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ON THE CONTINUITY AND GAUSSIAN CHAOS OF SELF-SIMILAR PROCESSES

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ABSTRACT. Let $\{X(t), t \geq 0\}$ be a stochastic integral process represented by stable random measure or multiple Ito-Wiener integrals. Under some conditions, we prove the continuity and self-similarity of these stochastic integral processes. As an application, we get Gaussian chaos which has some shift continuous function.

1. Introduction

We are interested in self-similar process which has continuous sample paths and want to know the properties of self-similar processes via stochastic integral representations of the type

$$X(t) = \int_R Q_t(u) dM_\alpha(u),$$

under some conditions on kernel Q_t, where M_α is a symmetric α stable random measure ([5],[6]). The useful examples of these types are fractional Brownian motion and linear fractional stable motion which are the two major families of self-similar time series to investigate the intensity of long-range dependence ([7],[8],[9]).

On the other hand, consider the integral processes defined by multiple Ito-Wiener integrals

$$Y_m(t) = \int \cdots \int Q_t(u_1, u_2, \cdots, u_m) dB(u_1) dB(u_2) \cdots dB(u_m), \quad t \geq 0,$$

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where, the right hand side is the m-multiple Ito-Wiener integral with respect to the standard Brownian motion $\{B(u), u \geq 0\}$. ([3],[4]) prove a functional iterated logarithm law for a certain class of self-similar processes and ([2]) furnish us with general information on Gaussian chaos which were introduced by N. Weiner.

In chapter 2, we give some definitions and get that some $S\alpha S$ distribution has the same distribution as the first arrival time of Brownian motion. In chapter 3, we show that $\{X(t), t \geq 0\}$ is H-self-similar and has continuous sample paths under some conditions on the kernel Q_t. In chapter 4, we consider the stochastic integral processes represented by multiple Ito-Wiener integral. We prove the continuity and self-similarity of this integral process $\{Y_m(t), t > 0\}$. Finally, we get Gaussian chaos which has some shift continuous function.

2. Preliminaries

DEFINITION 2.1. A random variable is said to have an α-stable distribution if there are parameters $0 < \alpha \leq 2$, $\sigma \geq 0$, $-1 \leq \beta \leq 1$ and μ real such that its characteristic function has the following form

$$E[\exp i\theta X] = \begin{cases} \exp\{-\sigma^\alpha|\theta|^\alpha(1 - i\beta(\text{sign }\theta)\tan\frac{\pi\alpha}{2}) + i\mu\theta\} & \text{if } \alpha \neq 1, \\ \exp\{-\sigma|\theta|(1 + i\beta\frac{2}{\pi}(\text{sign }\theta)\ln|\theta|) + i\mu\theta\} & \text{if } \alpha = 1. \end{cases}$$

We call σ scale parameter, β skewness parameter and μ shift parameter and the parameters are unique. We denote $S_\alpha(\sigma, \beta, \mu)$ an α-stable random variable. If random variable X is a symmetric α-stable, i.e., $\beta = \mu = 0$, then we write $X \sim S\alpha S(\sigma)$.

EXAMPLE 2.1. The Levy distribution $S_{1/2}(\sigma, 1, \mu)$ whose density

$$\left(\frac{\sigma}{2\pi}\right)^{1/2} \frac{1}{(x-\mu)^{3/2}} \exp\left\{-\frac{\sigma}{2(x-\mu)}\right\}$$

is concentrated on (μ, ∞). If $X \sim S_{1/2}(\sigma, 1, 0)$, then for $x > 0$,

$$P(X \le x) = 2\left(1 - \Phi\left(\sqrt{\frac{\sigma}{x}}\right)\right),$$

where Φ denote the cumulative distribution function of $N(0,1)$ distribution.

THEOREM 2.1. *Let $\{B(t), t \ge 0\}$ be a standard Brownian motion with a.s. continuous paths. Let T_α denote the first time it reaches the level $a > 0$. Then T_α has the Levy distribution*

$$S_{1/2}(a^2, 1, 0).$$

Proof. From reflection principle of Brownian motion, we know that $P(T_a < t) = 2P(B(t) > a)$. Thus

$$\begin{aligned} P(T_a < t) &= 2P(N(0,t) > a) \\ &= 2P(t^{1/2}N(0,1) > a) \\ &= 2P(N(0,1) > at^{-1/2}) \\ &= 2\left(1 - \Phi\left(\frac{a}{\sqrt{t}}\right)\right). \end{aligned}$$

By Example 2.1 , T_a has Levy distribution. □

DEFINITION 2.2. A stochastic process $\{X(t), t \ge 0\}$ is H-self-similar $(H - ss)$ if for some $H > 0$,

$$\{X(ct), t \ge 0\} \stackrel{d}{=} \{c^H X(t), t \ge 0\} \quad \text{for any } c > 0,$$

where, $\stackrel{d}{=}$ means the equality of all finite dimensional distributions.

DEFINITION 2.3. A random variable X with values in set C of continuous functions is a Gaussian chaos if for some $c_{ij}(\cdot) \in C$ $(i, j = 1, 2, \cdots)$,

$$X = \sum_{i,j} c_{ij}(\cdot) g_i g_j$$

where, $\{g_i\}_{i \ge 1}$ is i.i.d. $N(0,1)$ random sequence.

Denote by (Ω, F, P) with underlying probability space and $L^0(\Omega)$ the set of all real random variables defined on it. Let $(\mathbf{E}, E, m)$ be a $\sigma-$finite measure space and let

$$E_0 = \{A \in E, m(A) < \infty\}$$

be the subset of E that contains sets of finite m-measure.

DEFINITION 2.4. An independentently scattered σ-additive set function

$$M_\alpha : E_0 \to L^0(\Omega)$$

such that for each $A \in E_0$,

$$M_\alpha(A) \sim S\alpha S(m(A))$$

is called an SαS random measure on $(\mathbf{E}, E)$ with control measure m.

3. H-ss represented by stable random measure

Consider the processes

$$X(t) = \int_R Q_t(u) dM_\alpha(u),$$

where Q_t is of the form

$$Q_{t+h}(u) - Q_t(u) = Q_h(u) \quad \text{a.e.} \quad \text{in } u \text{ and for any } t, h \geq 0,$$

and

$$Q_t(u) = t^{H-\frac{1}{\alpha}} f\left(\frac{u}{t}\right),$$

where f is a measurable function.

LEMMA 3.1. *If f is a measurable function and if $f \in L^\alpha$, then the process X defined by*

$$X(t) = \int_R Q_t(u) dM_\alpha(u), \quad t \geq 0$$

is an $S\alpha S$ process.

Proof. For each $t \geq 0$,

$$\int \left| t^{H-\frac{1}{\alpha}} f\left(\frac{u}{t}\right) \right|^{\alpha} du = t^{H\alpha} \int |f(v)|^{\alpha} dv < \infty \quad \text{for each } t$$

Thus, Lemma follows from [5, chapter 3.4]. □

THEOREM 3.2. *$\{X(t), t \geq 0\}$ is an H-ss process with stationary increments and has continuous sample paths when $H > \frac{1}{\alpha}$.*

Proof. Note that $dM_\alpha(cu) = c^{1/\alpha} dM_\alpha(u)$. Since

$$\begin{aligned} X(ct) &= \int Q_{ct}(u) dM_\alpha(u) \\ &= \int (ct)^{H-\frac{1}{\alpha}} f\left(\frac{u}{ct}\right) dM_\alpha(u) \\ &\stackrel{d}{=} c^H \int t^{H-\frac{1}{\alpha}} f\left(\frac{v}{t}\right) dM_\alpha(v) \\ &= c^H X(t), \end{aligned}$$

the self-similarity of $X(t)$ holds. Similarly, $dM_\alpha(u+h) = dM_\alpha(u)$ and assumption on Q_t implies the stationary increments.

It is enough to show that for any $-\infty < a < b < \infty$, $\{X(t), a \leq t \leq b\}$ has continuous sample paths. By the H-self-similarity of the process and the stationarity of its increments, we have

$$E|X(t) - X(s)|^p = |t-s|^{Hp} E| \int f(u) dM_\alpha(u)|^p$$

for any $0 < p < \alpha$ and hence the metric generated by the process is of the form

$$d_X(t,s) = c_\alpha |t-s|^H.$$

Let $N(\epsilon)$ be the smallest number of open d_X balls of radius ϵ needed to cover $[a,b]$. Since

$$N(\epsilon) \leq c\epsilon^{-\frac{1}{H}}, \quad \text{for any } 0 < \epsilon \leq c_\alpha (b-a)^H,$$

the assumption $H > \frac{1}{\alpha}$ ensures that the condition of [5, Theorem 12.2.1] holds. Therefore, $\{X(t), t \geq\}$ has continuous sample paths. □

Let

$$f(u) = a\{(1-u)_+^{H-\frac{1}{\alpha}} - (-u)_+^{H-\frac{1}{\alpha}}\} + b\{(1-u)_-^{H-\frac{1}{\alpha}} - (-u)_-^{H-\frac{1}{\alpha}}\},$$

where, a and b are some constants. Then $X(t)$ is a fractional Brownian motion (FBM) if $\alpha = 2$ and a linear fractional stable motion (LFSM) if $\alpha < 2$. Let us apply Theorem 3.2 to FBM and LFSM which are two major self-similar time series to investigate long-range dependence from modeling view points.

Since

$$Q_t(u) = a\{(t-u)_+^{H-\frac{1}{\alpha}} - (-u)_+^{H-\frac{1}{\alpha}}\} + b\{(t-u)_-^{H-\frac{1}{\alpha}} - (-u)_-^{H-\frac{1}{\alpha}}\}$$

is in $L^\alpha(R)$, $X(t)$ is $S\alpha S$ process by Lemma 3.1. For the self-similarity,

$$\begin{aligned}
X(ct) &= \int a\{(ct-u)_+^{H-\frac{1}{\alpha}} - (-u)_+^{H-\frac{1}{\alpha}}\} \\
&\quad + b\{(ct-u)_-^{H-\frac{1}{\alpha}} - (-u)_-^{H-\frac{1}{\alpha}}\} dM_\alpha(u) \\
&= \int a\{(ct-cu)_+^{H-\frac{1}{\alpha}} - (-cu)_+^{H-\frac{1}{\alpha}}\} \\
&\quad + b\{(ct-cu)_-^{H-\frac{1}{\alpha}} - (-cu)_-^{H-\frac{1}{\alpha}}\} dM_\alpha(cu) \\
&\stackrel{d}{=} c^{H-\frac{1}{\alpha}} c^{\frac{1}{\alpha}} \int a\{(t-u)_+^{H-\frac{1}{\alpha}} - (-u)_+^{H-\frac{1}{\alpha}}\} \\
&\quad + b\{(t-u)_-^{H-\frac{1}{\alpha}} - (-u)_-^{H-\frac{1}{\alpha}}\} dM_\alpha(u) \\
&= c^H X(t).
\end{aligned}$$

By $dM_\alpha(u+h) = dM_\alpha(u)$, the stationary increment property holds. And, Theorem 3.2 implies $\{X(t), t \geq 0\}$ has continuous sample paths when $H > \frac{1}{\alpha}$.

4. H-ss represented by Ito-Wiener integrals

Let $\{Q_t(u_1, u_2, \cdots, u_m), t \geq 0\}$ be a square integrable symmetric function on R^m and be assumed to be of the form

$$Q_t(u_1, u_2, \cdots, u_m) = t^{H-\frac{m}{2}} f\left(\frac{u_1}{t}, \cdots, \frac{u_m}{t}\right),$$

where, f is a bounded continuous function on R^m such that

$$\int \cdots \int_{R^m} f^2(u_1, u_2, \cdots, u_m) du_1 \cdots du_m < \infty.$$

Consider the processes represented by the multiple Ito-Wiener integrals

$$Y_m(t) = \int \cdots \int_{R^m} Q_t(u_1, \cdots, u_m) dB(u_1) \cdots dB(u_m),$$

where, the right-hand side is a m-multiple Wiener integral with respect to standard Brownian motion $\{B(u), u \geq 0\}$.

LEMMA 4.1. *For any stochastic process $\{Z(t), t \geq 0\}$, if there exist constants $A > 0$ and $0 < H < 1$ such that*

$$[E|Z(t+h) - Z(t)|^2]^{1/2} \leq Ah^H, \quad (t \geq 0, \ 0 \leq h \leq 1),$$

then the processes $\{Z(t), t \geq 0\}$ may be supposed to have continuous sample paths.

Proof. See [3, Lemma 6.2]. □

THEOREM 4.2. *$\{Y_m(t), t \geq 0\}$ is self-similar with index H, $0 < H < 1$.*

Proof. For the self-similarity of $\{Y_m(t), t \geq 0\}$, we know that

$$\begin{aligned} Y_m(ct) &= \int \cdots \int_{R^n} Q_{ct}(u_1, u_2, \cdots) dB(u_1) dB(u_2) \cdots dB(u_m) \\ &= \int \cdots \int_{R^n} (ct)^{H-m/2} f\left(\frac{u_1}{ct}, \cdots \frac{u_m}{ct}\right) dB(u_1) \cdots dB(u_m). \end{aligned}$$

Let $\dfrac{u_1}{c} = v_1, \cdots, \dfrac{u_m}{c} = v_m$. Then

$$\begin{aligned} Y_m(ct) &= \int \cdots \int_{R^n} (ct)^{H-m/2} f\left(\frac{v_1}{t}, \cdots \frac{v_m}{t}\right) dB(cv_1) \cdots dB(cv_m) \\ &\stackrel{d}{=} c^H \int \cdots \int_{R^n} t^{H-m/2} f\left(\frac{v_1}{t}, \cdots \frac{v_m}{t}\right) dB(v_1) \cdots dB(v_m) \\ &= c^H Y_m(t). \end{aligned}$$

□

THEOREM 4.3. $\{Y_m(t), t \geq 0\}$ *has continuous sample paths under the following condition : there exists* A *such that*

$$\int \cdots \int \left[(1+h)^{H-\frac{m}{2}} f\left(\frac{u_1}{1+h}, \cdots, \frac{u_m}{1+h}\right) - f(u_1, \cdots, u_m)\right]^2$$

$$du_1 \cdots du_m \leq Ah^{2H}$$

for $0 \leq h \leq 1$.

Proof. It suffices to show that

$$[E|Y_m(t+h) - Y_m(t)|^2]^{1/2} \leq Ah^H$$

for some A, h $(0 \leq h \leq 1)$.

$$\begin{aligned} & E|Y_m(t+h) - Y_m(t)|^2 \\ = \; & E\left|\int \cdots \int_{R^m} Q_{t+h}(u_1, u_2, \cdots, u_m) - Q_t(u_1, u_2, \cdots, u_m)\right. \\ & \left. dB(u_1) dB(u_2) \cdots dB(u_m)\right|^2 \\ = \; & E\left|\int \cdots \int_{R^m} (t+h)^{H-\frac{m}{2}} f\left(\frac{u_1}{t+h}, \frac{u_2}{t+h}, \cdots, \frac{u_m}{t+h}\right)\right. \\ & \left. - t^{H-\frac{m}{2}} f\left(\frac{u_1}{t}, \frac{u_2}{t}, \cdots, \frac{u_m}{t}\right) dB(u_1) dB(u_2) \cdots dB(u_m)\right|^2. \end{aligned}$$

Doob-Meyer decomposition Theorem ([1, Theorem 4.10]) implies

$$
\begin{aligned}
&E|Y_m(t+h) - Y_m(t)|^2 \\
= &\int \cdot \int_{R^m} \left[(t+h)^{H-\frac{m}{2}} f\left(\frac{u_1}{t+h}, \frac{u_2}{t+h}, \cdots, \frac{u_m}{t+h}\right) \right. \\
&\left. -t^{H-\frac{m}{2}} f\left(\frac{u_1}{t}, \frac{u_2}{t}, \cdots, \frac{u_m}{t}\right)\right]^2 du_1 du_2 \cdots du_m \\
= &\int \cdot \int t^{2H-m} \left[\left(1+\frac{h}{t}\right)^{H-\frac{m}{2}} f\left(\frac{u_1/t}{1+h/t}, \frac{u_2/t}{1+h/t}, \cdots, \frac{u_m/t}{1+h/t}\right) \right. \\
&\left. -f\left(\frac{u_1}{t}, \frac{u_2}{t}, \cdots, \frac{u_m}{t}\right)\right]^2 du_1 du_2 \cdots du_m \\
= &\; t^{2H} \int \cdot \int \left[\left(1+\frac{h}{t}\right)^{H-\frac{m}{2}} f\left(\frac{v_1}{1+h/t}, \frac{v_2}{1+h/t}, \cdots, \frac{v_m}{1+h/t}\right) \right. \\
&\left. -f(v_1, v_2, \cdots, v_m)\right]^2 dv_1 dv_2 \cdots dv_m \\
\leq &\; t^{2H} A \left(\frac{h}{t}\right)^{2H} = Ah^{2H}.
\end{aligned}
$$

Therefore,

$$[E|Y_m(t+h) - Y_m(t)|^2]^{\frac{1}{2}} \leq A^{\frac{1}{2}} h^H.$$

By Lemma 4.1, $\{Y_m(t), t \geq 0\}$ has continuous sample paths. □

For each $t \geq 0$, let $Q_t(u_1, u_2) = t^{H-1} f\left(\frac{u_1}{t}, \frac{u_2}{t}\right)$ be a continuous kernel on $[0,1] \times [0,1]$ such that

$$\int_0^1 f\left(\frac{u}{t}, \frac{u}{t}\right) du$$

is continuous at each $t \geq 0$. Let

$$Y_2(t) = \int_0^1 \int_0^1 Q_t(u_1, u_2) dB(u_1) dB(u_2)$$

be a double Ito-Wiener integral such that the stochastic process $\{Y_2(t), t \geq 0\}$ has a continuous version.

LEMMA 4.4. *Let $\{h_i, i \geq 1\}$ be a family of Haar functions.*

(i) If $i = j$, then

$$\int_0^1 \int_0^1 h_i(u_1)h_j(u_2)dB(u_1)dB(u_2) = 2^{k+1}B^2\left(\frac{1}{2^{k+1}}\right) - 1$$

(ii) If $i \neq j$, then

$$\int_0^1 \int_0^1 h_i(u_1)h_j(u_2)dB(u_1)dB(u_2)$$

$$= \int_0^1 h_i(u_1)dB(u_1) \cdot \int_0^1 h_j(u_2)dB(u_2)$$

Proof. (i) By definition of Haar function and reflection principle of Brownian motion, it follows

$$\int_0^1 \int_0^1 h_i(u_1)h_j(u_2)dB(u_1)dB(u_2)$$

$$= 4 \cdot 2^k \int_{\frac{j-1}{2^k}}^{\frac{j-1/2}{2^k}} \int_{\frac{j-1}{2^k}}^{u_2} dB(u_1)dB(u_2)$$

$$-2 \cdot 2^k \left[B\left(\frac{j}{2^k}\right) - B\left(\frac{j-1/2}{2^k}\right)\right] \cdot \left[B\left(\frac{j-1/2}{2^k}\right) - B\left(\frac{j-1}{2^k}\right)\right],$$

Here,

$$
\begin{aligned}
& 4 \cdot 2^k \int_{\frac{j-1}{2^k}}^{\frac{j-1/2}{2^k}} \int_{\frac{j-1}{2^k}}^{u_2} dB(u_1)dB(u_2) \\
= & \ 4 \cdot 2^k \int_0^{\frac{1}{2^{k+1}}} \int_0^{u_2} dB(u_1)dB(u_2) \\
= & \ 2^{k+1} \cdot 2 \int_0^{\frac{1}{2^{k+1}}} dB(u_1)dB(u_2) \\
= & \ 2^{k+1} \left[B^2\left(\frac{1}{2^{k+1}}\right) - \frac{1}{2^{k+1}} \right] \\
= & \ 2^{k+1} B^2\left(\frac{1}{2^{k+1}}\right) - 1.
\end{aligned}
$$

Therefore, we get

$$
\begin{aligned}
& \int_0^1 \int_0^1 h_i(u_1)h_j(u_2)dB(u_1)dB(u_2) \\
= & \left[\sqrt{2^k} \left\{ B\left(\frac{j-1/2}{2^k}\right) - B\left(\frac{j-1}{2^k}\right) \right\} \right. \\
& \left. - \sqrt{2^k} \left\{ B\left(\frac{j}{2^k}\right) - B\left(\frac{j-1/2}{2^k}\right) \right\} \right]^2 - 1 \\
= & \left[\int_0^1 h_i(u)dB(u) \right]^2 - 1.
\end{aligned}
$$

(ii) By independent increment property of Brownian motion,

$$
\int_0^1 \int_0^1 h_i(u_1)h_j(u_2)dB(u_1)dB(u_2)
$$

$$
= \int_0^1 h_i(u_1)dB(u_1) \cdot \int_0^1 h_j(u_2)dB(u_2). \qquad \square
$$

THEOREM 4.5. *Let $\{Y_2(t), t \geq 0\}$ be stochastic processes which have continuous version as stated above. Then there exists fixed shift continuous function $F \in C$ such that*

$$(Y_2 + F)(t) = \sum_{i,j \geq 1} c_{ij}(t) g_i g_j$$

where, $\{c_{ij}(t); i, j \geq 1\}$ is a sequence of C and $\{g_i, i \geq 1\}$ is an i.i.d. sequence of $N(0,1)$ random variables.

Proof. Since $Q_t(u_1, u_2)$ is continuous on $[0,1] \times [0,1]$, Q_t has the following Fourier-Haar expansion,

$$Q_t(u_1, u_2) = \sum_{i,j=1}^{\infty} c_{ij}(t) h_i(u_1) h_j(u_2)$$

where,

$$c_{ij}(t) = \int_0^1 \int_0^1 Q_t(u_1, u_2) h_i(u_1) h_j(u_2) du_1 du_2.$$

Then

$$\begin{aligned} Y_2(t) &= \int_0^1 \int_0^1 Q_t(u_1, u_2) dB(u_1) dB(u_2) \\ &= \lim_{N \to \infty} \sum_{i,j=1}^{N} c_{ij}(t) \int_0^1 \int_0^1 h_i(u_1) h_j(u_2) dB(u_1) dB(u_2). \end{aligned}$$

Since $\int_0^1 h_i(u) dB(u)$ is a standard normal random variable for each $i = 1, 2, \cdots$, we can put $g_i = \int_0^1 h_i(u) dB(u)$. By Lemma 4.4,

$$\begin{aligned} Y_2(t) &= \lim_{N \to \infty} \sum_{i,j=1}^{N} c_{ij}(t)(g_i g_j - \delta_{ij}) \\ &= \sum_{i,j=1}^{\infty} c_{ij}(t) g_i g_j - \sum_{i=1}^{\infty} c_{ii}(t). \end{aligned}$$

We know that $\int_0^1 \left(\frac{u}{t}, \frac{u}{t}\right) du$ is continuous and

$$t^{H-1} \int_0^1 f\left(\frac{u}{t}, \frac{u}{t}\right) du = \sum_{i=1}^{\infty} c_{ii}(t).$$

Let $F(t) = t^{H-1} \int_0^1 f\left(\frac{u}{t}, \frac{u}{t}\right) du$. Therefore, we get $F \in C$ and

$$Y_2(t) + F(t) = \sum_{i,j=1}^{\infty} c_{ij}(t) g_i g_j.$$

□

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