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Design Aids for a Reinforced Concrete Beam with the Minimum Cost Concept

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Abstract

In reinforced concrete design, structural member sizes and amount of reinforcing steel areas are usually selected based on the structural designers' experience. Most existing charts provided for the design of reinforced concrete structural members were developed mainly based on force equilibrium conditions and some serviceability criteria. Sections selected from these charts may not result in an economic solution in terms of material costs as well as construction costs. Practical design aids are developed and suggested in this study for the economical design of reinforced concrete beam under flexural loading. With the beam width fixed, the depth of a beam, positive steel areas and negative steel areas are found from Khun-Tucker necessary conditions with Lagrangian multipliers to minimize the sectional cost of a beam. The developed design aids might be useful in selecting optimum reinforced concrete beam sections. Theoretical derivations and use of the developed design aids are described in this paper.

Keywords : optimum design, reinforced concrete beam, design aids

1. INTRODUCTION

Most design charts or aids commonly being used for reinforced concrete structural elements are developed basically with code equations (Notes on ACI 318-95). These equations require two conditions to be met: force equilibrium between the externally applied loads and member resistance as well as the serviceability criteria. In practice of designing reinforced concrete members, these equations are well condensed to various types of design aids for the sake of the structural designers' convenience in selecting proper members sizes and reinforcing steel areas. The charts relating M_u/bd^2 with ρ (where M_u, b, d and ρ represent design moment, beam width, beam depth and steel ratio, respectively) for the reinforced concrete beam design would be an example of a popular design aid, among others. Although many different design aids are available for different types of reinforced concrete members, the effect of costs for the constituent materials and construction of reinforced concrete members is not included in these design aids. This implies that depending on the individual structural designers' experience, different member costs may result.

For the realization of the optimum design of reinforced concrete structures, various technical methods have been suggested (Arsenis & Koumousis, 1994; Jenkins, 1994; Fellow & Franklin, 1997; Choi and Kwak, 1990). These methods, however, require computational efforts with mathematical tools such as nonlinear programming or combinatorial approaches to arrive at the desired global optimum solutions. Although these methods have their own merits, further development or refinement seems to be needed for their practical applications, especially for reinforced concrete structures. The design aids need to be simple and yet practical in their use from the point of the structural designers' view. As a bridge between the traditional design aids without the optimum concept and sophisticated numerical optimum algorithms, rather simple design charts considering cost effects are developed in this study for the reinforced concrete beam. For the rational design, conventional charts which are mainly developed from force equilibrium only, must be modified to include cost effects in determining beam sizes and reinforcing steel areas. Costs for a reinforced concrete beam include steel cost, concrete cost, forming cost and cost of increasing the building height in order to house the beam.

Friel(1974) suggested an equation for the optimum ratio of steel to concrete. In his study, a beam width is fixed, and then for this given beam width, the optimum beam depth and steel ratio are developed from Khun-Tucker necessary conditions for singly reinforced concrete sections. The total cost of the beam for a unit length is considered. Formulas for the optimum results have been derived, typical curves have been drawn, and an example has been included. In order to use the results, the optimum ratio of steel to concrete is found and is used to determine the beam dimensions.

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For the generality and practicability in designing a reinforced concrete beam with the minimum cost concept, basic equations developed by Friel(1974) are further extended in this study. The developed equations are applicable to continuous beams usually subject to both positive and negative moments. Explicit equations for the optimum beam depth, positive steel area and negative steel area are obtained from Khun-Tucker necessary conditions. These equations are then converted to design charts for the practical design of a reinforce concrete beam.

2. DERIVATION OF OPTIMUM REINFORCED CONCRETE BEAM SECTION SUBJECT TO BOTH POSITIVE AND NEGATIVE MOMENTS

In this section, the theoretical development of finding the optimum depth and reinforcing steel areas are presented for the reinforced concrete beam subject to positive and negative moments. Consider a beam section shown in Figure.1. The objective function for the cost of a unit length of a beam subject to both positive and negative moments can be described as:

$$\begin{aligned} \text{Minimize } C &= C_s + C_c + C_f + C_h \\ &= (C_s^+ + C_s^-) + C_c + C_f + C_h \\ &= (A_s^+ + A_s^-) w_s p_s + bdk_v p_c \\ &\quad + (2d+b)p_f + dk_h \end{aligned} \quad \text{(Eq. 1)}$$

$$\text{Subject to } \phi M_n^+ = \phi M_u^+ \text{ and } \phi M_n^- = \phi M_u^-, \quad \text{(Eq. 2)}$$

where C_s = cost of steel
$= A_s w_s p_s$;
C_c = cost of concrete
$= bdk_v p_c$;
C_f = cost of forming
$= (2d+b)p_f$; and
C_h = cost of increasing the building in order to house the beam
$= dk_h$.

In the above equations, A_s^+, A_s^- = steel areas for positive and negative moments, respectively, w_s = density of steel, p_s = price of steel, b = width, d = depth, k_v = volume constant, p_c = price of concrete, p_f = price of forming, and k_h =cost of increasing the building height, respectively.

In order to apply the Khun-Tucker necessary conditions, two Lagrangian multipliers $(\lambda^+$ and $\lambda^-)$ are used. The above constrained optimization problem is then converted to an unconstrained optimization problem:

$$\text{Minimize } \overline{C} = C + \lambda^+ g^+ + \lambda^- g^-, \quad \text{(Eq. 3)}$$

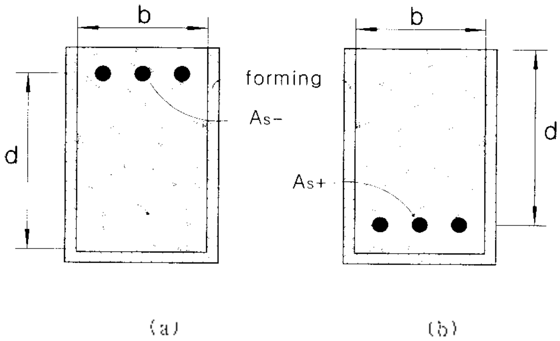


Figure 1. Reinforced concrete beam section: (a) section under negative moment; and (b) section under positive moment

where $g^+ = \phi A_s^+ f_y \left(d - \dfrac{A_s^+ f_y}{1.7 f_c' b} \right) - M_u^+$;

$g^- = \phi A_s^- f_y \left(d - \dfrac{A_s^- f_y}{1.7 f_c' b} \right) - M_u^-$; and

λ^+, λ^- = Lagrange's multipliers.

The above equations have five independent design variables, A_s^+, A_s^-, λ^+, λ^- and d. The Khun-Tucker necessary conditions lead to the following five simultaneous nonlinear system of equations:

$$\frac{\partial \overline{C}}{\partial A_s^+} = w_s p_s + \phi\lambda^+ f_y d - \frac{2\phi\lambda^+ f_y^2}{1.7 f_c' b} A_s^+ = 0 \quad \text{(Eq. 4a)}$$

$$\frac{\partial \overline{C}}{\partial A_s^-} = w_s p_s + \phi\lambda^- f_y d - \frac{2\phi\lambda^- f_y^2}{1.7 f_c' b} A_s^- = 0 \quad \text{(Eq. 4b)}$$

$$\frac{\partial \overline{C}}{\partial d} = k_p + \phi\lambda^+ A_s^+ f_y + \phi\lambda^- A_s^- f_y = 0 \quad \text{(Eq. 4c)}$$

$$\frac{\partial \overline{C}}{\partial \lambda^+} = \phi A_s^+ f_y \left(d - \frac{A_s^+ f_y}{1.7 f_c' b} \right) - M_u^- = 0 \quad \text{(Eq. 4d)}$$

$$\frac{\partial \overline{C}}{\partial \lambda^-} = \phi A_s^- f_y \left(d - \frac{A_s^- f_y}{1.7 f_c' b} \right) - M_u^- = 0. \quad \text{(Eq. 4e)}$$

In order to solve the above equations, two more equations with two additional variables, α and β, are introduced:

$$\lambda^- = \alpha\lambda^+ \quad \text{(Eq. 5a)}$$

$$A_s^- = \beta A_s^+ . \quad \text{(Eq. 5b)}$$

From Eq. (Eq. 4c), we get ;

$$\phi\lambda^+ f_y = -\frac{k_p}{A_s^-(1+\alpha\beta)}, \quad \text{(Eq. 6)}$$

where $k_p = b \times p_c + 2 \times p_f$.

After substituting the above equation into (Eq. 4a) and then dividing both sides by $\dfrac{b}{k_p}$, we have:

$$\frac{bd}{A_s^{+}(1+\alpha\beta)} = \frac{bw_s p_s}{k_p} + \frac{2f_y}{1.7f_c'} + \frac{1}{1+\alpha\beta}$$

or

$$\rho^{+} = \frac{A_s^{+}}{bd} = \left[\frac{2f_y}{1.7f_c'} + \frac{bw_s p_s}{k_p}(1+\alpha\beta)\right]^{-1} \qquad \text{(Eq. 7)}$$

(Eq. 4d) is simplified to the following equation by dividing both sides by $b \cdot d^2$;

$$k_u^{+} = \frac{M_u^{+}}{bd^2} = \phi\rho^{+} f_y\left(1-\frac{\rho^{+} f_y}{1.7f_c'}\right). \qquad \text{(Eq. 8)}$$

From the above (Eq. 8), d can be expressed as ;

$$d = \sqrt{\frac{M_u^{+}}{\phi\rho^{+} f_y b}\left(\frac{1.7f_c'}{1.7f_c' - \rho^{+} f_y}\right)}. \qquad \text{(Eq. 9)}$$

Since $(A_s^{+}) = (\rho^{+} bd)$ and using the results from (Eq. 8) and (Eq. 9), we get;

$$\begin{aligned}(A_s^{+}) &= (\rho^{+}) \cdot b \cdot \frac{M_u^{+}}{k_u^{+}} \\ &= \frac{M_u^{+}}{\phi f_y}\left[\frac{1.7f_c' k_p b}{f_y k_p + 1.7f_c' bw_s p_s(1+\alpha\beta)}\right].\end{aligned} \qquad \text{(Eq. 10)}$$

Now, we relate the remaining two equations (Eq. 4b) and Eq. 4c) to the two additional equations(Eq. 5a and Eq. 5b).

Substituting $\lambda^{-} = \alpha\lambda^{+}$ (Eq. 5a), $A_s^{-} = \beta A_s^{+}$ (Eq. 5b) and $\phi\lambda^{+} f_y = \frac{k_p}{A_s^{+}(1+\alpha\beta)}$ (Eq. 6) into (Eq. 4b) and then dividing both sides by $\frac{b}{\alpha k_p}$, we obtain:

$$\frac{bd}{A_s^{+}(1+\alpha\beta)} = \frac{bw_s p_s}{\alpha k_p} + \frac{2f_y}{1.7f_c'}\left(\frac{\beta}{1+\alpha\beta}\right). \qquad \text{(Eq. 11)}$$

From (Eq. 11), ρ^{+} defined as $\rho^{+} = \frac{A_s^{+}}{b \cdot d}$ can be given as ;

$$\rho^{+} = \left[\frac{bw_s p_s}{\alpha k_p}(1+\alpha\beta) + \frac{2f_y}{1.7f_c'}\beta\right]^{-1}. \qquad \text{(Eq. 12)}$$

It is required that ρ^{+} in (Eq. 12) be equal to the one in (Eq. 7) ;

$$\begin{aligned}&\frac{2f_y}{1.7f_c'} + \frac{bw_s p_s}{k_p}(1+\alpha\beta) \\ &= \frac{2f_y}{1.7f_c'}\beta + \frac{bw_s p_s}{\alpha k_p}(1+\alpha\beta)\end{aligned} \qquad \text{(Eq. 13)}$$

This equation then becomes a quadratic equation for α :

$$\beta\alpha^2 + [(\zeta+1)(1-\beta)]\alpha - 1 = 0, \qquad \text{(Eq. 14)}$$

where $\zeta = \frac{2\bar{f}k_p}{1.7bw_s p_s} = \frac{2k_p f_y}{1.7bw_s p_s f_c'}$; and $\bar{f} = \frac{f_y}{f_c'}$.

The solution to the above equation is:

$$\alpha = \frac{-(\zeta+1)(1-\beta) \pm \sqrt{(\zeta+1)^2(1-\beta)^2 + 4\beta}}{2\beta} \qquad \text{(Eq. 15)}$$

Now, (Eq. 4c), after being divided by $b \cdot d^2$ becomes;

$$k_u^{-} = \frac{M_u^{-}}{bd^2} = \phi\beta\rho^{+} f_y\left(1-\frac{\beta\rho^{+} f_y}{1.7f_c'}\right). \qquad \text{(Eq. 16)}$$

The depth of the beam d is then equal to:

$$d = \sqrt{\frac{M_u^{-}}{\phi\rho^{+} f_y \beta b}\left(\frac{1.7f_c'}{1.7f_c' - \beta\rho^{+} f_y}\right)}. \qquad \text{(Eq. 17)}$$

By letting (Eq. 9) be equal to (Eq. 17) for an identical prismatic beam, we have ;

$$\frac{M_u^{+}}{1.7f_c' - \rho^{+} f_y} = \frac{M_u^{-}}{\beta(1.7f_c' - \rho^{+} f_y \beta)}. \qquad \text{(Eq. 18)}$$

Solving for ρ^{+}, we get ;

$$\rho^{+} = \frac{1.7\,\mathrm{f}\,(\mathrm{m} - \beta)}{\mathrm{m} - \beta^2}, \qquad \text{(Eq. 19)}$$

where $m = \frac{M_u^{-}}{M_u^{+}}$ and $\mathrm{f} = \frac{f_c'}{f_y}$.

Substituting the above ρ^{+} into (Eq. 7), and then rearranging the resulting equation, we get ;

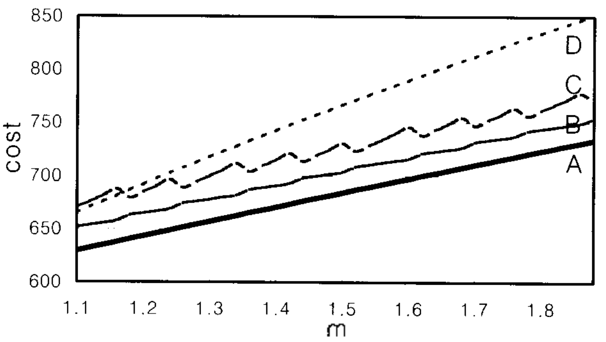


Figure. 2 Costs comparison with various design conditions for varying *m*.

$$\alpha = \frac{(m-\beta^2)k_p - 2(m-\beta)k_p - 1.7\mathrm{f}\, bw_s p_s (m-\beta)}{(m-\beta)\, 1.7\mathrm{f}\, bw_s p_s \beta} \quad \text{(Eq. 20)}$$

This must be equal to α in (Eq. 15) and from this condition, we arrive at the final equation in terms of one unknown variable, β:

$$\beta^2 + \frac{w_1w_2 + mw_1w_2 - 2w_1 - 4k_p}{2k_p - w_1w_2}\beta + \frac{2mk_p + 2mw_1 - mw_1w_2}{2k_p - w_1w_2} + \frac{(m-\beta)w_1\sqrt{w_2^{\,2}(1-\beta)^2 + 4\beta}}{2k_p - w_1w_2} = 0 \quad \text{(Eq. 21)}$$

where $w_1 = 1.7\,\mathrm{f}\, bw_s p_s$ and $w_2 = \zeta + 1$.

This equation is useful since only one design variable, β, needs to be found rather than solving all five nonlinear simultaneous equations of (Eq. 4a) through (Eq. 4e). Once this β is found for the given material costs and construction costs, all other design variables can be found: i.e., using β obtained from the above nonlinear equation, α can be found from (Eq. 20); using these two values, ρ' in (Eq. 12) can be obtained; and then with this ρ^-, the beam depth d can be obtained from (Eq. 9). Note that the steel area for negative moment can also be obtained from β, which was defined as the ratio of negative steel area to positive steel area.

The validity of the above solutions is illustrated in Figure. 2. For a given moment, the cost becomes minimum at the optimum depth for all moment ratios, $m\left(=\frac{M_{larger}}{M_{smaller}}\right)$. (see line A in Figure.2).

It also shows that an optimally designed beam based on the larger moment only (line D in Figure.2) leads to a more expensive design compared with the design from the optimization of both the larger and smaller moments at the same time.

Lines C and D in Figure.2 represent beams having shallower and deeper depths than the optimum one. It is shown that these lines are above line A in Figure.2, which implies more expensive designs than the optimum one (line A).

3. DEVELOPMENT OF DESIGN AIDS FOR REINFORCED CONCRETE BEAM

The design aids for practical application of the mentioned concept to reinforced concrete design have been developed (Figure.3). According to the design aids shown in Figure. 3, designers can select the optimum reinforcement area and beam depth depending on current material costs and construction costs. Each design aid is constructed for discrete beam width, specific concrete compressive strength, steel yield strength and steel cost. Designers need to estimate the total cost for concrete and forming (k_p). Based on this total cost and ratio of larger moment to smaller moment (m) being applied to a beam section, the positive steel ratio (ρ^+) can be found from Figure.3.

Using this positive steel ratio (ρ^+), the beam depth (d) can also be obtained. The negative steel area or its ratio (ρ^-) can be found from the same design aids.

4. EXAMPLE

The use of the design aids presented in the previous section is illustrated in this section. Two examples are given in the following.

4.1 A Beam subject to Positive and Negative Moments

Consider a 30cm width beam subject to positive moment and negative moment by $20\,\mathrm{t}\cdot\mathrm{m}$ and $30\,\mathrm{t}\cdot\mathrm{m}$, respectively. Concrete strength and yield strength of the reinforcing steels are assumed to be $f_c' = 240\,kg/cm^2$ and $f_y = 4000\,kg/cm^2$, respectively. Let the costs of concrete, beam forming and reinforcing steels be equal to 700 won/t, 121,660 won/m^3 and 19,200 won/m^2, respectively. From these, k_p can be found to be 7.44 won/cm^2.

From the design aid shown in Figure.3(c), optimum values can be found as follows:

(1) Calculate $m = \frac{\text{larger}\, M_u}{\text{smaller}\, M_u} = \frac{M_u^-}{M_u^+} = \frac{30\mathrm{t}\cdot\mathrm{m}}{20\mathrm{t}\cdot\mathrm{m}} = 1.5$

(2) Find $\rho^- = 0.0116$ from Figure.3(c) using m and k_p.
(steps (1) and (2) in Figure.3(c))

(3) Find $\frac{bd^2}{M_u}$ in Figure.3(c) and calculate d.

From Figure.3(c), $\frac{bd^2}{M_u} = 0.027$ (step (3) in Figure.3(c)).
Therefore,

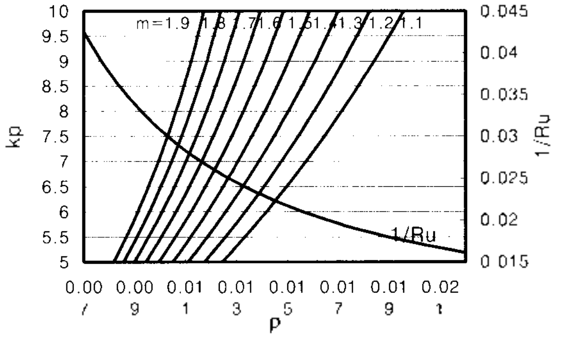
$$d = \sqrt{\frac{bd^2}{M_u}\times\frac{M_u^+}{b}} = \sqrt{0.027\times\frac{2\times10^6}{30}} = 42.37cm.$$

(4) Calculate $M = \frac{bd^2}{M_u^-} = 0.018$, and find ρ^- using Figure.3(c) (steps (4) and (5) in Figure. 3(c)).
$\rho^- = 0.019$.

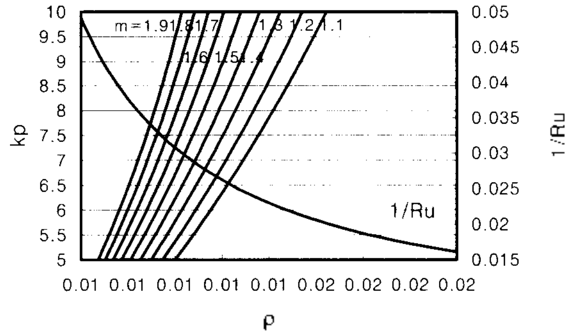
(5) Optimal steel areas are:

$$A_s^+ = \rho^+ \times b \times d = 0.0116\times30\times42.37 = 14.74cm^2$$

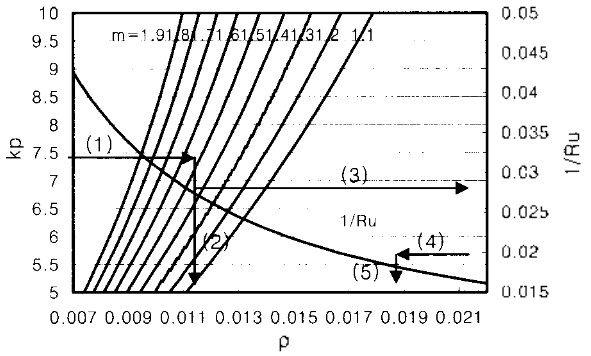
$$A_s^- = \rho^- \times b \times d = 0.0190\times30\times42.37 = 24.15cm^2.$$



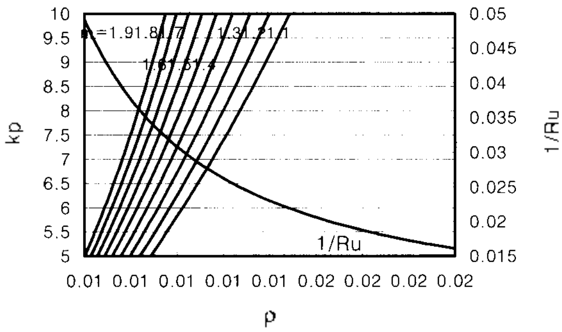
(a) $f_c' = 240kg/cm^2, b = 30cm, p_s = 600won/kg$



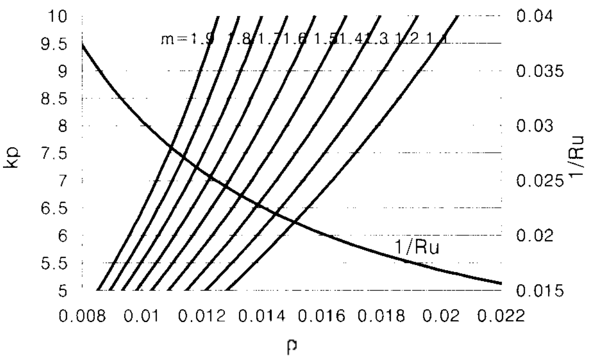
(b) $f_c' = 240kg/cm^2, b = 40cm, p_s = 600won/kg$



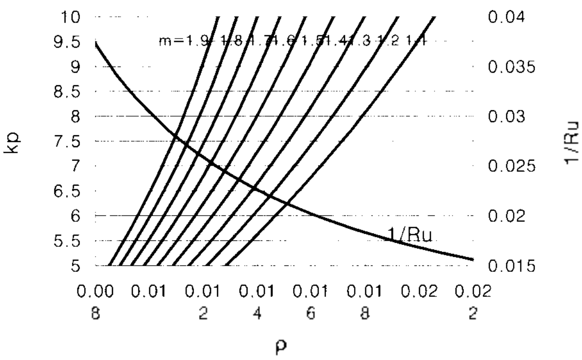
(c) $f_c' = 240kg/cm^2, b = 30cm, p_s = 700won/kg$



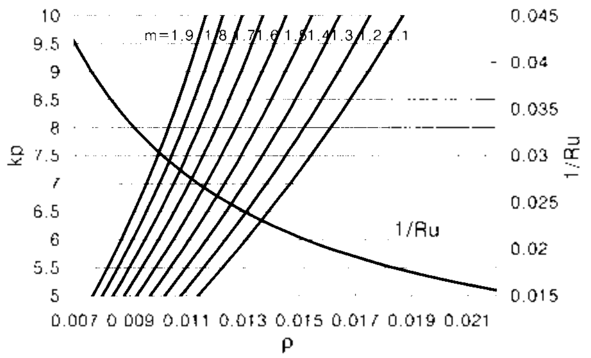
(d) $f_c' = 240kg/cm^2, b = 40cm, p_s = 700won/kg$



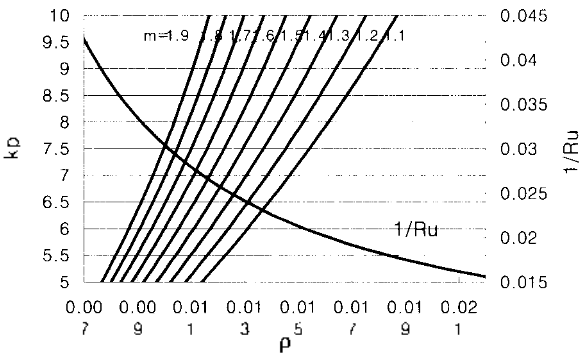
(e) $f_c' = 270kg/cm^2, b = 30cm, p_s = 600won/kg$



(f) $f_c' = 270kg/cm^2, b = 40cm, p_s = 600won/kg$



(g) $f_c' = 270kg/cm^2, b = 30cm, p_s = 700won/kg$



(g) $f_c' = 270kg/cm^2, b = 40cm, p_s = 700won/kg$

Figure.3 Design aids with minimum cost concepts ($f_y = 400kg/cm^2$)

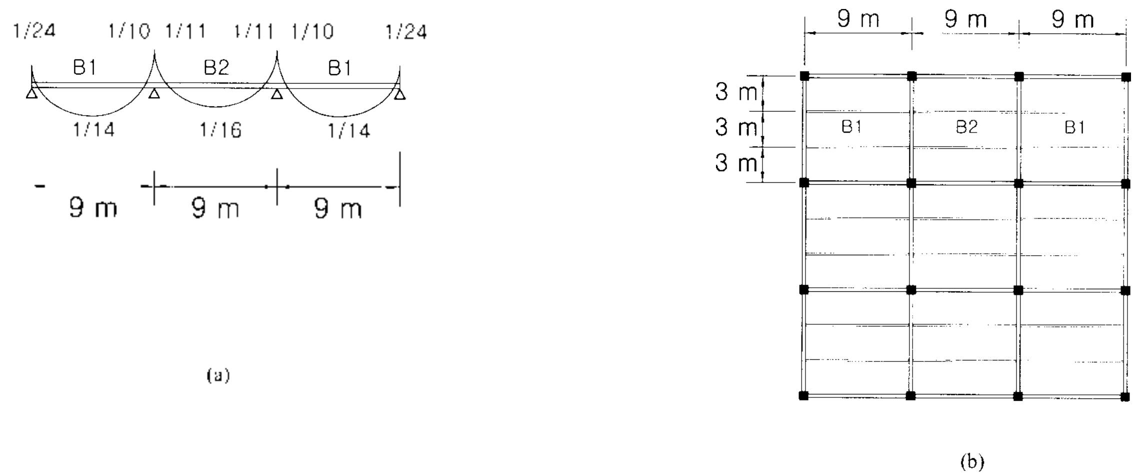


Figure.4 . Typical floor plan for the office building: (a) moment coefficients for a continuous beam from the ACI code1); and (b) structural floor plan showing beams and girders.

4.2 Continuous Beam

Figure. 4 shows a typical floor plan for the office building. Beams can be assumed to be continuous and supported by the girders.

For practical beam design, moments applied to these beams can be easily estimated from the moment coefficients suggested by the ACI Code (Note on ACI 318-95). These values are shown in Figure.4(a). Uniform service loads are assumed to be 250 kg/m^2 for live loads and 520 kg/m^2 for dead loads. The ultimate uniform load, therefore, is equal to:

$$w_u = 1.7 \times 250kg/m^2 + 1.4 \times 520kg/m^2 = 1{,}153kg/m^2 \quad \text{(Eq. 22)}$$

Costs for concrete, forming and reinforcing steels are assumed to be 700 won/t, 121,660 won/m^3 and 19,200 won/m^2, respectively. Concrete strength and yield strength of the reinforcing steel are $f_c' = 240kg/cm^2$ and $f_y = 4000kg/cm^2$ in this example.

Since each beam has different moment ratios, different beam depths can be expected as a result of optimization for each beam (B_1 through B_3). For practical reasons, however, all beams need to have identical depth. The beam width is assumed to be 30cm.

Figure. 3(c) is used as a design aid for this continuous beam design. Similar steps as in example 4.1 are taken. Table 1 summarizes the design results. In table 1, the initial solution is obtained by treating each beam individually.

This solution is then modified to the modified design in which the modified beam depths are adjusted to have the same dimension by referring to each beam depth from the initial design stage. Although this modification results in a marginal increase in beam cost per unit length, it is observed to be within the acceptable range near the optimum.

5. CONCLUSION

Practical design aids for structural designers are presented for finding the optimum depth and reinforcing steel areas of reinforced concrete beam subject to positive and negative moments. The Khun-Tucker necessary condition is used in finding optimum values of design variables. Although this condition is complicated by the five simultaneous nonlinear system of equations. These equations are simplified to a single explicit nonlinear equation. Iterative procedures like the Regula-Falsi method are employed in order to solve for the key parameter, β .

The suggest design aids can be useful in reinforced concrete beam design by the following reasons:

1) Although many research outcomes are available for optimization techniques for reinforced concrete structures or members, the suggested design aids might be the first attempt providing structural designers with a practical design guide in the form of design aids;
2) Compared to conventional beam design charts, the suggested design aids lead structural designers to more optimized beam section; and
3) Conventional beam design charts require a predetermined beam width and depth to find reinforcement areas for a given external moment, and need to be referred to twice to design a beam subject to both positive and negative moments. The suggested design aids, however, need the beam width and the ratio of the lager moment to the smaller moment only. These can be entered to an appropriate design aid to find the necessary amount of positive and negative steel areas. This feature might lessen the structural designers' effort in the design process.

Table 1. The optimum results for example 2.

		M_u^+ (kg.cm)	M_u^- (kg.cm)	d (cm)	A_s^+ (cm^2)	A_s^- (cm^2)	ρ^+	ρ^-	Cost (won/cm)	Total Cost (won/cm)
Initial Design	B1 (left)	2.00×10^6	1.17×10^6	34.0	20.3	10.6	0.019	0.010	481	1,605
	B1 (right)	2.00×10^6	2.80×10^6	41.3	15.3	23.1	0.012	0.019	576	
	B2	1.75×10^6	2.55×10^6	39.1	14.1	22.2	0.012	0.019	548	
Modified Design	B1 (left)	2.00×10^6	1.17×10^6	41.3	15.3	8.4	0.012	0.009	496	1,622
	B1 (right)	2.00×10^6	2.80×10^6	41.3	15.3	23.1	0.012	0.019	576	
	B2	1.75×10^6	2.55×10^6	41.3	13.14	20.4	0.011	0.016	550	

REFERENCES

1. *Note on ACI 318-95 Building Code Requirements for Structural Concrete with Design Applications*.
2. Arsenis, S.J., & Koumousis, V.K. (1994). *Genetic algorithms in a multi-criterion optimal detailing of reinforced concrete members*. (pp. 223-240) Edinburgh, Scotland: CIVIL-COMP Ltd.
3. Jenkins, W.M (1994). *A space condensation heuristic for combinatorial optimization*. (pp.215-224) Edinburgh, Scotland: CIVIL-COMP Ltd.
4. Fellow & Franklin Y.C. (1997, Sep) Multiobjective optimization design with pareto genetic algorithm. *J. of Structural Engineering*, 123 (9), 1252-1261.
5. Choi, C.K., & Kwak, H.G. (1990, Oct) Optimum RC member with predetermined discrete section. *ASCE, J. of the Structural Engineering*, 116 (10), 2634-2655.
6. Friel, L.L. (1974, Nov) Optimum singly reinforced concrete sections. *ACI Journal*, 556-558.
7. Arora, J.S. (1989). *Introduction to Optimum Design, Readings*. McGROW-HILL Ltd.
8. Thanedar, P.B., & Vanderplaats, G.N. (1995, Feb) Survey of discrete variable optimization for structural design. *ASCE, J. of the Structural Engineering*, 121 (2), 301-306.
9. Balling, R.J., & Xiaoping, Y. (1997, Feb) Y. Optimization of reinforced concrete frame. *J. of Structural Engineering*, 193-202.
10. Gurujee, C.S & Yadava, R.S.S., (1997, May) Optimal design of trusses using available section. *ASCE, J. of the Structural Engineering*, 123 (5), 685-688.