Developing and Comparing Site Index Curves Using Polymorphic and Anamorphic Equations for Douglas-fir[1]

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多變態와 變態 方程式을 利用한 美松의 地位指數 曲線 推定과 比較[1]

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ABSTRACT

This research describes the construction of a site index equation and curves for Douglas-fir plantation (*Pseudotsuga menziesii* Mirb. Franco) in Nelson, New Zealand. The data sets of 146 Permanent Sample Plots (PSP) were used to build the model, and it was developed using the difference equation method. Parameter estimates were obtained using the non-linear routine of the SAS, PROC NLIN procedure. Of the models tested, a variant of the Schumacher polymorphic yield function showed the higher precision of fitting. About 95% of the observations used to fit the model could be predicted within ± 1.2 m of the actual values. Therefore, polymorphic family of site index curves, which reflect different shapes for the different site index classes, were derived from the Schumacher equation. It was found that the polymorphic site index equation was more accurate than the anamorphic equation in this study.

Key words : site index, difference equation, yield prediction, polymorphic and anamorphic equations, model

要 約

본 연구는 뉴질랜드 빌슨 지역에 조림된 美松 (*Pseudotsuga menziesii* Mirb. Franco)의 地位指數 식과 地位指數 곡선 구성에 관한 것이다. 地位指數 모델 구성에 146개의 영구 표본 플롯 (PSP)이 사용되었고, Difference 방정식을 이용하여 地位指數 식을 유도하였다. 母數 추정은 SAS의 PROC NLIN을 이용한 비선형 루틴에 의하여 수행하였다. Schumacher 多變態 생장식이 사용된 여러 생장식 중 가장 정밀한 추정을 보여주었고, 모델 추정에 사용된 95% 관측치는 실측치의 ±1.2 m 이내의 추정치를 나타내었다. 따라서, 地位指數 등급의 相異에따라 地位指數 곡선의 변형을 반영하는 多變態 地位指數 곡선을 Schumacher 생장식으로부터 유도하였다. 또한 多變態 방정식과 變態 방정식의 비교 결과는, 多變態 방정식으로부터 보다 정확한 地位指數 식을 이끌어 냄을 알 수 있었다.

INTRODUCTION

The most frequently used representation of site productivity is site index, which is the expected top height at a specified index age (Avery and Burkhart, 1994). Theoretically height growth is sensitive to differences in site quality, slightly affected by varying density levels and species com-position, relatively stable under varying thinning intensities, and strongly correlated with volume. For the species, of which height growth is significantly influenced by stand density variation, estimation of site quality from stand height data will provide poor results unless the effect of stand density is taken into account. Fortunately, for many important plantation species, height growth

is rarely affected by variation in stand density (Assmann, 1970; Lanner, 1985). Consequently site quality estimation procedures based on stand height information are the most commonly used techniques for evaluating site productivity.

Site index curves can be classified into three types : 1) anamorphic curves; 2) polymorphic-disjoint curves; and 3) polymorphic-nondisjoint curves according to the nature of the height/age curve families they generate (Clutter *et al.*, 1983; Borders *et al.*, 1984). Anamorphic curves have the shape parameter (β) eliminated, constraining the resultant curves to have similar shape, but with different asymptote parameters (α). Thus, for any two curves in an anamorphic family, the height of one curve at any age is a constant proportion of the height of the other curve at the same age. Polymorphic equations have the shape parameter (β) which allows each curve the freedom to change in shape, while asymptote parameters (α) may or may not vary over the curves in the family (Rennolls, 1995). Hence, there is no constant proportionality like in anamorphic curves, but the curves do not cross each other within the age of interest in a polymorphic-disjoint family curve. While in polymorphic-nondisjoint family curves, one curve can cross the other within the range of interest.

The major weakness of anamorphic curves is the assumption of a common shape for all site classes. For some species, the height curve shape varies with site quality. Higher quality lands generally exhibit more pronounced sigmoid shapes and lower quality lands produce flatter height growth patterns. Polymorphic families of site index curves produce differing shapes for different site index classes (Avery and Burkhart, 1994). Thus, polymorphic curves generally reflect height growth trends across a wide range of site qualities more accurately than anamorphic curves.

The aims of this study, therefore, were 1) to construct site index curves for Douglas-fir using the difference equation method, and 2) to review that developing a site index equation using the polymorphic form is more logical than that using the anamorphic form.

MATERIAL AND METHODS

In order to derive a site index equation and curves for Douglas-fir grown in the Nelson region of New Zealand, data were used from 146 permanent sample plots, which are maintained by New Zealand Forest Research Institute. Plot sizes ranged from 0.01 to 0.2 hectare with a mean of 0.05 hectare. Mean age of trees was 27 years, while minimum and maximum ages were 8 and 60 years, respectively. A summary of relevant plot statistics is given in Table 1.

Table 1. A summary of permanent sample plots data

Variables	Mean	Min.	Max.
Age (years)	27	8	60
Top height (m)	22.9	5.6	45.7
Stocking (n/ha)	935	93	3533
Basal area(m²/ha)	41.2	2.3	115.3

The method used for deriving the site index equation was the difference equation (Borders *et al.*, 1984), which has been widely used for growth and yield modeling studies. The difference equation method for determining site index usually requires permanent sample plots or stem analysis data. The procedure of formulating a difference equation is flexible and can be used with height and age equations to produce anamorphic and polymorphic family curves (Clutter *et al.*, 1983). For example, height H_2 at age T_2 is expressed as a function of T_2, height H_1 at age T_1 and T_1. A procedure of developing the difference equation is shown below using the Schumacher log reciprocal function (Clutter *et al.*, 1983).

A modified Schumacher height equation is given as :

$$\ln(H) = \alpha + \beta / T^{\gamma} \tag{1}$$

where,
  H = height in meters
  T = age in years
  α, β and γ = coefficients to be estimated.

The heights are given by equations (2) and (3) at time T_1 and T_2 :

$$\ln(H_1) = \alpha + \beta / T_1{}^r \qquad (2)$$

$$\ln(H_2) = \alpha + \beta / T_2{}^r \qquad (3)$$

Making the subject in equations (2) and (3), and equating the two resultant equations leads to equation (4) :

$$T_2{}^r (\ln(H_2) - \alpha) = T_1{}^r (\ln(H_1) - \alpha) \qquad (4)$$

Solving this equation for $\ln(H_2)$ gives equation (5) :

$$\ln(H_2) = \ln(H_1)^{\left(\frac{T_1}{T_2}\right)^r} + \alpha \left(1 - \left(\frac{T_1}{T_2}\right)^r\right) \qquad (5)$$

Taking the exponent of equation (5) gives equation (6) :

$$H_2 = H_1^{\left(\frac{T_1}{T_2}\right)^r} \exp\left(\alpha \left(1 - \left(\frac{T_1}{T_2}\right)^r\right)\right) \qquad (6)$$

When T_2 in equation (6) is set equal to an index age (e.g. 40 years for Douglas-fir and 20 years for radiata pine in New Zealand), then H_2 will be an explicit definition of site index.

The main standard analytical procedures used in this study were non-linear least-squares regression based on PROC NLIN in SAS. Analysis of residual patterns, through PROC UNIVARIATE was used to confirm the goodness of fit equations. When regressions were fitted to data in this study :

1) the dependent and independent variables conformed to biologically and mathematically realistic relationships;
2) the functions used were of an appropriate form to represent the intended relationship; and

3) a good fit was produced without bias in the regression coefficients;

as Temu (1992) and Liu Xu (1990) described in their growth and yield studies.

Various sigmoid functions were fitted to the data set using PROC NLIN (SAS Inc, 1990) and the derivative-free algorithmic methods for non-linear least-squares (Ralston and Jennrich, 1979). The general functional forms of equations used are presented in Table 2. In the table the term "exp" represents exponential functions.

In order to determine the best model, candidate models were assessed by seeking a minimum mean square error, as well as observing the plot of residuals. Also, the following aspects were considered :

1) the residual patterns should have no bias;
2) the regression coefficient estimates of the 95 % confidence interval should have the same sign;
3) the average mean of residual and skewness should be close to 0, and kurtosis should lie between 2 and $+\infty$;
4) extreme values of residuals between positive and negative values should be similar.

The above tests, therefore, were used together, not just on their own, to avoid biased results and provide good fits to the equations.

RESULTS AND DISCUSSION

Firstly, several polymorphic models were assayed such as the log-reciprocal equation (Schumacher, 1939; Woollons and wood, 1992), Chapman-Richards (Pienaar and Tunbull, 1973; Goulding, 1979),

Table 2. General form of projection equations applied to data

Equation name	Equation Forms
Schumacher polymorphic	$H_2 = \exp(\ln(H_1) (T_1/T_2)^\beta + \alpha (1-(T_1/T_2)^\beta))$
Chapman-Richards polymorphic	$H_2 = (\alpha/\gamma)^{[1/(1-\beta)]} (1-(1-(\gamma/\alpha)H_1^{1-\beta}) \exp(-\gamma(1-\beta)(T_2-T_1))^{[1/(1-\beta)]}$
Gompertz polymorphic	$H_2 = \exp(\ln(H_1) \exp(-\beta(T_2-T_1) + \gamma(T_2^2-T_1^2) \alpha + (1-\exp(-\beta(T_2-T_1) + \gamma(T_2^2-T_1^2))))$
Weibull polymorphic	$H_2 = H_1 \exp(-\beta(T_2^r-T_1^r)) + \alpha(1-\exp(-\beta(T_2^r-T_1^r))$
Schumacher anamorphic	$H_2 = H_1 \exp(-\beta(1/T_1^r -1/T_2^r))$
Hossfeld anamorphic	$H_2 = 1/((1/H_1) + \beta(1/T_2^r -1/T_1^r))$
Chapman-Richards Anamorphic	$H_2 = H_1((1-\exp(-\beta T_2))/(1-\exp(-\beta T_1)))^r$
Gompertz anamorphic	$H_2 = H_1 \exp(-\beta(\exp(\gamma T_2)-\exp(\gamma T_1)))$

Table 3. Coefficients for general polymorphic equations fitted to data

Model Name	Coefficients			MSE
Schumacher polymorphic	5.3076	0.4476	–	0.489
Chapman-Richards polymorphic	0.6207	0.3710	0.0484	0.500
Gompertz Polymorphic	4.121	0.0547	0.0003	0.497
Weibull polymorphic	115.94	0.0287	0.0183	0.493

Gompertz (Whyte and Woollons, 1990) and Weibull (Yang *et al.*, 1978; Goulding and Shirley, 1979). The fitted coefficients and mean square error are shown in Table 3.

Most of the polymorphic equations generally fitted well without bias in residuals pattern. Comparing residuals pattern and mean square error values the Schumacher polymorphic equation (7), was found to represent the best fit (Table 3).

$$H_2 = \exp(\ln(H_1)(T_1/T_2)^\beta + \alpha(1-(T_1/T_2)^\beta)) \quad (7)$$

This equation contained desirable functions commonly used in growth and yield models, such as compatibility, consistency; and path-invariance (Clutter *et al.*, 1983). As T_2 approaches infinitely, H_2 approaches the upper asymptote, when T_1 equals T_2 then H_1 equals H_2 (consistency property), and the projection from T_1 to T_3 yields the same result as the projection from T_1 to T_2 followed by projection from T_2 to T_3 (path-invariance property).

A plot of residual values against predicted values is given in Figure 1; the data were evidently well balanced, with no apparent bias or systematic patterns and showed well goodness of fit. PROC UNIVARIATE in SAS showed that residual statistics were satisfactory as it contained 0.31 value for skewness and 1.94 value for kurtosis.
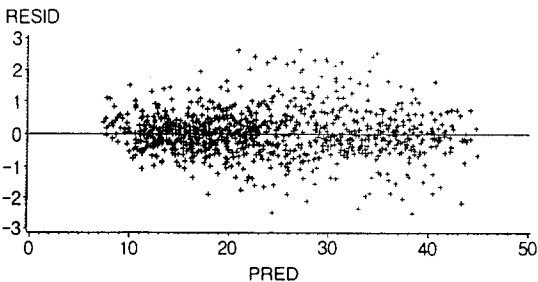
A Shapiro-Wilk test for normality was totally accepted (0.96). The equation gave a maximum residual of 2.6 m, a minimum residual of 2.5 m, a mean residual of 0.026 m and 95% of residuals lay 1.2 m.

The site index equation (8) can be derived from top height equation (7) by setting H_2=site index (S) when T_2=40 years, which is the base age for Douglas-fir in New Zealand (Burkhart and Tennent, 1977; Mountfort, 1978).

$$S = \exp(\ln(H_1)(T_1/40)^\beta + \alpha(1-(T_1/40)^\beta)) \quad (8)$$

Where, $\alpha = 5.3076$
$\beta = 0.4476$

Site index curves can then be generated by rearranging equation (8) and making H_1 the subject. Substituting S with any required site index values (e.g. 20, 30, 40, and 50) results in polymorphic height growth curves. Figure 3 shows a set of site index curves resulted from equation (9).

$$H_1 = \left[\frac{S}{\exp(\alpha(1-(T_1/40^\beta)))}\right]^{1/\left(\frac{T_1}{40}\right)^\beta} \quad (9)$$

Fig. 1. A plot of residuals against the predicted for Schumacher polymorphic height equation
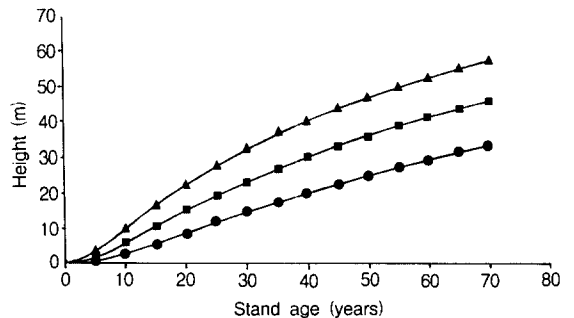

Fig. 2. Site index curves of Douglas-fir derived from a height equation

The site index equation, developed previously, for Douglas-fir grown in the South Island was an anamorphic form of the equation which was developed by Temu (1992). Hence, several other frequently used anamorphic equations were also investigated for a comparative purpose. These were :

$$H_2 = H_1 \exp(-\beta (1/T_1^{\,r} - 1/T_2^{\,r})) \qquad (10a)$$
$$H_2 = H_1((1-\exp(-\beta T_1))/(1-\exp(-\beta T_2)))^r \qquad (10b)$$
$$H_2 = 1/((1/H_1) + \beta (1/T_2^{\,r} - 1/T_1^{\,r})) \qquad (10c)$$
$$H_2 = H_1 \exp(-\beta (\exp(\gamma T_2) - \exp(\gamma T_1))) \qquad (10d)$$

These equations are Schumacher, Chapman-Richard, Hossfeld and Gompertz functions, respectively. Equation (10d) showed to be unsuitable with residuals pattern, while equations (10a) and (10c) proved inferior to the Chapman-Richard equation that had the lowest mean square error value. Hence, the anamorphic form of Chapman-Richards equation, (10b), was employed as the final site index equation because this combination produced the best fit among several anamorphic forms fitted. The coefficients of the anamorphic equations fitted are presented in Table 4 with corresponding mean square values.

Table 4. Coefficients for general anamorphic equation fitted to data

Model Name	Coefficients		MSE
Schumacher anamorphic	$-$ 8.8869	0.3849	0.534
Chapman-Richards anamorphic	$-$ 0.0272	1.4554	0.516
Hossfeld anamorphic	$-$ 2.2895	1.3053	0.644
Gompertz anamorphic	$-$ -1.0077	0.0132	2.128

A plot of residual against predicted values for equation (10b) shown in Figure 3 fitted well without apparent bias, but slightly less satisfactory than the residual plot in Figure 1. The mean of the average residuals and mean absolute values were 0.04 m and 0.54 m, respectively, indicating that the equation slightly under-estimated tree height with an average deviation of 0.54 m.

Skewness and kurtosis values were 0.25 and 2.02, respectively. Equation (10b) was able to contain all residuals within 2.9 m. This equation had the maximum residual of 2.9 m and the minimum residual of 2.5 m, while 95 % of residuals were belonged to the range of 1.2 m.

When T_2 in equation (10b) is set equal to the desired index age, H_2 is the site index by definition.

$$S = H_1((1-\exp(-\beta T_1)) / (1-\exp(-\beta T_2)))^r \qquad (11)$$
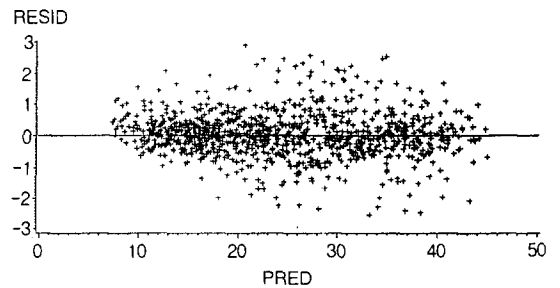

Fig. 3. A plot of residuals against the predicted for Chapman-Richard anamorphic height equation

The result showed that both the polymorphic and the anamorphic equations provide estimates of height growth that were generally acceptable, with the average error within 0.55 m of the observed values. Statistics of residuals with the polymorphic equation were slightly better than those of the anamorphic equation.

Even though there were no significant differences in statistics between the anamorphic and polymorphic equations, the latter was preferred. The reason for this was that polymorphic curves generally reflect height growth trends over a wide range of site qualities more accurately than anamorphic curves.

CONCLUSIONS

It was clear that both the polymorphic and the anamorphic models (7) and (10b), provided satisfactory models of the site index equation for Douglas-fir, but the polymorphic form, equation (7), was somewhat superior. This was ensured by comparing the respective residual mean squares,

where the polymorphic equation was lower in value, as well as better residual patterns and residual statistics. It is unrealistic to expect a unique function to perform consistently better than others with forest growth and yield data. However, the initial selection of appropriate equations is most important for success of the goodness of fit models. In this research, the polymorphic equation that reflects different shapes for the different site index classes was found to be the best model.

ACKNOWLEDGEMEMTS

LITERATURE CITED

1. Assmann, E. 1970. The principles of forest yield study, Translated by S.H. Gardiner, Pargamon Press, Oxford; 506 pp.

2. Avery, T.E., and H.E. Burkhart. 1994. Forest measurement. McGraw-Hill Book Co., New York. 408 pp.

3. Borders, B.E., R.L. Bailey, and K.D. Ware. 1984. Slash pine index from a polymorphic model joining (splining) non-polynominal segments with an algebraic difference method. Forest Science. 30(2) : 411-423.

4. Burkhart, H.E., and R.B. Tennent. 1977. Site index equations for radiata pine in New Zealand. New Zealand Journal of Forestry Science. 7(3) : 408-416.

5. Clutter, J.L., J.C. Forsion, L.V. Pienaar, G.H. Brister, and R.L. Bailey. 1983. Timber management; a quantitative approach. John Wiley and Sons, New York. 333 pp.

6. Goulding, C.J. 1979. Validation of growth models for *Pinus radiata* in New Zealand. New Zealand Journal of Forestry. 24(1) : 108 -124.

7. Goulding, C.J., and J.W. Shirey. 1979. A method to predict the yield of log assorments for long term planning. In Elliott, D.A.(Ed).

Mensuration of management planning of exotic forest plantations. New Zealand Forestry Service and Forestry Research Institute Symposium No. 20 : 301-314.

8. Lanner, R.M. 1985. On the intensity of height growth to spacing. Forest Ecology and Management. 13 : 143-148.

9. Liu, Xu. 1990. Growth and yield of Douglas fir plantations in the Central North Island of New Zealand. Ph.D Thesis. School of Forestry, University of Canterbury, New Zealand. 244 pp.

10. Mountfort, C.J. 1978. Growth of Douglas fir. New Zealand Forestry Service and Forestry Research Institute Symposium No. 15 : 375-411.

11. Pienaar, L.V., and K.J. Turnbull. 1973. The Chapman-Richards generalization of Von Bertalanffy's model for basal area growth and yield in even-aged stands. Forest Science. 19 : 2-22.

12. Ralston, M.L., and R.I. Jennrich. 1979. Dud, a derivative free algorithm for nonliner least squares. Technometrics 20(1) : 7-13.

13. Rennols, K. 1995. Forest height growth modelling. Forest Ecology and Management. 71 : 217-225.

14. SAS Institute Inc. 1990. SAS/STAT Procedures guide, Version 6, NC.

15. SAS Institute Inc. 1990. SAS/STAT User's guide, Version 6, Cary, NC.

16. Schumacher, F.X. 1939. A new growth curve and its application to timber-yield studies. Journal of Forestry 37 : 819-820.

17. Temu, M.J. 1992. Forecasting yield of Douglas fir in the South Island of New Zealand, Ph.D Thesis, School of Forestry, University of Canterbury, New Zealand, 246 pp.

18. Whyte, A.G.D., and R.C. Woollons. 1990. Modelling stand growth of radiata pine thinned to varying densities. Canadian Journal of Forest Research. 20 : 1069-1076.

19. Woollons, R.C., and G.R. Wood. 1992. Utility and performance of five sigmoid yield-age functions fitted to stand growth data. Improving yield forecasting reliability through aggregated modelling. In : G.B. wood, and

B.J. Turner (Eds). Proceedings IUFRO-Integrating Information Over space and Time; Australian National University Canberra, Jan; 13-17, 1992 : 71-80.

20. Yang, R.L., A. Kozak, and H.J.G. Smith. 1978. The potential of Weibull-type functions as flexible growth functions. Canadian Journal of Forest Research. 8 : 424-431.