# LMI Based $H_{\infty}$ Active Vibration Control of a Structure with Output Feedback : Experiment Results LMI에 기초한 구조물의 $H_{\infty}$ 등동진동제어 : 실험적 고찰

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**Key Words**: Active Vibration Control, Structural Parameter, Simultaneous Optimization, Output Feedback Linear Matrix Inequality, Control Performance

요약: 제어이론분야에서의 발전은 그러한 이론을 다방면으로 응요할 수 있는 분야를 더더욱 폭넓게 제공해 주고 있다. 자동화와 관련된 분야뿐만 아니라, 건축 및 토목분야에서도 고도의 제어어이론을 응용한 예를 쉽게 접할 수 있게 되었으며, 지진등에 의한 구조물의 진동을 억제하려는 방책이 그 옝ㅣ다. 이에 관한 많은 연구에서도 알 수 있듯이, 일반적으로 구조물의 수학적 모델에만 의존하여, 즉 구조물의설계파라미터는 이미 설계되어져 있다는 가정하에 제어계를 설계하고 있다. 그러나 이러한 설계법에 있어서는 설계자로 하여금 구조물의 설계파라미터를 조정할 수 있는 자유도는 전혀 주어지지 않게 되며단지 제어계의 파라미터를 조정하는 자유도만 허용된다. 이러한 문제점을 극복하기 위해 구조계 및 제어기의 설계파라미터를 동시에 조절할 수 있는 자유도가 허용되는 '구조계/제어계의 동시 최적화' 기법이 있다. 따라서 본 논문에서는 교량의 주탑 및 해양구조물 등의 진동제어 문제에 이러한 설계기법을이용하여 주어진 설계사양을 만족하도록 구조계 및 제어계의 파라미터를 최적화 한다. 특히 본 논문에서는 제어계 설계 문제에 있어서의 일반적인 경우를 고려하여 상태의 일부가 관측된다고 가정하고 출력 피드백의 경우에 대해 고찰하고 있다. 이때의 설계사양은 선형행렬부등식(LMI)으로 주어지며, 실험을 통하여 본 논문에서 소개하는 설계기법의 유효성을 검증한다.

### 1. Introduction

System design technology has taken on more interdisciplinary nature. This has been caused by more demanding performance criteria and design specification of all types of machines and structures in various fields. Passive control alone may not meet the high specifications. On the other hand, pure active control may be very expensive to realize. This has led researchers to integrate the passive and active control design in a certain optimal sense to satisfy the high

demanding performance requirements<sup>1,5)</sup>. The and modeling control problems independent. The structure design and control design are not structure structure design and control design are not separable and necessarily are iterative. This paper introduces an iterative algorithm to integrate structure and control design. Specifically, the algorithm simultaneously finds: i) the optimal values of the stiffness, damping ratios, and actuator location parameter, and ii) an optimal stabilizing state feedback and output feedback controller such that the active contol energy is minimized subject to : a) the prespecifed RMS constraints on the outputs, and b) the constraints on the structure parameters. The algorithm provides a systematic approach to tune the structure parameters and design an active controller. Especially, this algorithm is

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applied to active vibration control for a structure, and it is easily extended to antirolling control system design of ocean vehicles, platforms etc..

### 2. Problem Formulation

Consider a linear time-invariant dynamical model for a structure illustrated in Fig. 1 with the following representation:

$$M\ddot{q} + D(\alpha)\dot{q} + K(\beta)q = b_1w + b_2(\delta)u$$
  
 $z_1 = u, \quad z_2 = C\begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad y = z_2$  (1)

where  $q \in \mathbb{R}^{n_e}$  is the displacement vector, q and q are the velocity and acceleration vectors, respectively.

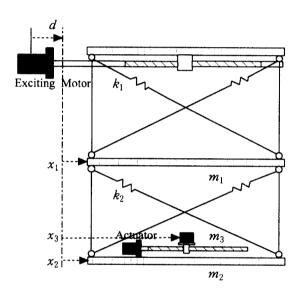


Fig. 1 Schematic diagram of a controlled structure

And,  $u \in R^{n_u}$  is the control input vector,  $z_1 \in R^{n_n}$  and  $z_2 \in R^{n_u}$  is the output vector to be regulated,  $y \in R^{n_v}$  is vector of measurements,  $w \in R^{n_v}$  is the disturbance.

$$D(\alpha) = D + \Delta D(\alpha) = D + \sum_{i=1}^{d} \alpha_i D_i$$

$$K(\beta) = K + \Delta K(\beta) = K + \sum_{j=1}^{k} \beta_j K_j$$

$$b_2(\delta) = b_2 + \Delta b_2(\delta) = b_2 + \sum_{j=1}^{k} \delta_j b_j$$
(2)

where  $\alpha = [\alpha_1, \alpha_2, ..., \alpha_d]$  is the set of system parameters that can be designed to adjust the

system damping,  $\beta = [\beta_1, \beta_2, ..., \beta_k]$  is the set of system parameters that can be designed to adjust the system stiffness,  $\delta = [\delta_1 \delta_2 \cdots \delta_l]$  is a vector of actuator and location parameters, the matrices  $D_i$ ,  $K_j$ , and  $b_l$  are the corresponding basis matrices. The matrices M, D, K,  $b_1$ ,  $b_2$  and C are constant matrices with appropriate dimensions that represent the nominal structure design. Also

$$\alpha \in [\alpha_m, \alpha_M], \beta \in [\beta_m, \beta_M]$$

where  $\alpha_m$ ,  $\alpha_M$ ,  $\beta_m$  and  $\beta_M$  are specified constants that represent the structure design constraints ( $\alpha_m$ ,  $\beta_m$ ) denote minimum values and  $\alpha_M$ ,  $\beta_M$  denote the maximum values, respectively). Define  $p = [\alpha \ \beta]$ . Then the state space representation for the dynamical system (1) is given by

$$\dot{x} = A(p) x + B_1 w + B_2(\delta) u$$

$$z_1 = u$$

$$z_2 = C x$$
(3)

where

$$x = \begin{bmatrix} q \\ q \end{bmatrix}, \quad A(p) = A + \Delta A(p)$$

$$A = \begin{bmatrix} 0 & I_{n_s} \\ -M^{-1}K & -M^{-1}D \end{bmatrix}$$

$$\Delta A(p) = \begin{bmatrix} 0 & 0 \\ -M^{-1}\Delta K & -M^{-1}\Delta D \end{bmatrix}$$

$$B_2(\delta) = B_2 + \Delta B_2(\delta),$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ M^{-1}b_2 \end{bmatrix}, \quad \Delta B_2(\delta) = \begin{bmatrix} 0 & 0 \\ M^{-1}\Delta b_2(\delta) \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ M^{-1}b_1 \end{bmatrix}$$

$$p = \begin{bmatrix} \alpha_1, \dots, \alpha_d, \beta_1, \dots, \beta_k \\ p_m = \begin{bmatrix} \alpha_{1m}, \dots, \alpha_{dm}, \beta_{1m}, \dots, \beta_{km} \\ p_M = \begin{bmatrix} \alpha_{1M}, \dots, \alpha_{dM}, \beta_{1M}, \dots, \beta_{kM} \end{bmatrix}$$

Here, the system matrix A(p) is a linear function of p. The matrices A,  $B_1$ ,  $B_2$ , C are real constant with appropriate dimensions, and  $\Delta$  means the variation around the nominal value. We shall make the following assumptions for the system (3).

**Assumptions**: For any  $p \in [p_m \ p_M]$  and  $\delta \in [\delta_m \ \delta_M]$ ,

- (i) The pair  $[A(p), B_2(\delta)]$  is stabilizable,
- (ii) The pair  $[A(p), B_1]$  is stabilizable.

## 3. LMI Based $H_{\infty}$ Control with Output Feedback

In this section, we give the formulation of LMI based output feedback control in detail<sup>8)</sup>. So that the result to be introduced in this section is natural extension of the state feedback case. Here, the objective is to make clear this formulation for practical use.

Let us consider a LTI system described by

where x, w, u,  $z_1$  and  $z_2$  take values in finite dimensional vector spaces :  $x \in R^n$ ,  $w \in R^r$ ,  $u \in R^m$ ,  $z_1 \in R^p$  and  $z_2 \in R^q$ . The system parameters A,  $B_1$   $B_2$ ,  $C_1$ ,  $C_2$ ,  $D_{11}$ ,  $D_{12}$ ,  $D_{21}$  and  $D_{22}$  are matrices of appropriate dimensions.

Let us consider system (4) and given a proper real rational controller represented by

$$\dot{x_c} = A_c x_c + B_c y 
 u = C_c x_c + D_c y$$
(5)

where  $y = z_2$  is considered.

**Lemma 2**<sup>9)</sup> The system (4) is stabilizable with  $H_{\infty}$  disturbance attenuation  $\gamma$  via output feedback (5) if and only if there exist symmetric matrices R, S satisfying the following LMI system:

$$\begin{bmatrix} \frac{N_R}{0} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} AR + RA^T & RC_1^T & B_1 \\ C_1R & -\gamma I & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{bmatrix} \begin{bmatrix} \frac{N_R}{0} & 0 \\ 0 & I \end{bmatrix} < 0$$

$$\begin{bmatrix} \frac{N_S}{0} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} AS + SA^T & SB_1^T & C_1^T \\ B_1R & -\gamma I & D_{11}^T \\ C_1 & D_{11} & -\gamma I \end{bmatrix} \begin{bmatrix} \frac{N_S}{0} & 0 \\ 0 & I \end{bmatrix} < 0$$

$$\begin{bmatrix} R & 0 \\ 0 & S \end{bmatrix} \ge 0$$
(6)

where  $N_R$  and  $N_S$  denote bases of the null spaces of  $(B_2^T, D_{12}^T)$  and  $(C_2, D_{21})$  respectively. In addition, there exist such controllers of order k < n (reduced order) if and only if the above three LMIs hold for some R, S that further satisfy

$$rank (I - RS) \le k \tag{7}$$

Suppose that some solution (R, S) of the LMI systems (6), (7) has been computed. We here give a method to construct an  $H_{\infty}$  controller from the obtained data. And, we collect the controller parameters into a single variable  $\mathcal{E}$  which is to be used in next formulation.

$$\mathcal{Z} = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \tag{8}$$

Then the matrices of closed-loop system are obtained by

$$A_{cl} = A_0 + \widetilde{B} \widetilde{\Xi} \widetilde{C}, \quad B_{cl} = B_0 + \widetilde{B} \widetilde{\Xi} \widetilde{D}_{2l}$$

$$C_{cl} = C_0 + \widetilde{D}_{12} \widetilde{\Xi} \widetilde{C}, \quad D_{cl} = D_0 + \widetilde{D}_{12} \widetilde{\Xi} \widetilde{D}_{2l}$$
(9)

where

$$A_{0} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{0} = \begin{bmatrix} B_{1} \\ 0 \end{bmatrix}, \quad C_{0} = \begin{bmatrix} C_{1} & 0 \end{bmatrix}$$

$$\widetilde{B} = \begin{bmatrix} 0 & B_{2} \\ I & 0 \end{bmatrix}, \quad \widetilde{C} = \begin{bmatrix} 0 & I \\ C_{2} & 0 \end{bmatrix}, \quad \widetilde{D}_{12} = \begin{bmatrix} 0 & D_{12} \end{bmatrix}$$

$$\widetilde{D}_{21} = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix}$$
(10)

From the state-space realization of the plant and controller, let

$$\begin{aligned}
\dot{x}_{cl} &= A_{cl} x_{cl} + B_{cl} w \\
z_1 &= C_{cl1} x_{cl} + D_{cl1} w \\
z_2 &= C_{cl2} x_{cl} + D_{cl2} w
\end{aligned} (11)$$

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad C_{cl} = \begin{bmatrix} C_{ch} \\ C_{cl2} \end{bmatrix}, \quad D_{cl} = \begin{bmatrix} D_{ch} \\ D_{cl2} \end{bmatrix}$$

be the corresponding closed-loop state-space equation.

We are now in the position to state the procedure:

1. Compute two full-column-rank matrices  $N, N \in \mathbb{R}^{n \times k}$  satisfying  $MN^T = I - RS \tag{12}$ 

2. Solve the following linear equation about  $X_{cl}$ :

$$\begin{bmatrix} S & I \\ N^T & 0 \end{bmatrix} = X_{cl} \begin{bmatrix} I & R \\ 0 & M^T \end{bmatrix}$$
 (13)

3. Solve the matrix inequality

$$A_{cl}^{T}X_{cl} + X_{cl}A_{cl} \quad X_{cl}B_{cl} \quad C_{cl}^{T} \\
B_{cl}^{T}X_{cl} \quad -\gamma I \quad D_{cl}^{T} \\
C_{cl} \quad D_{cl} \quad -\gamma I \right]$$

$$= \begin{bmatrix} A_{0}^{T}X_{cl} + X_{cl}A_{0} & X_{cl}B_{0} & C_{0}^{T} \\
B_{0}^{T}X_{cl} & -\gamma I & D_{0}^{T} \\
C_{0} & D_{0} & -\gamma I \end{bmatrix}$$

$$+ \begin{bmatrix} \widetilde{C}_{1}^{T} \\ \widetilde{D}_{2l}^{T} \\ 0 \end{bmatrix} \mathcal{E}^{T} \begin{bmatrix} \widetilde{B}^{T}X_{cl} & 0 & \widetilde{D}_{12}^{T} \end{bmatrix}$$

$$+ \{ \begin{bmatrix} \widetilde{C}_{1}^{T} \\ \widetilde{D}_{2l}^{T} \\ 0 \end{bmatrix} \mathcal{E}^{T} \begin{bmatrix} \widetilde{B}^{T}X_{cl} & 0 & \widetilde{D}_{12}^{T} \end{bmatrix} \}^{T} < 0$$

for the matrix variable  $\Xi$ . Then, the matrices  $A_c$ ,  $B_c$ ,  $C_c$  and  $D_c$  are obtained from the solution  $\Xi$ .

This procedure will be applied to compute  $H_{\infty}$  output feedback controller for simultaneous optimization design.

# The algorithm to Integrate structure and Control System Design

Here, our problem of finding a solution to inequality (14) can be embedded in the parameterized family of problems:

$$\Omega(X_{cl}, p, y_c(\delta, \Xi), \gamma) > 0$$
 (15)

For this, we consider the following subproblems.

### 4.1 $(X_{cl}, p)$ Optimization

Here, we consider the optimal selection of the

structure parameter p.

Suppose the solution of inequality (14),  $(X_{cln}, p_n, y_{cn}(\delta, \Xi), \gamma_n)$  is given for the nominal system (4) and let the matrix  $y_{cn}(\delta, \Xi)$  be fixed. So, we consider the following Matrix Inequality:

$$\begin{bmatrix} A_{cl}^{T}(\mathbf{p})X_{cln} + X_{cln}A_{cl}(\mathbf{p}) & X_{cln}B_{cl} & C_{cl}^{T} \\ B_{cl}^{T}X_{cln} & -\gamma I & D_{cl}^{T} \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0$$

$$(16)$$

**Theorem 1** Suppose  $y_{cn}(\delta, \Xi)$  which denotes the actuator parameter and dynamic controller is given for the nominal system. Then the  $(X_{cl}, p)$  Optimization problem is represented as follows:

min 
$$\gamma$$

$$(X_{cl.}, p, \gamma)$$

$$(17)$$
subject to Inequality (14) and  $p_m$ 

The optimal solution is denoted as  $(X_{c/n+1}, p_{n+1}, \gamma_{n+1})$ . Then the sub-optimal dynamic controller and actuator parameters are determined from  $y_{cn}(\delta, \Xi)$ , yield  $\gamma_{n+1} \leq \gamma_n$ .

### 4.2 $(X_{cl}, y_c(\delta, \Xi))$ Optimization

Here, we consider the selection for the actuator type and location parameter  $\delta$  and dynamic controller parameter  $\Xi$ .

Suppose the solution of the  $(X_{cl}, p)$  design loop,  $(X_{cln+1} p_{n+1}, \gamma_{n+1})$  is given for the system with fixed  $y_{cn}(\delta, \Xi)$ . Then the  $(X_{cl}, y_c(\delta, \Xi))$  optimization problem is to find a controller subject to  $H_{\infty}$  constraints. This problem can be formulated as following theorem:

**Theorem 2** Let the solution  $(X_{cln+1} p_{n+1}, \gamma_{n+1})$ , to the  $(X_{cl}, p)$  design loop be given. Then the  $(X_{cl}, y_c(\delta, \Xi))$  optimization is given as follows:

Therefore the optimal solution is given by  $(X_{cln+2}, y_{cn+1}(\delta, \Xi), \gamma_{n+2})$ . Then  $\gamma_{n+2} \leq \gamma_{n+1}$ .

**Proof**: See references 4), 5), 6), 7).

This problem gives the optimal(in general suboptimal) value for the parameter  $\delta$  and new controller  $\mathcal{E}$ , satisfying the  $H_{\infty}$  constraint. Now, an algorithm is given to integrate structure and control system design. The basic idea is to iterate the  $(X_{cl}, p)$  optimization and the  $(X_{cl}, y_c(\delta, \mathcal{E}))$  optimization loop.

**Algorithm** Consider the system represented by equation (4) and set  $p_0 = 0$ ,  $\delta_0 = 0$ , n = 0. Suppose  $\Xi$  is given for the nominal system and the numerical tolerance  $\varepsilon > 0$ .

Step 1: Solve the 
$$(X_{cl}, p)$$
 design loop  
with fixed  $y_{cn}(\delta, \Xi)$  to get  
 $(X_{cln+1} p_{n+1}, \gamma_{n+1})$ 

Step 2: Solve the 
$$(X_{cl}, y_c(\delta, \Xi))$$
 optimization problem to get  $(X_{cln+2}, y_{cn+1}(\delta, \Xi), \gamma_{n+2})$ 

Step 3: If 
$$|\gamma_{n+1} - \gamma_{n+2}| > \varepsilon$$
, go to Step 1,  
otherwise, output  
 $(X_{c/n+2}, p_{n+1}, y_{cn+1}(\delta, \Xi), \gamma_{n+2})$ 

Step 4: The actuator and controller parameters are obtained as follows:  $y_{cn+1}(\delta, \Xi)$ .

Table Estimated Parameters

Parameters		Values	Unit
mass	$m_1$	150.30	kg
	$m_2$	116.50	kg
	$m_3$	to be designed	kg
damping coef.	$c_1$	29.12	N/(m/s)
	$c_2$	14.22	N/(m/s)
	$c_3$	11.33	N/(m/s)
stiffness coef.	$k_1$	28,812.00	N/m
	$k_2$	25,855.00	N/m
motor torque coef.	k <sub>T</sub>	to be designed	N/A

### 5. Experiment

The model of the design object shown in Fig. 1 is considered. The equation of motion for this system is given by

$$m_{1}(\ddot{x}_{1} + \dot{d}) = -c_{1}\dot{x}_{1} - k_{1}x_{1} - c_{2}(\dot{x}_{1} - \dot{x}_{2}) - k_{2}(x_{1} - x_{2}) m_{2}(\ddot{x}_{2} + \dot{d}) = -f - c_{2}(\dot{x}_{2} - \dot{x}_{1}) - k_{2}(x_{2} - x_{1})$$
(19)  
$$-c_{3}(\dot{x}_{2} - \dot{x}_{3}) m_{3}(\ddot{x}_{3} + \dot{d}) = f - c_{3}(\dot{x}_{3} - \dot{x}_{2})$$

where  $m_i$ ,  $c_i$  (i=1,2,3) and  $k_j$  (j=1,2) are mass, damping and stiffness, respectively. Especially,  $m_3$  is the active mass to suppress the vibration. And  $x_i$  denotes the state displacement. If we let

$$x_{1d} = x_1 + d$$
,  $x_{2d} = x_2 + d$ ,  $x_{3d} = x_3 + d$ ,  $f = k_T u$ 

then the equation (19) is represented by

$$m_{1} \dot{x}_{1d} = -(c_{1} + c_{2}) \dot{x}_{1d} + c_{2} \dot{x}_{2d} - (k_{1} + k_{2})x_{1d} + k_{2}x_{2d} + c_{1}\dot{d} + k_{1}d$$

$$m_{2} \dot{x}_{2d} = c_{2} \dot{x}_{1d} - (c_{2} + c_{3}) \dot{x}_{2d} + c_{3} \dot{x}_{3d} + k_{2} \dot{x}_{1d} + k_{2}x_{2d} - k_{T}u$$

$$m_{3} \dot{x}_{3d} = c_{3} \dot{x}_{2d} - c_{3} \dot{x}_{3d} + k_{T}u$$

$$(20)$$

where  $k_T$  and  $u_d$  denote the torque constant and control input, respectively. And consider

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$$x_{4d} = \dot{x}_{1d}, \ \ x_{5d} = \dot{x}_{2d}, \ \ x_{6d} = \dot{x}_{3d}$$
 (21)

then, the system description is given by

$$\dot{x} = A(p)x + B_2(\delta)u + E_1\dot{d} + E_2d$$

$$z_1 = u$$

$$z_2 = C x$$
(22)

where

$$x = [x_{1d} \ x_{2d} \ x_{3d} \ x_{4d} \ x_{5d} \ x_{6d}]^T$$
 (23)

Here, it is assumed that the passive parameter  $p(m_1, m_2, k_1 \text{ and } k_2)$  is fixed. So, optimization parameters are given by  $\delta(m_3, k_T)$  and  $\mathcal{E}$  (controller parameter). Then the system matrices of equation (22) are represented by

$$B_{2}(\delta) = \begin{bmatrix} 0 & 0 & 0 & 0 & -k_{T}/m_{2} & k_{T}/m_{3} \end{bmatrix}^{T}$$

$$E_{1} = \begin{bmatrix} 0 & 0 & 0 & c_{1}/m_{1} & 0 & 0 \end{bmatrix}^{T}$$

$$E_{2} = \begin{bmatrix} 0 & 0 & 0 & k_{1}/m_{1} & 0 & 0 \end{bmatrix}^{T}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Suppose

$$B_1 = \begin{bmatrix} E_2 & E_1 \end{bmatrix}, \quad w = \begin{bmatrix} d \\ \dot{d} \end{bmatrix} \tag{25}$$

then, the equation (22) is rewritten as follows:

$$\dot{x} = A(\delta)x + B_1w + B_2(\delta)u$$

$$z_1 = u$$

$$z_2 = Cx$$
(26)

Suppose that we have the freedom to redesign the values for active mass  $m_3$  and torque constant  $k_T$ . That is, the actuator redesign parameters are

$$\delta_1 = k_T, \quad \delta_2 = m_3$$

The constraint on the redesign parameter is given by

$$\delta_{im} = 0.1$$
,  $\delta_{iM} = \infty$ ,  $i = 1, 2$ .

With the numerical tolerance 0.1, the algorithm of section 4 gives the optimal variation of  $\delta(k_T, m_3)$  with the parameter  $\gamma(>0)$  which is illustrated in Fig. 2. From this result,  $k_T=1.8$  and  $m_3=2.2$  are chosen for optimal values. Where  $\gamma=0.0086$  and the controller parameter is given by

$$\mathcal{Z} = \left[ \begin{array}{cc} A_c & B_c \\ C_c & D_c \end{array} \right]$$

where

$$A_c = \begin{bmatrix} -1.008 \times 10^2 & 1.562 \times 10^2 & 5.556 \times 10 & 2.645 \times 10 & 2.223 \times 10 & 8.508 \\ 4.457 & -6.621 \times 10 & -2.720 & -6.903 & -1.005 \times 10 & -6.897 \\ 6.239 \times 10 & 6.615 \times 10 & 2.848 & 1.766 & 1.075 \times 10 & -4.881 \\ -2.572 \times 10^2 & 3.405 \times 10^2 & 1.290 & 4.598 \times 10 & 4.866 \times 10 & 2.278 \\ 3.767 & -1.475 & -3.248 & 3.465 \times 10^2 & -1.644 \times 10^2 & -1.644 \times 10^2 \\ -1.016 \times 10^2 & 1.665 \times 10^2 & 5.998 & 2.760 \times 10 & 2.381 \times 10 & 8.465 \end{bmatrix}$$

$$B_c = \begin{bmatrix} -5.653 \times 10 \\ 4.167 \times 10 \\ -7.252 \times 10 \\ -1.049 \times 10^2 \\ -1.080 & -6.398 \times 10 \end{bmatrix}, C_c = \begin{bmatrix} 6.504 \times 10^{-6} \\ -1.016 \times 10^{-7} \\ -4.737 \times 10^{$$

The impulse response(the displacement of under plate which is observed using laser

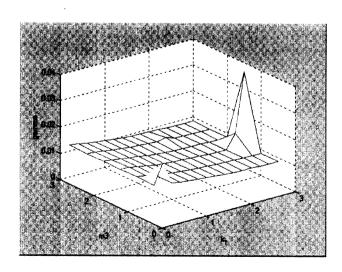


Fig. 2 Performances with redesign process (iteration index with respect to  $k_T$ ,  $m_3$ ,  $\gamma$ )

sensor) of the open-loop structure is shown in Fig. 3. Fig. 4 and Fig. 5 denote the impulse response and control input of the closed-loop system associated with the redesigned closedloop structure and the controller (simultaneous optimization design method). The frequency responses of the system with control and without control are shown in Fig. 6.

### 6. Concluding Remarks

In this paper, an algorithm for integrating structure and control system design of output feedback case is considered. The optimization

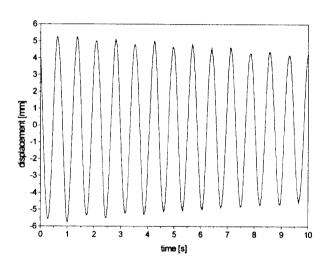


Fig. 3 Impulse response (uncontrolled case)

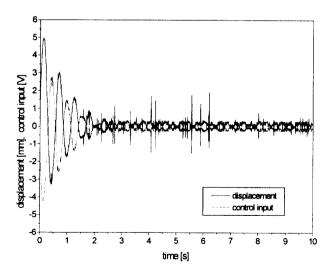


Fig. 4 Impulse response and control input (controlled case : simultaneous optimization control)

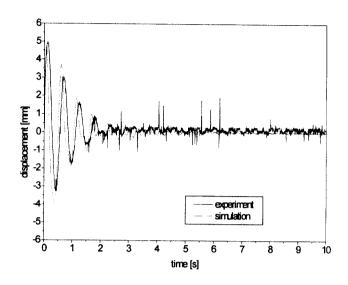


Fig. 5 Impulse responses (simulation and experiment results)

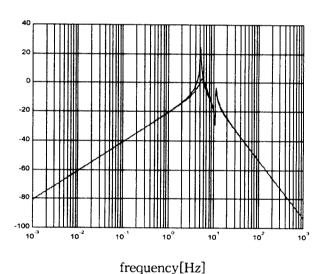


Fig. 6 Frequency responses (--:open loop, --:closed loop)

problem is divided into two subproblems with iteration between the two. The first subproblem gives the optimal values for the structure passive parameters. The second subproblem gives the optimal values for the actuator location parameters and controller with constraint. This approach has been applied to the vibration control system design problem of a structure. In practical, from the experiment results, it is clear that this approach is very useful to the control system design such that

the high demanding performance requirements are achieved.

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