

# Direct Drive 모터의 적응 퍼지 슬라이딩 모드제어

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## Adaptive Fuzzy Sliding Mode Control of a Direct Drive Motor

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### ABSTRACT

본 논문에서는 새로운 적응 퍼지 슬라이딩 모드제어 방법을 제시하였다. 제어기는 정확한 수학적 모델이 없이도 점근적으로 시스템을 안정화시킬 수 있으며 적분항을 포함시킴으로써 정상상태에서의 오차를 좀더 줄일 수가 있다. 직접구동모터는 감쇄기어가 없어서 부하나 외란 토크의 변화에도 모터 역학에 직접적으로 많은 영향을 줄 수가 있다. 제어기의 실제성능을 확인하기 위하여 불확실한 부하나 변수를 갖는 직접구동모터의 위치제어에 적용하였다.

**Key Words** : Fuzzy control(퍼지제어), Adaptive control(적응제어), Sliding mode control (슬라이딩모드제어)

### 1. Introduction

Since its inception in 1965<sup>[1]</sup> and particularly within the past decade, fuzzy logic has been the subject of significant debate and analysis among scientists with a large array of backgrounds from medicine to engineering and to psychology. Although, through numerous simulations and applied examples, fuzzy logic has been shown to be a powerful technique in integrating human knowledge and intuition for control of complex systems, there is not yet an adequate systematic method of rigorous analysis and design.

Recently, several researchers have combined the three concepts of fuzzy logic, adaptive control, and sliding mode control. In this combination, the robustness of fuzzy logic systems, the adaptation

capability of adaptive control, and the disturbance rejection of sliding mode control are collected in one control strategy, namely Fuzzy Sliding Mode Control. Furthermore, rigorous analysis of this type of control strategy has proven to be stable. Wang<sup>[2]</sup> introduced the concept of fuzzy basis functions and used the mathematical framework for stability analysis of adaptive fuzzy controllers for nonlinear systems. This mathematical representation of fuzzy rule-sets was later used by others to derive rigorous stability results for fuzzy systems. The fuzzy logic systems<sup>[2]</sup> are capable of uniformly approximating any nonlinear function over compact input space to any degree of accuracy. In [3], a fuzzy sliding mode control law is described for nonlinear systems by setting an upper bound for system uncertainty. The fuzzy logic rule-set is used to estimate the

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unknown upper bound. Lin<sup>[4]</sup> changes the defining parameters of the membership functions in the fuzzy rule-set according to a sliding mode adaptive routine.

Under normal applications of fuzzy sliding mode control, the presence of system disturbances may result in steady state error since the error vector is deviated from the sliding surface. This paper presents a new approach to adaptive fuzzy sliding mode control of systems with more robustness to system disturbances. The proposed control scheme does not require an accurate mathematical model of the system. It incorporates an integral term in the sliding surface which eliminates steady state error. The feedback controlled loop is guaranteed to be stable. The feedback derivative term is added so that the differentiation of the Lyapunov function is assured negative definite. As a result, asymptotic stability is proven.

To verify the actual performance of the proposed controller, it is then applied to a position control of a direct drive motor with payload and parameter uncertainty. Direct drive motors have received increasing attention since they have no backlash or dead zone which are caused by gears. Since they are used in high-precision robot and machine tool applications, they must have high resistance to external disturbances. However, since direct drive motors do not have reduction gears, the variation of the load and the disturbance torque directly influence parameter changes in motor dynamics. In fact, the absence of gear reduction leads to great sensitivity for the motor to variations in the load inertia. As a result, a linear controller cannot provide a good response under varying load conditions. Variable Structure System(VSS) type self-tuning control<sup>[5]</sup>, Bang-Bang control<sup>[6]</sup>, and adaptive control<sup>[7,8]</sup> have been proposed to handle such problems. The paper describes adaptive fuzzy sliding mode control method for control of a direct drive motor for more robustness to system disturbances.

Two sets of simulations are performed. In one simulation, the desired trajectory is a set point. In

the other simulation, the desired trajectory is a sinusoidal function. In both of these simulations, the parameters of the direct drive motor **J**(motor and load inertia) and **D**(coefficient of viscous friction) are allowed to vary. Yet, very good tracking results are observed.

The paper is organized as follows. In section 2, the basic configuration of fuzzy logic systems is described<sup>[2]</sup>. Section 3 illustrates the proposed sliding mode control law as applied to control of a direct drive motor. Two sets of simulations with different initial conditions and desired trajectories are illustrated in section 4.

## 2. Fuzzy Logic Systems and Fuzzy Basis Functions

A fuzzy logic knowledge base consists of a set of fuzzy IF-THEN rules which themselves consist of a set of linguistic variables associated with inputs and outputs and fuzzy operators such as AND and OR. Since a multi-output system can always be separated into a group of single output systems, without loss of generality, let's consider a Multi-Input Single-Output(MISO) rule structure such as below,

$$R^{(l)}: \text{IF } x_1 \text{ is } A_1^l \text{ and } \dots \text{ and } x_n \text{ is } A_n^l, \quad (1)$$

$$\text{THEN } y \text{ is } B^l$$

where  $x = (x_1, \dots, x_n)^T \in V \subset R^n$  is compact and  $y \in W \subset R$  represent the linguistic inputs and output of the fuzzy logic system respectively.  $A_i^l$  and  $B^l$  are labels of the fuzzy sets in  $V$  and  $W$  respectively.  $i = 1, 2, \dots, n$  corresponds to the input number where  $n$  is the number of inputs, and  $l = 1, 2, \dots, m$  corresponds to the rule number where  $m$  is the number of rules.

There are many alternatives to implementation of fuzzy rules. In this paper, the product form of t-norm fuzzy implication is used.

$$A_1^{l'} \times A_2^{l'} \times \dots \times A_n^{l'} \rightarrow B^l \quad (2)$$

and

$$\begin{aligned} \mu_{A_1^{l'} \times \dots \times A_n^{l'} \rightarrow B^l}(\mathcal{X}, y) = & \mu_{A_1^{l'}}(\mathcal{X}_1) \star \dots \\ & \star \mu_{A_n^{l'}}(\mathcal{X}_n) \star \mu_{B^l}(y) \end{aligned} \quad (3)$$

where  $\star$  denotes the t-norm product operator and corresponds to the conjunction "and" in the linguistic rule-representation.

Let  $A_x$  be an arbitrary fuzzy set in  $V$ , then each fuzzy rule determines a fuzzy set,  $A_x \circ R^{(l)}$ , in  $R$  based on the following sup-star compositional rule of inference,

$$\begin{aligned} \mu_{A_x \circ R^{(l)}}(y) = \sup_{x \in U} [ & \mu_{A_x}(\mathcal{X}) \star \\ & \mu_{A_1^{l'} \times \dots \times A_n^{l'} \rightarrow B^l}(\mathcal{X}, y)] \end{aligned} \quad (4)$$

All  $m$  fuzzy rules are then combined to determine a final fuzzy through the fuzzy disjunction:

$$\begin{aligned} \mu_{A_x \circ (R^{(1)}, \dots, R^{(m)})}(y) \\ = \mu_{A_x \circ R^{(1)}}(y) + \dots + \mu_{A_x \circ R^{(m)}}(y) \end{aligned} \quad (5)$$

where  $+$  denotes the t-conorm which is commonly defined as fuzzy union, algebraic sum, or bounded sum, In this paper, however, the center-average defuzzifier is used to aggregate rules. The center-average-defuzzifier is defined as

$$y(\mathcal{X}) = \frac{\sum_{l=1}^m \bar{y}^l (\mu_{A_x \circ R^{(l)}}(\bar{y}^l))}{\sum_{l=1}^m (\mu_{A_x \circ R^{(l)}}(\bar{y}^l))} \quad (6)$$

where  $\bar{y}^l$  is the point in the  $R$  at which  $\mu_{B^l}(y)$  achieves its maximum value (assume that  $\mu_{B^l}(\bar{y}^l)=1$ ). In [2], it is shown that using center-average-defuzzifier, product inference, and singleton defuzzifier, the above equation reduces to,

$$y(\mathcal{X}) = \frac{\sum_{l=1}^m \bar{y}^l (\prod_{i=1}^n \mu_{A_i^{l'}}(\mathcal{X}_i))}{\sum_{l=1}^m (\prod_{i=1}^n \mu_{A_i^{l'}}(\mathcal{X}_i))} \quad (7)$$

The above equation can be stated as,

$$y(\mathcal{X}) = \underline{\theta}^T \underline{\xi}(\mathcal{X}) \quad (8)$$

where

$$\underline{\theta} = (\bar{y}^1, \dots, \bar{y}^m)^T,$$

$$\underline{\xi}(\mathcal{X}) = (\xi^1(\mathcal{X}), \dots, \xi^m(\mathcal{X}))^T$$

and

$$\xi^l(\mathcal{X}) = \frac{\prod_{i=1}^n \mu_{A_i^{l'}}(\mathcal{X}_i)}{\sum_{j=1}^m (\prod_{i=1}^n \mu_{A_i^{j'}}(\mathcal{X}_i))} \quad (9)$$

where the  $\xi^l(\mathcal{X})$  is called the fuzzy basis function,  $\underline{\theta}$  are adjustable parameters, and  $\mu_{A_i^{l'}}$  are given membership functions.

It has been proved [2,10] that fuzzy logic system in the form of eq.(7) are universal approximators. So the above fuzzy logic system is capable of uniformly approximating any nonlinear function over compact input space to any degree of accuracy.

### 3. Adaptive Fuzzy Sliding Mode Control Law

Let's consider the dynamic model of a direct drive motor as below,

$$\begin{aligned} \ddot{\mathcal{X}} = & -\frac{D}{J} \dot{\mathcal{X}} + \frac{1}{J} u \\ = & f(\mathcal{X}) + bu \end{aligned} \quad (10)$$

where,

$J$  - Inertia moment of the system load and rotor

$D$  - Coefficient of viscous friction term

$\mathcal{X}$  - Angular displacement of the motor (output)

$u$  - Output torque of the motor (control input)

The state vector

$$\mathcal{X} = (\mathcal{X}_1, \mathcal{X}_2)^T = (\mathcal{X}, \dot{\mathcal{X}})^T \in R^2$$

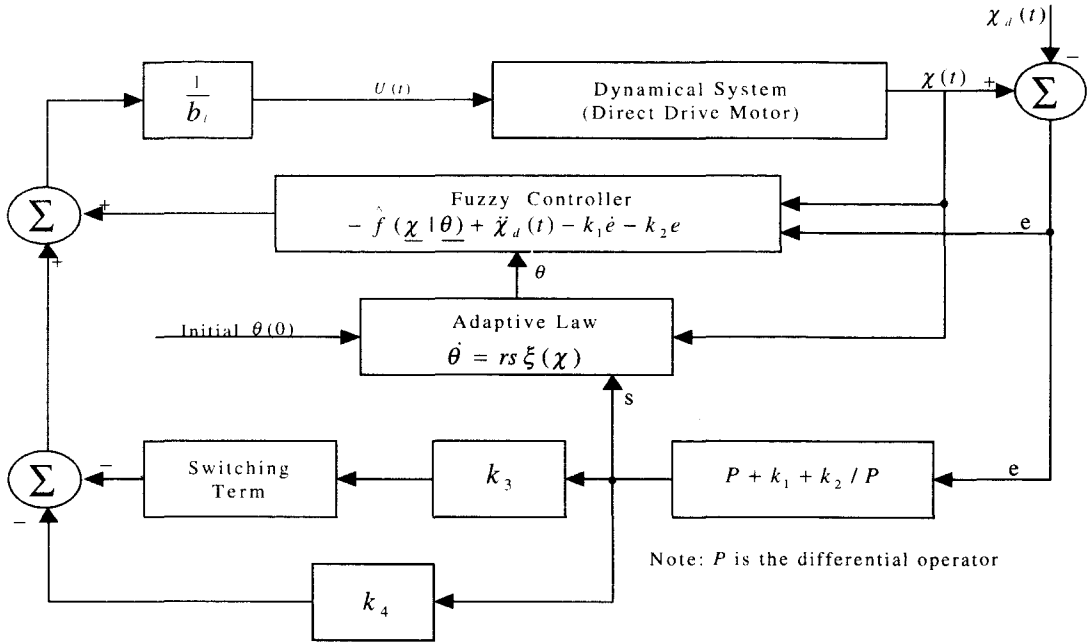


Fig. 1 A block diagram of the adaptive fuzzy sliding mode control system

is available for measurement. The function  $f(\chi)$  is not exactly known. The control gain  $b$  is also not exactly known, but it is lower bounded by a known  $b_l$ , that is  $0 < b_l \leq b$ ,  $b = b_l + \Delta b$ . Now, let  $e(t) = \chi(t) - \chi_d(t)$  where  $\chi_d(t)$  is the desired trajectory. Then, the sliding surface may be defined by,

$$s = \dot{e} + k_1 e + k_2 \nu \quad (11)$$

where

$$\nu = \int_0^t e(\tau) d\tau \quad (12)$$

$\nu$  allows including an integration term to reduce steady state error. By taking a derivative of both sides of eq.(11),

$$\begin{aligned} \dot{s} &= \ddot{e} + k_1 \dot{e} + k_2 e \\ &= \ddot{\chi}(t) - \ddot{\chi}_d(t) + k_1 \dot{e} + k_2 e \quad (13) \\ &= f(\chi) + bu - \ddot{\chi}_d(t) + k_1 \dot{e} + k_2 e \end{aligned}$$

If the function  $f(\chi)$  and the gain  $b$  are

known, we can easily obtain the optimal sliding mode controller [9]. Due to the poor knowledge of  $f(\chi)$  and  $b$  we replace  $f(\chi)$  by the fuzzy logic system  $\hat{f}(\chi|\theta)$  which is in the form of eq.(8) and consider the term  $k_3 \cdot \text{sgn}(s)$  to reduce the unknown disturbance.

The resulting controller is as follows.

$$\begin{aligned} u(t) &= \frac{1}{b_l} [-\hat{f}(\chi|\theta) + \ddot{\chi}_d(t) \\ &\quad - k_1 \dot{e} - k_2 e] - \frac{1}{b_l} k_3 \cdot \text{sgn}(s) - \frac{1}{b_l} k_4 s \quad (14) \end{aligned}$$

which reduces to,

$$\begin{aligned} u(t) &= \frac{1}{b_l} [-\hat{f}(\chi|\theta) + \ddot{\chi}_d(t) \\ &\quad - k_1 \dot{e} - k_2 e - k_3 \cdot \text{sgn}(s) - k_4 s] \quad (15) \end{aligned}$$

Where the term  $k_4 s$  is added so that  $\dot{V}(t)$  is assured more negative. Figure 1 illustrates a realization of the proposed adaptive fuzzy sliding

mode control law. The term  $k_3 \cdot \text{sgn}(s)$  gives rapid switching on the sliding surface. It depends on the sampling frequency. Direct drive motors cause large chattering at this sampling frequency. In order to reduce the chattering, we use the technique that makes the term  $\text{sgn}(s)$  continuous as follows,

$$\text{sgn}(s) \rightarrow \frac{s}{|s| + \delta} \quad (16)$$

where  $\delta > 0$ .

Substituting eq.(15) into eq.(13) yields,

$$\begin{aligned} \dot{s} = & f(x) + (b_l + \Delta b) \frac{1}{b_l} [-\hat{f}(x|\theta)] \\ & + \ddot{x}_d(t) - k_1 \dot{e} - k_2 e - \frac{k_3 s}{|s| + \delta} - k_4 s \end{aligned} \quad (17)$$

reduces to,

$$\begin{aligned} \dot{s} = & f(x) - \hat{f}(x|\theta) - k_4 s + \frac{\Delta b}{b_l} F \\ & - \frac{b}{b_l} k_3 \cdot \frac{s}{|s| + \delta} \end{aligned} \quad (18)$$

where

$$\begin{aligned} F = & -\hat{f}(x|\theta) + \ddot{x}_d(t) \\ & - k_1 \dot{e} - k_2 e - k_4 s \end{aligned} \quad (19)$$

Define the minimum approximation error  $\omega$ ,

$$\omega = f(x) - \hat{f}(x|\theta^*) \quad (20)$$

where  $\theta^*$  is the optimal parameter. Then,

$$\begin{aligned} \dot{s} = & \hat{f}(x|\theta^*) - \hat{f}(x|\theta) + \omega + \frac{\Delta b}{b_l} F \\ & - \frac{b}{b_l} k_3 \cdot \frac{s}{|s| + \delta} - k_4 s \end{aligned} \quad (21)$$

If we choose  $\hat{f}$  to be the fuzzy logic systems in the form of eq.(8), then it can be rewritten as

$$\begin{aligned} \dot{s} = & \underline{\Phi}^T \underline{\xi}(x) + \omega + \frac{\Delta b}{b_l} F \\ & - \frac{b}{b_l} k_3 \cdot \frac{s}{|s| + \delta} - k_4 s \end{aligned} \quad (22)$$

where  $\underline{\Phi} = \theta^* - \theta$ , and  $\underline{\xi}(x)$  is the fuzzy basis function eq.(9).

Consider the following Lyapunov function,

$$V = \frac{1}{2} s^2 + \frac{1}{2r} \underline{\Phi}^T \underline{\Phi} \quad (23)$$

then

$$\begin{aligned} \dot{V} = & s \dot{s} + \frac{1}{r} \underline{\Phi}^T \dot{\underline{\Phi}} \\ = & s(\underline{\Phi}^T \underline{\xi}(x) + \omega + \frac{\Delta b}{b_l} F \\ & - \frac{b}{b_l} k_3 \cdot \frac{s}{|s| + \delta} - k_4 s) + \frac{1}{r} \underline{\Phi}^T \dot{\underline{\Phi}} \quad (24) \\ = & \frac{1}{r} \underline{\Phi}^T (rs \underline{\xi}(x) - \dot{\underline{\theta}}) - \frac{1}{b_l} (k_3 b \frac{s^2}{|s| + \delta} \\ & - \Delta b \cdot s \cdot F - b_l \cdot s \omega) - k_4 s^2 \end{aligned}$$

Choose  $k_3$ (gain) and  $\delta$  so that it satisfies the inequality,

$$k_3 \frac{s^2}{|s| + \delta} > |s(F + \omega)| \quad (25)$$

Then

$$\dot{V} < \frac{1}{r} \underline{\Phi}^T (rs \underline{\xi}(x) - \dot{\underline{\theta}}) - k_4 s^2 \quad (26)$$

The term  $s\omega$  is of the order of minimum approximation error. Because of the Universal Approximation Theorem [2], we can expect that  $\omega$  should be small. If we choose the adaptive law.

$$\dot{\underline{\theta}} = rs \underline{\xi}(x) \quad (27)$$

then we have,

$$\dot{V} < -k_4 s^2 \quad (28)$$

Hence, the asymptotic stability of the proposed method is guaranteed.

#### 4. Simulation

For the adaptive fuzzy sliding mode control derived in Section 3, we investigate the performance of the system under practical environmental conditions in order to show the effectiveness of the

proposed controllers. We used parameters for the 6-inch megatorque motor<sup>[5]</sup> which is a popular direct drive motor. This motor has a resolver which is used to sense its rotor angle. The dynamic model of the above direct drive motor is simulated using Matlab's command "ode23" with a sampling period of 1 ms. The parameters used in the two simulations are the following:

- The reference trajectory for the first simulation:

$$\chi_{d_1}(t) = \frac{1}{4} \left[ \frac{4\pi t}{5} - \sin \frac{4\pi t}{5} \right], 0 \leq t \leq 2.5$$

in seconds. Initial Condition:  $\chi(0) = 0.0$

- The reference trajectory for the second simulation:

$$\chi_{d_2}(t) = \sin(t), 0 \leq t \leq 10.0$$

in seconds. Initial Conditions:  $\chi(0) = 0.5$

- Maximum torque =  $39.2 \text{ N} \cdot \text{m}$
- Rotor inertia =  $0.0077 \text{ kg} \cdot \text{m}^2$
- Coefficient of viscous friction =  $0.31 \text{ N} \cdot \text{s/m}$
- $k_1 = 12$ ,  $k_2 = 36$ ,  $k_3 = 10$ ,  $k_4 = 5$
- $\delta = 0.01$ ,  $r = 100000$ ,  $b_l = 45$

The results of these simulations are shown in Figures 2 and 4 and show the output tracks the reference trajectories  $\chi_{d_1}(t)$  and  $\chi_{d_2}(t)$  very closely. Figures 3 and 5 show the errors of position angles for the control system immediately after changing the rotor inertia  $0.02 \text{ kg} \cdot \text{m}^2$  and the coefficient of viscous friction  $0.6 \text{ N} \cdot \text{s/m}$  at  $t=1$  second. Figure 6 show phase portait of the trajectory. In spite of these sudden changes, the adaptive fuzzy sliding mode controller continues to perform very well and show that the proposed controller does not need exact knowledge of the load inertia and the coefficient of viscous friction.

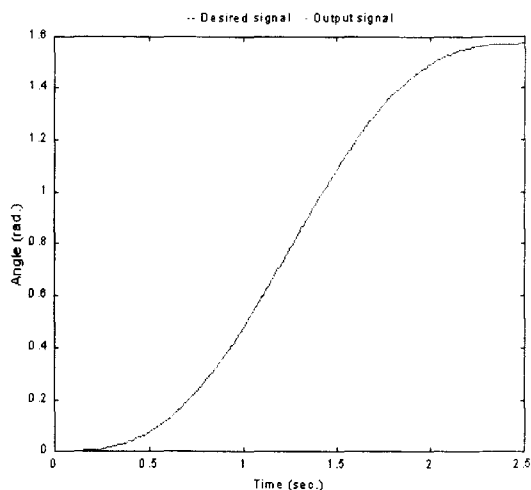


Fig. 2 Trajectory of the Angular Position with Respect to

$$\chi_{d_1} = \frac{1}{4} \left[ \frac{4\pi t}{5} - \sin \frac{4\pi t}{5} \right]$$

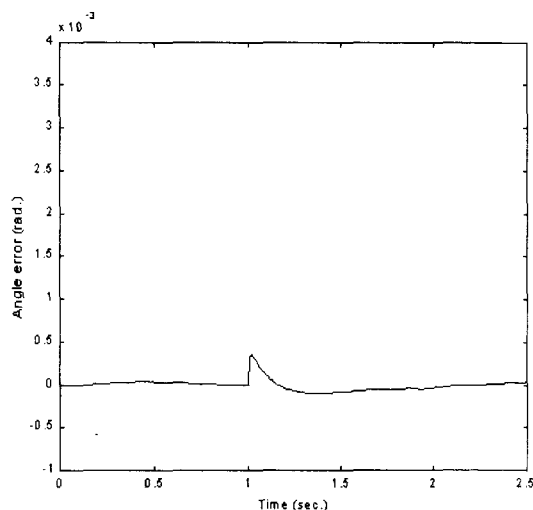


Fig. 3 Error Trajectory of the Angular Position with Respect to

$$\chi_{d_1} = \frac{1}{4} \left[ \frac{4\pi t}{5} - \sin \frac{4\pi t}{5} \right]$$

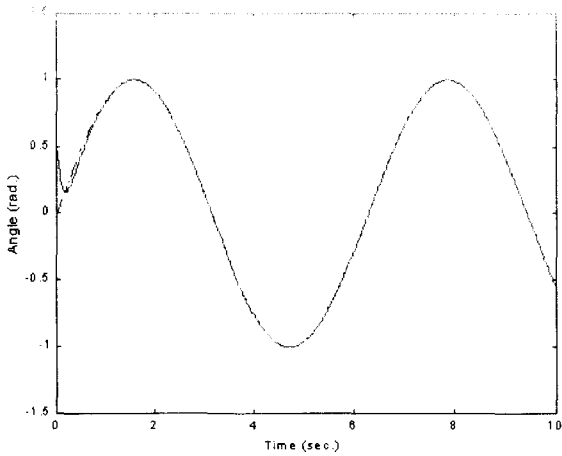


Fig. 4 Trajectory of the Angular Position with Respect to

$$\chi_{d_1}(t) = \sin(t)$$

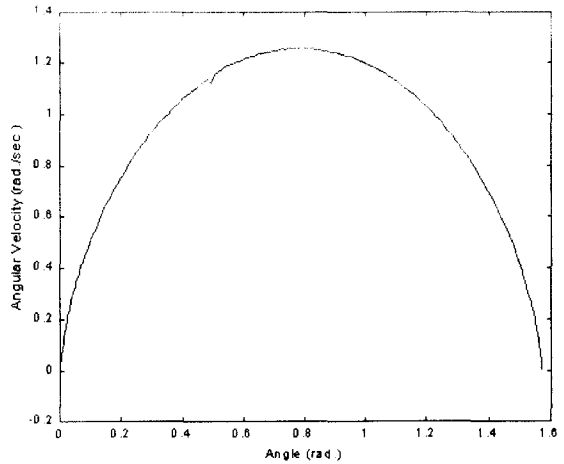


Fig. 6 Phase Portrait of the Trajectory.

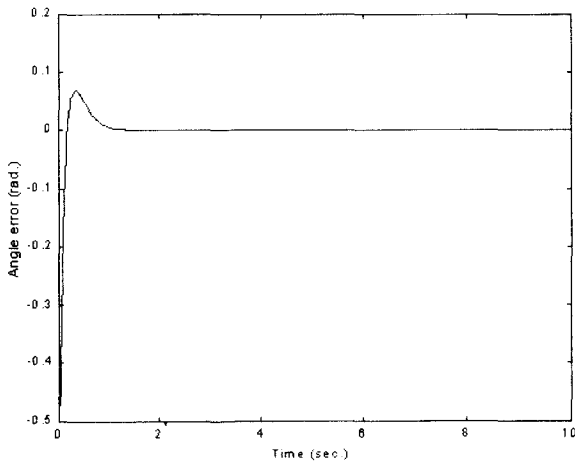


Fig. 5 Error Trajectory of the Angular Position with Respect to

$$\chi_{d_2}(t) = \sin(t)$$

## 5. Conclusion

This paper presents a state feedback sliding mode fuzzy adaptive control method for position control of a direct drive motor. The adaptive law adjusts the parameters of a fuzzy controller which multiply a vector of fuzzy basis functions. In addition, It incorporates an integral term in the sliding surface which eliminates steady state error. The proposed control scheme does not require an accurate mathematical model of the system. The method is applied to position control of a direct drive motor with payload and parameter uncertainty. Since direct drive motors do not have reduction gears, the variation of the load and the disturbance torque directly influence parameter changes in motor dynamics. Two sets of simulations demonstrate that the error in trajectory following is minimal even while introducing large errors in estimated parameters of the direct drive motor.

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