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Estimation of Site Index Curves for Loblolly Pine(*Pinus taeda* L.) and Slash Pine(*Pinus elliottii* Engelm.) Plantations[1]*

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테에다소나무林과 엘리오티소나무林의 造林地에 대한 地位指數 曲線 推定에 關한 硏究[1]*

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ABSTRACT

Loblolly(*Pinus taeda* L.) and slash(*Pinus elliottii* Engelm.) pines are the most important timber producing species in the Southern United States. Site index equations to estimate site index curves (base age 25 years) for loblolly pine and slash pine plantations have been developed based on long-term repeated measurement data sets. To check magnitude of errors in estimating site index, each cumulative measurement cycle data sets and all combined data sets were used to recalculate site index values.

The Chapman-Richards' growth function was selected for stand height prediction. Anamorphic base age invariant site index curves were presented based on this height prediction equation. Statistics used in the evaluation were mean of the differences and mean of the absolute differences. For plantation ages less than 5 years, site index values showed very sensitive for both species based on the evaluation test.

Key words : Pinus taeda, Pinus elliottii, *The Chapman-Richards' growth function, site index curves, magnitude of errors.*

要 約

美國 南部 地域의 主要 經濟 造林樹種인 테에다소나무(*Pinus taeda* L.)와 엘리오티소나무(*Pinus elliottii* Engelm.) 造林地에 대한 地位指數式을 開發하였으며, 地位指數의 값들에 대한 誤差의 크기를 糾明하기 위하여 永久的으로 設置된 plot으로부터 測定된 data sets을 利用하였다.

分析에 利用된 資料는 3年을 測定週期(measurement cycle)로 하여 測定하였으며, 各各의 累積 測定週期 資料와 이들 모두를 綜合한 資料를 利用하였다.

最終的인 地位指數 豫測式의 誘導는 모든 測定 資料를 綜合하여 Chapman-Richards의 生長函數式을 使用하였으며, 이를 基礎로 하여 두 樹種의 同形(anamorphic) 地位指數 曲線을 作成하였다.

林令의 變化推移에 따른 地位指數값에 대한 誤差의 크기를 糾明하기 위하여, 最終的 測定週期의 累積 資料를 利用한 地位指數 豫測値와 各各의 週期別 累積 資料를 利用한 地位指數 豫測値 간의 差異에 대한 絶對 平均値를 使用하였다. 그 結果, 地位指數의 값들은 林令이 5年 以下에서 豫測되었을 境遇,

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두 樹種 모두 絶對値의 平均 誤差가 約 5 meter 정도로 크게 나타났다. 따라서 林令이 5年 以下인 幼令 林分에 대한 地位指數 값의 像測値들은 많은 誤差를 內包하고 있는 것으로 判斷된다.

INTRODUCTION

Forest managers have been interested in using growth and yield models to assist them in forest management planning. Site index is a practical and commonly used methods for quantifying site quality for even-aged forest stands and an essential component in growth and yield models.

Site index is an expression of forest site productivity defined as the average total height of the dominant(or codominant) trees in pure even-aged stands at an arbitrarily chosen index age. Typically this base index age is set at 25, 50, or 100 years. The base index age is specified somewhat less than rotation age(Goelz and Burk, 1992).

Numerous site index equations have been published for loblolly pine(Clutter and Lenhart, 1968 ; Lenhart, 1971 ; Popham et al., 1979 ; Pienaar and Shiver, 1980 ; Golden et al., 1981 ; Amateis and Burkhart, 1985 ; Lenhart et al., 1986 ; Lee, 1998) and for slash pine plantations(Bailey et al., 1973 ; Zarnoch and Feduccia, 1984 ; Borders et al., 1984 ; Lenhart et al., 1986 ; Lee, 1998) grown under different conditions in various geographic locations in the southern United State.

An evaluation of a variety of site index models by Cao(1993), using remeasurement data from pine plantations from Texas to Maryland for short-, medium-, and long-range height projection showed that the Chapman-Richards model was more suitable than Schumacher model.

The objective of this study is to develop anamorphic base age invariant site index equations to estimate site index for loblolly and slash pine plantations and to check magnitude of errors in estimating site index based on repeated measurement data sets(1982 - 1997) in the south-western of United States.

MATERIALS AND METHODS

1. The study area

The study area consists of 22 counties in East Texas, USA. Generally, the counties are located within the rectangle from 30° - 35° north latitude and 93° - 96° west longitude.

2. Data collection

Long-term data from East Texas Pine Plantation Research Project(ETPPRP) permanent plots located in loblolly and slash pine plantations throughout on the southwestern region of United States were analyzed in this study. The ETPPRP is a long-term comprehensive endeavor initiated by the College of Forestry at Stephen F. Austin State University in 1982 in conjunction with five participating forest industries. Over a 3-year period in 1982 - 84, 256 permanent plots were installed by the ETPPRP in these industrial pine plantations. Of the 256 plots, 177 were in loblolly pine plantations, and 79 were in slash pine plantations.

Each plot consists of two adjacent subplots separated by a 60ft(18m) buffer. Each subplot is 100ft(30m) by 100ft(30m) square. Surrounding the subplots is a 30ft(9m) wide buffer zone. Each subplot is a distinct sampling unit. For this study, the life of each planted pine trees had been tracked for 16 years(1982 - 1997) or 5 and 1/3 measurement cycles. Each measurement cycle is three years in length. For this study, all subplots were combined for model fitting and each measurement cycle data sets were utilized for evaluation purposes.

The character and nature of the observed loblolly and slash pine stands data sets is shown in Table 1.

3. The Chapman-Richards(Richards 1959, Chapman 1961) function

The basic height-age function employed was the Chapman-Richards function, which was extension of Von Bertalanffy's(1957) growth function.

The Chapman-Richards function based on a height-age model can take the form :

$$H = b_1(1 - \exp(-b_2 A))^{(1-b_3)} \quad \cdots\cdots\cdots\cdots (1)$$

Table 1. Descriptive statistics of *Pinus taeda* and *Pinus elliottii* stands data sets.

Species	Variable	N	Mean	Std. Dev.	Min.	Max.
P. taeda	Age (yrs)	1,713	11.5	5.6	1	30
	Height (m)	1,713	11.6	5.7	0.6	30.5
P. elliotti	Age (yrs)	764	11.1	5.4	1	27
	Height (m)	764	11.2	5.6	0.6	33.8

where H is height at a given age A(m), A= plantation age(years), b_1=the asymptote, b_2 the rate parameter, and b_3 the shape parameter.

Equation (1) has been used extensively in growth and yield studies in forestry for describing site index curves, height-age, diameter-age, basal area-age, and growth rate-age relationships (Pienaar and Turnbull, 1973 ; Clutter et al., 1983 ; Somers and Farrar, 1991 ; Payandeh and Wang, 1994). Because of its sigmoidal flexibility in shape and biological and statistical properties.

4. Evaluations

Evaluations were calculated based on the average mean differences(MD) and absolute mean differences(AMD). MD is measures of the bias in estimating mean heights, while AMD is measures of precision. The sum of the absolute deviation is a measure of precision in fitting height prediction. To check magnitude of errors in estimating site index, each cumulative measurement cycle data sets and all combined data sets were used to recalculate site index values based on the guide curve equation (1).

Comparisons of site index values between each cumulative measurement cycle data sets and all combined measurement cycle data sets showed magnitude of absolute mean deviation of site index values over ages for both species based on the absolute mean difference of site index values.

RESULTS AND DISCUSSION

1. Loblolly pine

The nonlinear regression was used to fit 1,713 age-height pairs using the PROC NLIN procedure in SAS(SAS Institute Inc., 1989). Multiple starting values for parameters were provided to ensure that the nonlinear least square solution was a global minimum rather than a local minimum.

Resulting parameter estimates and associated asymptotic standard error are :

$\widehat{b_1}$=25.00523 with S($\widehat{b_1}$)=0.67044, $\widehat{b_2}$=0.09368 with S($\widehat{b_2}$)=0.00577, and $\widehat{b_3}$=0.40758 with S($\widehat{b_3}$)=0.02566.

None of the asymptotic 95% confidence intervals for each of the parameter estimates contained zero, thus the conclusion was reached that the equation parameters are significant. In addition, it was estimated that the model explained about 87.3% of the variation in the average value of height.

A plotting of residuals against predicted and independent variables indicated that a random pattern around zero with no detectable trends. Table 2 shows the differences between observed and predicted height values by age classes.

The residual mean differences and the absolute residual mean differences were calculated for validation purposes. A paired t-test showed that expected value of the residuals were not significantly different from zero(p-value=0.9938). The absolute residual mean differences as a magnitude of error index showed 1.46 meter.

Substitution of the nonlinear regression coefficients into equation (1) gave this equation

$$H = 25.00523(1-\exp(-0.09368A))^{1.68799} \quad \cdots\cdots (2)$$

Using procedures described by Clutter et al. (1983), equation (2) is a guide curve, which can be used to develop anamorphic site index equation to estimate site index for any index age.

$$S_{25} = H\left(\frac{1-\exp(-0.09368IA)}{1-\exp(-0.09368A)}\right)^{1.68799} \quad \cdots\cdots (3)$$

where S is site index (m) for index age (IA).

For an index age 25 yrs, the above equation

Table 2. Differences between observed and predicted loblolly pine stand heights by age classes.

Age (yr)	Avg. diff. (m)	Difference in meter(Observed - Predicted Height)										All
		≤ -5	-4	-3	-2	-1	1	2	3	4	≥5	
		(no. of observations)										
1 - 2	0.11					26	23	1				50
3 - 4	-0.13				23	56	57	12				148
5 - 6	0.14		1	11	21	28	60	29	9	1		160
7 - 8	-0.14	1	7	19	32	37	58	23	10	7		194
9 - 10	0.03	5	7	12	35	47	55	30	20	10	1	222
11 - 12	0.21	7	5	20	20	48	36	25	29	13	7	210
13 - 14	-0.07	13	7	16	40	47	34	25	32	7	9	230
15 - 16	-0.03	8	9	12	26	55	18	25	11	12	9	185
17 - 18	-0.27	8	5	10	10	39	19	9	7	8	4	119
19 - 20	0.27	6	2	4	13	14	19	5	8	5	7	83
21 - 22	-0.33	3	2		7	14	9	3	2	3	1	44
23 - 24	0.49		2	3	5	10	10	5	3	3	3	44
25 - 26	-0.20	1	1		5	5	1	1	2	1	1	18
27 - 30	-0.60			1	1	3		1				6
All	-0.52	52	48	108	238	429	399	194	133	70	42	1,713

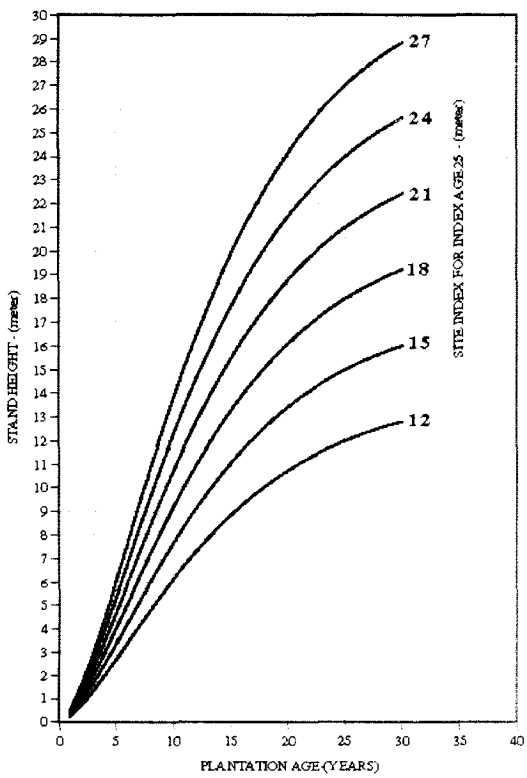

Fig. 1. Site index curves(base age 25 yrs) for unthinned loblolly pine plantation stands.

(3) can be algebraically rearranged as :

$$H = S_{25}\left(\frac{1-\exp(-0.09368A)}{0.90384}\right)^{1.68709} \quad \cdots\cdots (4)$$

Site index curves generated from equation (4) and illustrated in Fig. 1 for loblolly pine plantations ranging in age from 1 to about 30 years old.

2. Slash pine

The nonlinear regression was used to fit equation (1) to the 764 age-height pairs using the PROC NLIN procedure in SAS(SAS Institute Inc., 1989).

Multiple starting values for parameters were provided to ensure that the nonlinear least square solution was a global minimum rather than a local minimum.

Resulting parameter estimates and asymptotic standard error are :

$\widehat{b_1}$=36.87791 with S($\widehat{b_1}$)=3.85436, $\widehat{b_2}$=0.04731 with S($\widehat{b_2}$)=0.00829, and $\widehat{b_3}$=0.23028 with S($\widehat{b_3}$)=0.04550.

None of the asymptotic 95% confidence intervals for each of the parameter estimates contained zero, thus the conclusion was reached that the equation parameters are significant. The model explained about 89.2% of the variation in the average value of height. A plotting of residuals against predicted and independent variables indi-

cated that a random pattern around zero with no detectable trends.

Substitution of the coefficients into equation(1) resulted in :

$$H = 36.87791(1 - \exp(-0.04731A))^{1.29918} \quad \cdots\cdots (5)$$

To evaluate equation (5), the differences between observed and predicted height values by age classes are listed in Table 3.

A paired t-test showed that expected value of the residuals were not significantly different from zero(p-value=0.9837). The absolute residual mean differences showed 1.25 meter. Plots of residuals versus age as well as predicted height revealed no noticeable trends.

Using the same procedures described by Clutter et al.(1983), equation (5) was converted into anamorphic site index equation to estimate site index for index age=25 years as :

$$S_{25} = H\left(\frac{0.69357}{1-\exp(-0.04731A)}\right)^{1.29918} \quad \cdots\cdots (6)$$

Above equation (6) can be algebraically rearranged to draw site index curves.

$$H = S_{25}\left(\frac{1-\exp(-0.04731A)}{0.69357}\right)^{1.29918} \quad \cdots\cdots (7)$$

Site index curves generated from equation (7) are shown in Fig. 2 for slash pine plantations

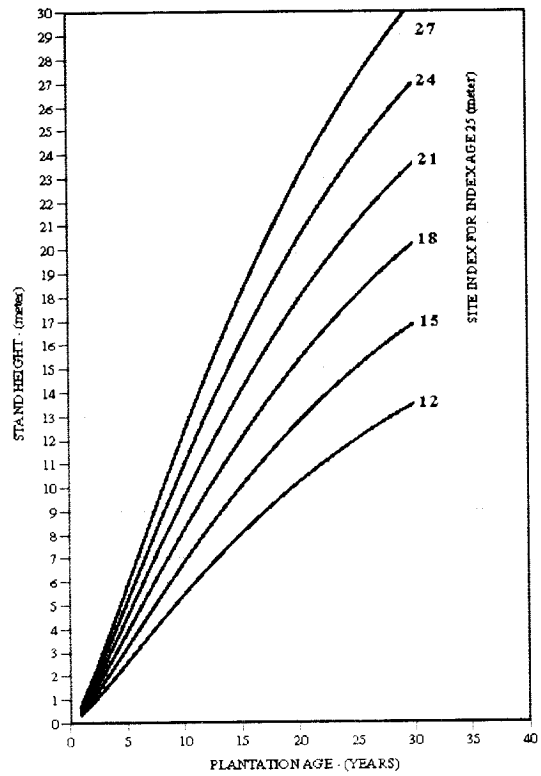

Fig. 2. Site index curves(base age 25 yrs) for unthinned slash pine plantation stands.

Table 3. Differences between observed and predicted slash pine stand heights by age classes.

Age (yr)	Avg. diff. (m)	Difference in meter(Observed - Predicted Height)										All
		≤-5	-4	-3	-2	-1	1	2	3	4	≥5	
		(no. of observations)										
1-2	0.01				3	14	17					34
3-4	-0.09			1	13	13	18	7				52
5-6	-0.11	4	2		6	25	32	17				86
7-8	0.04	3	2	1	15	19	29	19	4	2		94
9-10	0.07	3		1	7	30	27	13	4		1	86
11-12	0.09	8		4	11	30	21	25	9	4		112
13-14	0.14	4	3		12	22	23	22	9	1		96
15-16	-0.29	6		6	6	25	16	15	5		1	80
17-18	0.30	2	1	3	8	12	16	15	2	2	1	62
19-20	-0.68	4	1	1	2	6	4	6	2			26
21-22	-0.42	2		1	3	3	6	3	1	1		20
23-24	0.72	1	1				2	2		1	1	8
25-27	-0.68			2	1	1		2		1	1	8
All	-0.90	37	10	20	87	200	211	146	36	12	5	764

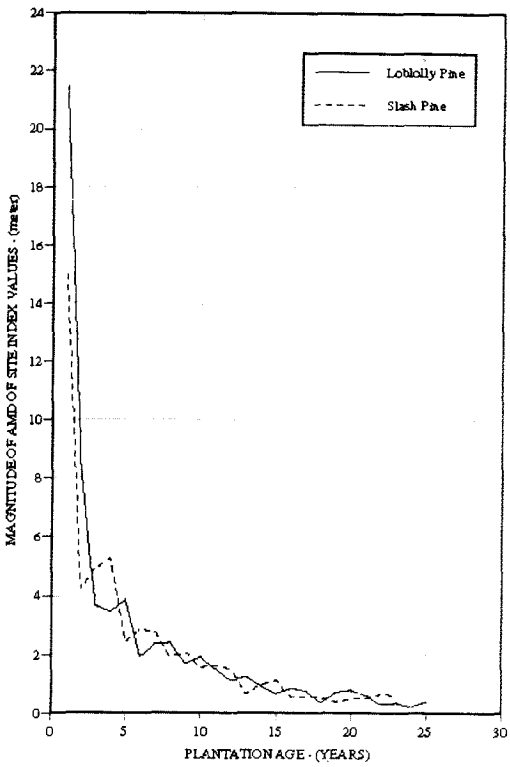

Fig. 3. Magnitudes of absolute mean deviation of site index values for both species.

ranging in age from 1 to about 27 years old.

3. Evaluations

To illustrate the sensitivity of estimating site index, each cumulative measurement cycle data sets and all combined data sets were used to recalculate site index values. Fig. 3 showed magnitude of absolute mean deviation of site index values over ages for both species.

For plantation ages less than 5 years, the expected height at an index age is very sensitive for both species based on absolute mean difference of site index values.

SUMMARY

Height growth function have been developed to predict the height of the dominant trees in loblolly and slash pine plantations. Guide curve was transformed into an anamorphic site index equation. The resulting site index equation can provide an indication of the productivity of the site based on plantation ages. However, for plantation ages less than 5 years, the expected height at an index age of 25 years is very sensitive to the young trees for both species based on the evaluation test. In spite of the some limitations(Avery and Burkhart, 1994), site index is very useful tool because it provides a simple numerical value that can be easily measured and understood by foresters.

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