The displacement vector may be large even for small strain tensors. For example, when a long thin rod is slightly bent, its end may move a considerable distance, even though the relative displacements of neighboring points in the rod are small. Hence in the description of rod with a fixed length we need a special treatment to adjust the length of the deformed rod to have the same length as the original one. When a strand of hair is approximated by linear segments with fixed lengths it is convenient to use polar coordinates to describe the relative angles between neighboring segments. Equation (1) can be approximated by the following equation for the slope:

$$\frac{\partial \theta}{\partial x} = \frac{M}{EI} \tag{2}$$

where θ is the slope of the elastic curve. This representation has an advantage in the simulation of hair in that there is no need to adjust the lengths.

Whereas the uniform force distribution was used by Anjyo, Usami and Kurihara [1], the uniform force is not suitable to model various hairstyles. Actually, non-uniform bending force is distributed over a strand of hair according to the hairstyle design. Consider a cantilever rod composed of n segments. The length of each segment is l_i for i=1,2,...,n as shown in Figure 1. Let a uniform downward force F_i per unit length be loaded on the i-th segment for i=1,2,...,n. Then the bending moment at a point x contained in the k-th segment is

$$M(x) = \sum_{i=k+1}^{n} F_i\, l_i (c_i - x) + \frac{1}{2} F_k \left(c_k + \frac{1}{2} l_k - x\right)^2 \tag{3}$$

where $c_i = \sum_{j=1}^{i-1} l_j + \frac{1}{2} l_i$ is the center of the i-th segment.

The bending moment at a point x with origin at the support increases by the loaded force on the remaining part of the rod, that is, $F(y)$ for $y \geq x$. Hence the bending moment becomes large when a point becomes close to the support and it results in large deflections near the support. We note that Equation (1) determines the slope of the elastic curve over all rod for given force distributed on the initial shape of the rod.

To design natural hair styling, it is necessary to give arbitrary stresses on each segment of the strand. This effect is achieved by (3). Note that the first term in the right-hand side of (3) represents the contribution of the forces loaded on the remaining segments. Thus if we concentrate on the design of hairstyle we can think that each segment is designed independently and ignore this summation. In other words, on the part of hair styling we regard each segment as an independent rod, or equivalently after each segment is deformed the loaded force on the remaining part is removed. In order to consider the effect of gravity, we add the gravity force g to F_i. Obviously on the part that is important in designing curly hair F_i can

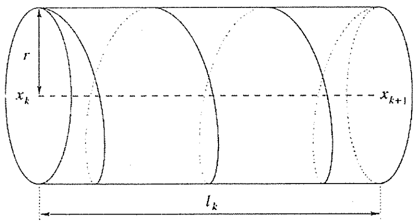


Figure 2: A cylindrical representation.

resist the gravity force. As a result, by an appropriate manipulation of the distributed forces we can effectively simulate the hairstyle modeling.

It is easy to generalize the problem into three dimensions. We shall use polar coordinates (r, θ, ϕ) where r is the distance from the origin, and θ and ϕ are the zenith angle and the azimuth angle, respectively. Considering the bending moment deformation, from (2) it follows that

$$\frac{\partial \theta}{\partial r} = \frac{M_\theta}{EI},$$
$$\frac{\partial \phi}{\partial r} = \frac{M_\phi}{EI},$$

where M_θ and M_ϕ are the components of the bending moment M in θ and ϕ directions, respectively.

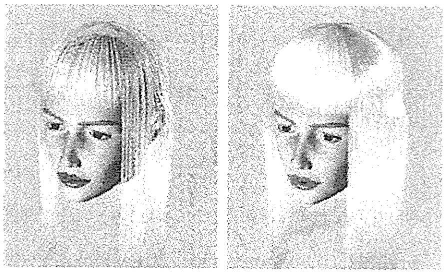
We can also simulate perming effect of hair. Two approaches are possible to deal with the styling of wavy hair. The one is to give an external force rotating about the straight hair in the beam model. As we march from the pore to the end, larger force should be loaded to simulate evenly curled hair. If the wave of a strand is small, direct application of the above hair styling can be used. However, when the wave is large, it is more convenient to use cylindrical representation and a strand is considered as a closely coiled helical spring.

The other approach is to model a strand coiled around cylinder and to deal with a cylinder instead of a strand itself. In this case we have only to describe the axis of the cylinder together with the radius, and the above method is directly applicable. x_i's which are used in the description of a strand are now used to describe the axis of the cylinder (see Figure 2). We can adjust the radius of the cylinder and the winding number for each segment. The radius of the cylinder is also used in collision detection. After the end points of each cylindrical segment are determined in hair styling a smoothly curved cylinder is constructed with these points as control points and hair is winded on this cylinder.

Figure 3: Dynamic collision detection.

3. Collision Detection

In hair style modeling, we have to consider collision detection. We ignore collisions between hairs and concentrate on collisions between hair and a human body. To simulate volumetric representation of the hair, dynamic collision detection is employed. For example, let $C_\omega(x, y, z) = x^2/a^2 + y^2/b^2 + z^2/c^2 - \omega = 0$ be the surface describing the head. Then $C_\omega(x, y, z) > 0$ is the outside of the head. If $C_\omega(x, y, z) < 0$, we project out the segment to the surface. Instead of detecting collisions between hair and $C_\omega(x, y, z) = 0$ we detect collisions between hair and $C_{\omega(\theta)}(x, y, z) = 0$ allowing to vary ω according to the location of the pore. When θ in polar coordinates of the pore is near zero $\omega(\theta)$ is increased. Then the hair strands whose pores are located at upper latitude generally covers the ones whose pores are located at the lower part. Even though a strand has a small flexural rigidity EI near the top of the head a volume of hair is created. This dynamic collision detection is effectual on the collision between hair and a human body, in that it looks like the collision between hairs. Also it enables us to consider the friction in the interaction between hairs. Actually, on the collision detecting surface we impose frictional coefficient for more realistic animation.



(a) (b)

Figure 4: The effect of grouping hair strands. (a) Representative strands (b) All strands

For more realistic animation, we have to consider precise geometry of a head. Locating an origin in the head we determine polar coordinates of the set of surface points which are distributed over the triangulated head evenly with respect to the angles. These precalculated polar coordinates of the surface points can be directly used in collision detection. We calculate the zenith and azimuth angles of a strand and find the surface point whose angles are nearest to the strand in the precalculated polar coordinates on the surface points. Comparing the distances of the strand and the corresponding surface point from the origin we can detect collision. This method speeds up collision detection effectively (See Figure 3 for the result of collision detection).

4. Hair Animation

To animate the styled hair, we use articulated sticks as a physical model of hair dynamics. The angular variables θ_i and ϕ_i satisfy the following ordinary differential equation:

$$\frac{\partial^2\theta_i}{\partial t^2} = \beta_i F_{\theta,i}(\theta_i, t), \qquad \frac{\partial^2\phi_i}{\partial t^2} = \beta_i F_{\phi,i}(\phi_i, t) \tag{4}$$

where β_i is the reciprocal number of the inertia moment of the i-th segment. Near the pore the hair moves relatively slowly because of hair's friction. By increasing β_i proportional to i we can simulate the effect of this self-interaction of hair.

The dynamics equation (4) is approximated

(a) (b)

Figure 5: Blending hair (a) before and (b) after.

by the finite difference equations;

$$\theta_i^{n+1} = 2\theta_i^n - \theta_i^{n-1} + (\Delta t)^2 \beta_i F_{\theta,i}(\theta_i, n\Delta t),$$
$$\phi_i^{n+1} = 2\phi_i^n - \phi_i^{n-1} + (\Delta t)^2 \beta_i F_{\phi,i}(\phi_i, n\Delta t),$$

where Δt is the time step.

To animate the hair we have to apply external forces to the hair. In [1] Anjyo, Usami and Kurihara introduced external force to describe wind blowing. However, they gave no relation between hairstyle modeling and animation As a result, during the animation the hairstyle is destroyed over time and cannot be recovered. Therefore their system can only simulate gentle wind. Even though Rosenblum, Carlson and Tripp III [2] simulated wild action of the head, they did not present the treatment of hairstyle modeling.

For realistic animation of hair we use mass-spring-hinge model partly to give a recovery force for the original hairstyle. For the part where hair styling is important, a large spring constant is imposed, and for the part where there is no hair styling no spring model is used. Let $\theta_{i,0}$ and $\phi_{i,0}$ be the angles of i-th segment determined by the styling process described in Section 2. We consider the following forces

$$F_{\theta,i}(\theta_i, t) = F_{\theta,e} - k_{\theta,i}(\theta_i - \theta_{i,0}), \qquad F_{\phi,i}(\phi_i, t) = F_{\phi,e} - k_{\phi,i}(\phi_i - \phi_{i,0}), \tag{5}$$

applied on the i-th segment of the hair where F_e is an external force exerted into the system and k_i is the spring constant. The external force includes gravity, wind force, frictional force, etc.

A direct application of the mass-spring-hinge model cannot avoid the transformation of the hairstyle. By examining (4) and (5) we can

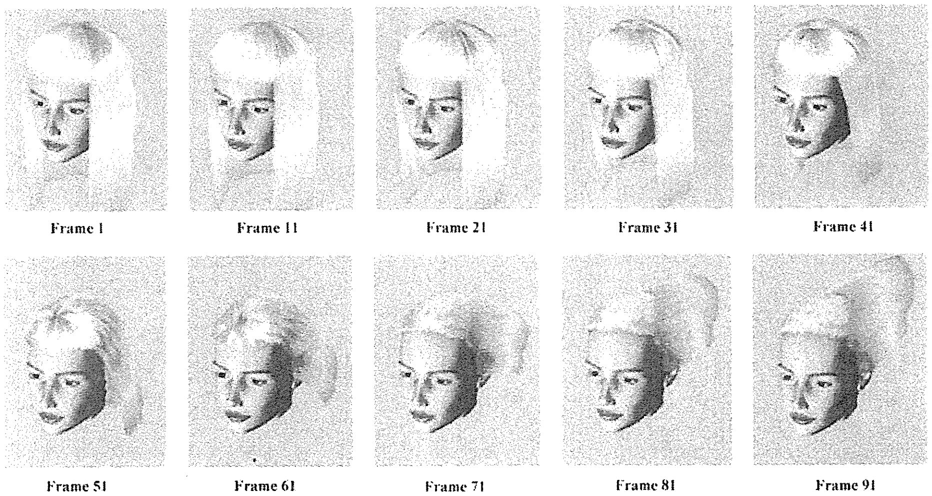


Figure 6: The wind blows in front of the face during frame 1 and frame 100 (30 frames per second).

see that the original hairstyle is not in equilibrium under influence of gravity and hence the shape of the styled hair is not preserved even with gravity solely, which is not realistic. Thus we change the initial angles in order that the original hairstyle is in equilibrium under gravity. To be more specific, let $\tilde{\theta}_{i,0} < \theta_{i,0}$ be such that

$$m_i g - k_{\theta,i}(\theta_{i,0} - \tilde{\theta}_{i,0}) = 0 \qquad (6)$$

where m_i is the mass of the i-th segment. When $k_{\theta,i}$ is prescribed, we can obtain $\tilde{\theta}_{i,0}$ in (6);

$$\tilde{\theta}_{i,0} = \theta_{i,0} - \frac{m_i g}{k_{\theta,i}}.$$

On the other hand, if we prefer to prescribe $\tilde{\theta}_{i,0}$, the spring constant $k_{\theta,i}$ follows from (6);

$$k_{\theta,i} = \frac{m_i g}{\theta_{i,0} - \tilde{\theta}_{i,0}}.$$

The same arguments hold for $\phi_{i,0}$. In other words, the angles $\tilde{\theta}_{i,0}$ and $\tilde{\phi}_{i,0}$ describe the shape of the original hairstyle without gravitational force.

For more fast animation, we divide hairs into representative groups (See Figure 4). In other words, when one representative strand is simulated, its neighboring strands move in conformity to its movement except the detection of collision. By increasing the number of its neighboring elements one can efficiently simulate the behavior of hair.

5. Numerical Simulation

For more realistic representation of the hair, we blended the hair over all strands. Figure 5 shows the effect of the blending near the pore and the end. While the alpha value near those parts is 0.5, the value along the rest parts is 0.9. So, near the pore, the color of hair is blended with the color of face.

The model used in the animation is depicted in the first frame of Figure 6. We used 850 hair groups each of which contains 25 strands. As a result, total number of effective strands is 21,250. Wind was generated in front of the face from frame 1 to frame 100. Figure 6 shows the resulting hair dynamics driven by wind field. After wind stops blowing gentle wind was generated from the back (see Figure 7). We can clearly see the effect of collision between nose and hair from frame 221 to frame 281.

For corresponding movies and more recent result, you may refer to the following URL:

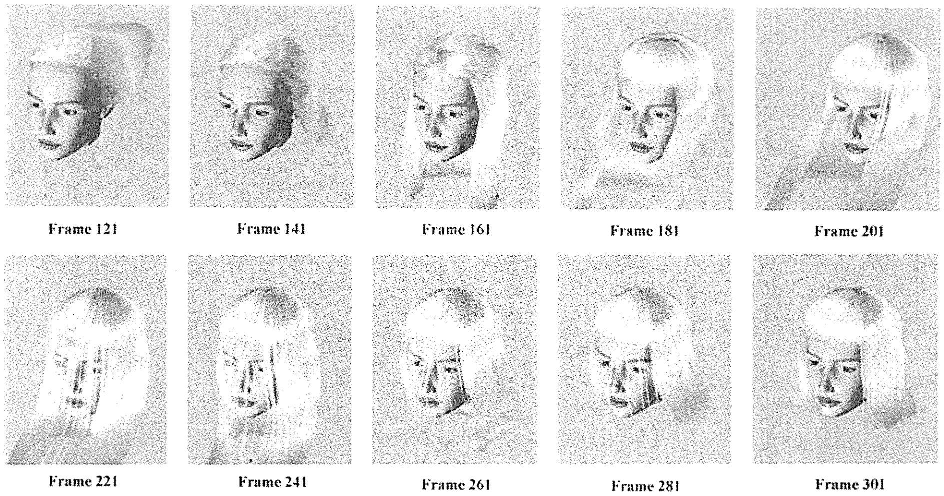


Figure 7: Animation between frame 121 and 301.

http://graphics.snu.ac.kr.

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Bibliography

[1] K. Anjyo, Y. Usami and T. Kurihara, *A simple method for extracting the natural beauty of hair.* Computer Graphics **26** (1992), 111-120.

[2] R. E. Rosenblum, W. E. Carlson and E. Tripp III, *Simulating the structure and dynamics of human hair: modelling, rendering and animation*, The Journal of Visualization and Computer Animation **2** (1991), 141-148.