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Derivation and Application of Survival Functions for Unthinned Forest Plantation[1*]

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未間伐 人工林에서 殘存林木 推定 函數의 誘導와 適用[1*]

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ABSTRACT

In this study, 15 survival functions in integral and difference forms for forest plantation were derived based on assumptions for the number of surviving trees and the differential forms of the mortality rate model. Then, performance of the models was evaluated by fitting to remeasurement data of unthinned white pine(*Pinus strobus*) forest plantation. As a result, three equations associated with a power function of age, t^{β}, are somewhat more suitable for describing the effect of self-thinning over time. On the other hand, a general survival function for Japanese larch(*Larix leptolepis*) forest plantation was derived in order to exam the effect of site quality on self-thinning procedures. The results indicate that the N_{min} is negatively correlated with site index and, even though the same initial stand density was assumed, the survival function curves differ in shapes associated with site index values.

Key words : Pinus strobus, Larix leptolepis, *survival functions, self-thinning, mortality*

要 約

이 연구에서는 잔존목의 본수에 대한 수식형태 및 고사율 추정 미분함수형태를 가정하여 인공림에 대한 잔존목을 추정하기 위한 15개의 함수들을 적분 및 지수함수형태로 유도하였다. 또한 이 모델들을 간벌이 되지 않은 스트로부스 잣나무 인공림의 반복측정 자료를 이용하여 모델의 적용성을 검토하였다. 그 결과 t^{β}와 같이 임령의 지수형태를 포함하는 3개의 함수들이 시간에 따른 자연 간벌효과를 설명하는데 상대적으로 유효한 것으로 나타났다. 한편 지위가 자연 간벌에 미치는 효과를 분석하기 위하여 낙엽송 임분의 잔존목 추정을 위한 함수를 유도하였다. 그 결과 N_{min}이 지위지수와 부의 상관관계를 가지는 것으로 나타났고, 초기 임분밀도를 같은 값으로 가정한 경우에도 잔존목 추정함수의 곡선이 지위지수별로 달라짐을 알 수 있었다.

INTRODUCTION

With the assumption that no mortality occurs in well-managed forest plantations, most of growth models neglect the problem of mortality. However, the assumption may not be appropriate to use in predicting growth of stands with higher initial density, because mortality in a dense stand is one of key factors influencing the change of timber growth.

To solve the problem, there have been efforts among forest scientists to develop stand growth

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models considering stand mortality as the function of stand age, competition and suppression. Drew and Flewlling(1977), Smith and Hann(1984), and Tang *et al.*(1994) used Reineke's stand density index(1933) or Yoda *et al.*'s 3/2 power law of self-thinning(1963) in estimating the mortality in pure even-aged stands. In these models, mortality was assumed as the results of competition and suppression rather than stand age. However, Khilmi(1957), Minowa(1983) and Hara(1984) interpreted stand mortality in a different way and developed stand growth models describing decrease in the number of surviving plants as the function of stand age for development of self-thinning.

On the other hand, some researchers focused their attention on relative mortality rate(RMR) models. Assuming the RMR models as a constant and hyperbola function of age, Clutter *et al.*(1983) derived two kinds of mortality functions. Clutter and Jones(1980) developed a mortality function for slash pine plantation by expressing the RMR model as a power function of age and the number of surviving trees. Piennar and Shiver(1981) developed survival function for slash pine based on assumption concerning the RMR as a power function of stand age. In developing the yield-prediction system for slash pine, Pienaar *et al.*(1990) introduced the site quality into the Clutter and Jones's model.

The objective of this study is to derive theoretical survival(or mortality) prediction equations by analyzing basic forms of the RMR models and to describe the self-thinning procedures in unthinned forest plantation. Also, to test the performance of models, the survival functions are fitted to remeasurement data of white pine(*Pinus strobus*) and data from yield tables of Japanese larch(*Larix leptolepis*).

DATA

In developing the survival(or mortality) equations, we have used the data set from a permanent plot of unthinned white pine reported by Spurr *et al.*(1957). The stand characteristics of the white pine plantation are summarized in Table 1. These survival data provide reliable description of self-thinning procedures for plantation and were used, in this study, for parameter estimates and evaluating model performance.

In developing the survival prediction models related to age and site index, also, we have used the data set from yield tables of Japanese larch plantation reported by Korea Forestry Research Institute(1989). The average site index for the 6 tables of Japanese larch plantation varies from 10 to 20m. Each table has the same initial density of 3000 stems per hectare when t_0=5 years.

SURVIVAL FUNCTIONS

Having promising properties of self-thinning, the RMR model can be integrated to obtain a corresponding survival function(or difference equation model) of the time course self-thinning for forest plantation. For even-aged stands, the mortality rate decrease to zero in a matured state, while the number of trees finally reaches the minimum asymptotic density(N_{min})(Li and Meng, 1995). In other words, as stand age becomes very large, N should approach to N_{min}. Therefore, in the survival function, it is necessary to constrain $N \to N_{min}$ when $t \to \infty$. The survival functions over time can be achieved by replacing N with two additional dependent variables, $N - N_{min}$ and $\ln N - \ln N_{min}$.

Assuming the RMR as a function of age(t) and

Table 1. Summary of stand characteristics for the unthinned white pine forest plantation.*

Age(years)	15	19	24	29	34	39	44	49	54
DBH(cm)	5.33	6.86	9.14	11.18	12.83	13.97	15.24	17.02	18.29
Height(m)	5.49	6.71	8.84	10.97	12.80	14.33	15.85	17.68	20.73
No. of trees (stems/ha)	10538.7	9098.1	6884.1	5070.4	4027.7	3343.2	2826.8	2285.6	2013.8

* N_0=11860 stems per hectare when t_0=7 years

the number of surviving trees(x), for a specified stand where SI is a constant, it can be expressed as ;

$$\frac{1}{x}\frac{dx}{dt} = f(t,\ x) \qquad (1)$$

where $\frac{1}{x}\frac{dx}{dt}$ = RMR at age t ; $x=N$, or $x=N-N_{min}$, or $x=\ln N-\ln N_{min}$

As shown in Table 2, we proposed 5 expressions of the RMR models : a constant, a power or an exponential function of t, and the more complex form which is proportional to current size(x), raised to some positive power γ, associated with a power or an exponential function of t. All of five RMR models should have a value less than 0, because the number of stems decreases with the development of stand.

According to Table 2, Clutter et al.(1983) developed the simplest form of the survival functions using an assumption that $x=N$ and the RMR model has constant values. Pienaar and Shiver(1981), Clutter and Jones(1980), and Khilmi (1957) also developed their own survival functions as shown in Table 2. Including the 4 survival functions already known through literature review, we have investigated and compared the performance of all 15 functions available from Table 2 in this study.

Using the combinations of columns and rows in Table 2, we obtained 15 integral forms and difference equation forms as the survival functions as shown in Table 3. In deriving the functions, we assumed that the initial condition is $N=N_0$(N_0 indicates planting-survival density), when $t=t_0$. The difference equation models in Table 3 have the following logical properties :

1) If $t_2 = t_1$, $N_2 = N_1$.
2) If $t_2 > t_1$, $N_2 < N_1$.
3) As t_2 becomes very large, N_2 will approach to 0(group A) or N_{min}(groups B and C).
4) If the model is used to predict N_2 at age t_2 and t_2 and N_2 are, then, used to predict N_3 at age t_3($t_3>t_2>t_1$), the results obtained will be equal to that given by a single projection from t_1 to t_3.

RESULTS AND DISCUSSION

1. Comparison of alternative survival functions

Using DUD method implemented in SAS(1985), we performed a nonlinear regression analysis for 15 types of the survival functional forms listed in Table 3. They were fitted to the data of the unthinned white pine forest plantation after Spurr *et al.*(1957). The number of surviving trees, N_0, at 7 years after planting was 11,860 stems per hectare for all models. The results are shown in Table 4, in which the goodness-of-fit is evaluated using the statistics of the residual mean square, RMS, the standard error of estimate, $S_{y.x}$, and the adjusted coefficient of determination, R_a^2. Also the observed and the predicted values of time course survival trees per hectare for the five equations of groups A and C are plotted as functions of stand age in Fig. 1.

According to the fit statistics in Table 4, the results indicate that all the equations possess a potential for estimating the future number of surviving trees. However, R_a^2 values of A1, B1

Table 2. Differential equations of RMR model to develop survival functions.*

Assumptions of RMR model		Assumption 1	Assumption 2	Assumption 3	Assumption 4	Assumption 5
Group	Definition of x	$\frac{1}{x}\frac{dx}{dt}=-\alpha$	$\frac{1}{x}\frac{dx}{dt}=-\alpha t^{\beta}$	$\frac{1}{x}\frac{dx}{dt}=-\alpha e^{\beta t}$	$\frac{1}{x}\frac{dx}{dt}=-\alpha t^{\beta}x^{\gamma}$	$\frac{1}{x}\frac{dx}{dt}=-\alpha e^{\beta t}x^{\gamma}$
A	$x=N$	Clutter *et al.* (1983)	Pienaar and Shiver(1981)	unknown	Clutter and Jones(1980)	unknown
B	$x=N-N_{min}$	unknown	unknown	unknown	unknown	unknown
C	$x=Ln(N/N_{min})$	Khilmi(1957)	unknown	unknown	unknown	unknown

* and are constants independent of t. >0 and >0 for general case.

Table 3. Integral forms and difference equation forms of the survival functions.*

Group	Eq.	Integral form	Difference equation model
A	A1	$N=N_0 e^{-k(t-t_0)}$	$N_2=N_1 e^{-k(t_2-t_1)}$
	A2	$N=N_0 e^{-k(t^b-t_0^b)}$	$N_2=N_1 e^{-k(t_2^b-t_1^b)}$
	A3	$N=N_0 e^{-b(e^{ct}-e^{ct_0})}$	$N_2=N_1 e^{-b(e^{ct_2}-e^{ct_1})}$
	A4	$N=(N_0^{-c}+k(t^b-t_0^b))^{-\frac{1}{c}}$	$N_2=(N_1^{-c}+k(t_2^b-t_1^b))^{-\frac{1}{c}}$
	A5	$N=(N_0^{-c}+k(e^{bt}-e^{bt_0}))^{-\frac{1}{c}}$	$N_2=(N_1^{-c}+k(e^{bt_2}-e^{bt_1}))^{-\frac{1}{c}}$
B	B1	$N=N_{min}+(N_0-N_{min})e^{-k(t-t_0)}$	$N_2=N_{min}+(N_1-N_{min})e^{-k(t_2-t_1)}$
	B2	$N=N_{min}+(N_0-N_{min})e^{-k(t^b-t_0^b)}$	$N_2=N_{min}+(N_1-N_{min})e^{-k(t_2^b-t_1^b)}$
	B3	$N=N_{min}+(N_0-N_{min})e^{-b(e^{ct}-e^{ct_0})}$	$N_2=N_{min}+(N_1-N_{min})e^{-b(e^{ct_2}-e^{ct_1})}$
	B4	$N=N_{min}+((N_0-N_{min})^{-c}+k(t^b-t_0^b))^{-\frac{1}{c}}$	$N_2=N_{min}+((N_1-N_{min})^{-c}+k(t_2^b-t_1^b))^{-\frac{1}{c}}$
	B5	$N=N_{min}+((N_0-N_{min})^{-c}+k(e^{bt}-e^{bt_0}))^{-\frac{1}{c}}$	$N_2=N_{min}+((N_1-N_{min})^{-c}+k(e^{bt_2}-e^{bt_1}))^{-\frac{1}{c}}$
C	C1	$N=N_{min}\left(\frac{N_0}{N_{min}}\right)^{e^{-k(t-t_0)}}$	$N_2=N_{min}\left(\frac{N_1}{N_{min}}\right)^{e^{-k(t_2-t_1)}}$
	C2	$N=N_{min}\left(\frac{N_0}{N_{min}}\right)^{e^{-k(t^b-t_0^b)}}$	$N_2=N_{min}\left(\frac{N_1}{N_{min}}\right)^{e^{-k(t_2^b-t_1^b)}}$
	C3	$N=N_{min}\left(\frac{N_0}{N_{min}}\right)^{e^{-b(e^{ct}-e^{ct_0})}}$	$N_2=N_{min}\left(\frac{N_1}{N_{min}}\right)^{e^{-b(e^{ct_2}-e^{ct_1})}}$
	C4	$N=N_{min}e^{\left((\ln\frac{N_0}{N_{min}})^{-c}+k(t^b-t_0^b)\right)^{-\frac{1}{c}}}$	$N_2=N_{min}e^{\left((\ln\frac{N_1}{N_{min}})^{-c}+k(t_2^b-t_1^b)\right)^{-\frac{1}{c}}}$
	C5	$N=N_{min}e^{\left((\ln\frac{N_0}{N_{min}})^{-c}+k(e^{bt}-e^{bt_0})\right)^{-\frac{1}{c}}}$	$N_2=N_{min}e^{\left((\ln\frac{N_1}{N_{min}})^{-c}+k(e^{bt_2}-e^{bt_1})\right)^{-\frac{1}{c}}}$

* N_{min}, b, c, and k are parameters to be estimated from data ; N_0 is planting-survival density(trees per hectare)

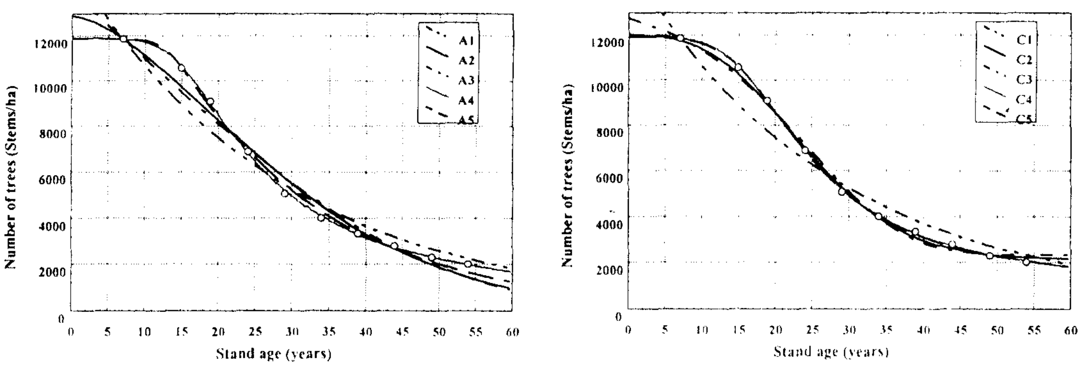
and C1 are relatively lower than that of other types of survival functions. The equations, A1, B1 and C1, were derived from the assumption 1 as shown in Table 2. Hence, it was concluded that this kind of the differential form of the RMR model is not as suitable as others in deriving the survival functions for white pine stands. While the definition of x in Table 2 does not have any effect on changing the accuracy of survival function.

On the other hand, even though the survival functions derived from the assumptions 4 and 5 of RMR model in Table 2 provide the best results of fitting to the present data, the parameter estimates for the corresponding functions derived by nonlinear least square are statistically unstable. As the results, the estimated values of parameter, k, in the equations A4, A5, B4, B5, C4, and C5 are too small to use.

In all 15 survival equations analyzed in this paper, the six equations, A2, A3, B2, B3, C2, and C3, are somewhat more suitable to describe the time course of self-thinning for plantation. Among them, the three models, A2, B2, and C2, associated with the power function of age are generally the best in each group.

Table 4. Parameter estimates and fit statistics for survival functions fitted to the unthinned white pine plantation.

Group	Eq.	Estimated parameters				Goodness of fit		
		N_{min}	k	b	c	RMS	$S_{y.x}$	R_a^2
A	A1		0.035399			690741.68	831.11	0.9269
	A2		0.003645	1.603834		313376.04	559.80	0.9668
	A3		0.022670	0.949401		424985.33	651.91	0.9550
	A4		8.83116510^{-21}	5.273961	3.311847	11185.18	105.76	0.9988
	A5		1.33218810^{-74}	0.812867	17.027847	84811.82	291.22	0.9911
B	B1	812.2070	0.040196			863747.18	929.38	0.8980
	B2	2249.0427	0.000196	2.579237		73956.80	271.95	0.9921
	B3	2380.2905	0.079572	0.135673		151841.99	389.67	0.9839
	B4	781.0515	8.85138910^{-16}	4.672896	2.239285	9771.32	98.85	0.9990
	B5	1557.5746	3.28561510^{-20}	0.332362	4.292943	12431.42	111.50	0.9987
C	C1	0.0001	0.001955			852722.96	923.43	0.9097
	C2	2156.8581	0.000034	2.927499		48382.40	219.96	0.9949
	C3	2345.1263	0.097128	0.044725		127120.75	356.54	0.9865
	C4	216.3602	3.00863410^{-11}	4.511942	7.657413	8852.93	94.09	0.9990
	C5	1048.3934	3.49327610^{-6}	0.294403	8.535850	8259.76	90.88	0.9992



(a) Group A (b) Group C

Fig. 1. Predicted number of trees using the survival functions in groups A and C for white pine plantation.

2. Effects of sites on self-thinning

To investigate the effect of site quality on the time course self-thinning for plantation stands, a separate survival function was fitted for each of the six site index(SI) class of Japanese larch by using equation(C2) as a base model. Table 5 shows the parameter estimates and their statistics by the class of site index.

According to the values of R^2 in Table 5, the survival function(C2) was successful in estimating the number of surviving trees, which accounted for more than 99.9%(R^2>0.999) of the observed variation in surviving trees, for each site index. Parameters N_{min} and b decrease exponentially with site index. On the other hand, parameter k increases exponentially with site index. The simple correlation coefficients between SI and parameters N_{min}, k, and b were -0.9864, 0.9934, and -0.9764, respectively.

Each parameter of the survival function(C2) was hypothesized to be a power function of SI. Then, a general self-thinning model applicable to different site index for Japanese larch plantation was developed as :

Table 5. Parameter estimates and fit statistics for the survival function, (C2), fitted to the yield tables of Japanese larch plantation in Korea.

Site index (m)	Estimated parametes			Goodness of fit		
	N_{min}	k	b	RSS	$S_{y.x}$	R^2
10	826.9119	0.0067063	1.6758026	3486.688	28.2506	0.99929
12	756.8344	0.0151682	1.4277749	1109.718	12.5909	0.99979
14	699.7939	0.0272046	1.2539908	199.279	5.3355	0.99996
16	653.3062	0.0420402	1.1262307	106.972	3.9092	0.99998
18	615.4453	0.0598748	1.0219575	219.429	5.5988	0.99996
20	590.3653	0.0754412	0.9552997	400.360	7.5626	0.99992

Table 6. Parameter estimates and fit statistics of self-thinning model(2) for Japanese larch plantation.

Parameter	Estimate	Asymptotic Std. Error	T ratio	Approx. Prob. >|T|	Goodness of fit			
					n	RSS	$S_{y.x}$	R^2
a_0	1588.586194	177.21304	8.9643	0.00000				
a_1	-0.299573	0.04289	-6.9847	0.00000				
k_0	0.000202	0.00003	6.7333	0.00000	60	20193.02	19.34	0.99933
k_1	1.818439	0.26455	6.8737	0.00000				
b_0	3.188616	0.57781	5.5185	0.00002				
b_1	-0.345176	0.07231	-4.7736	0.00010				

$$N = a_0 SI^{a_1}\left(\frac{3000}{(a_0 SI^{a_1})}\right)^{\exp\left(-k_0 SI^{k_1}\left(t^{b_0 SI^{b_1}} - 5^{b_0 SI^{b_1}}\right)\right)} \cdots\cdots\cdots(2)$$

where a_0, a_1, k_0, k_1, b_0, and b_1 are parameters to be estimated.

Parameter estimates and the results of the goodness-of-fit test of the self-thinning model (2) are listed in Table 6. Also the predicted number of surviving trees per hectare by site index is plotted as a function of stand age as shown in Fig. 2.

Based on Fig. 2, it was found that the minimum asymptotic density(N_{min}) and shape of survival curve were different for each site. N_{min} is negatively correlated with site index. With same initial density, the better the site condition is, the lower the N_{min} is. In equation(C2), parameter k is related to the mortality rate and parameter b determines the shape of survival curve. To model(2), the parameters k and b are negatively and positively correlated with SI, respectively, for Japanese larch plantation(Table 6). That means the surviving trees of stand growing in good site decreases rapidly and the curve is more concave than that of in poor site.

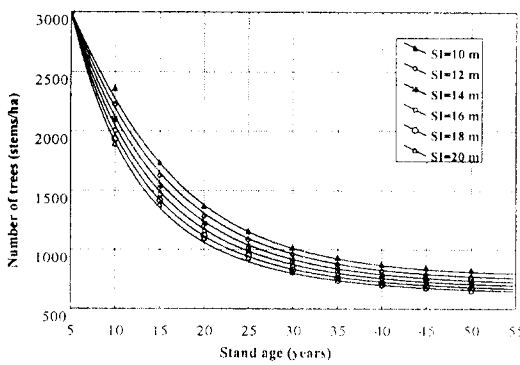


Fig. 2. The number of surviving trees for 6 site indexes predicted by the self-thinning model(2).

CONCLUSION

15 survival functions in integral and difference forms were derived for white pine forest plantation based on 5 differential forms and 3 different definitions of dependent variables($x=N$, $x=N-N_{min}$, and $x=\ln N-\ln N_{min}$) of the RMR model. The performance test indicates that, the more complex a model, the better the results. But, the parameter estimates for the survival functions derived from the assumptions 4 and 5 were sta-

tistically unstable and the estimated values of the parameter, k, in the functions were too small to use. The survival equations associated with a power function of age(t) were slightly better than the equations with an exponential function of age(e^{t}). As a result, the three equations, A2, B2, and C2, were somewhat more suitable for describing the time course of self-thinning of unthinned white pine forest plantation.

By re-parameterize the equation(C2), on the other hand, a general self-thinning model for Japanese larch forest plantation was developed in order to account for the effects of site quality on self-thinning procedures. It was found that each parameter of the equation could be defined as a power function of SI. The results of the goodness-of-fit test showed that the minimum asymptotic density(N_{min}) was negatively correlated with site index and SI affects the shapes of the survival function curves.

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