공차이론의 개발과 재봉기의 이송조절기구에 적용

신대영*, 전경진**, 송창섭***

A Development of Tolerance Theory and Its Application to Feeding Control Mechanism of a Sewing Machine

Dae Young Shin*, Keyoung Jin Chun**, Chang Seop Song***

ABSTRACT

기구의 운동에 있어서 공차는 필연적으로 운동 오차를 발생시키게 된다. 이상적인 기구의 운동을 위하여 공차를 작게 설정하는 것은 가공 또는 조립 비용을 증가시키는 반면에 제작 비용의 절감을 위하여 공차를 크게 설정하는 것은 기구의 운동 오차를 크게 증가시키게 된다. 따라서 기구의 운동오차를 고려한 효율적인 공차의 설정 및 배분이 중요하다. 이 논문은 민감도 해석을 이용하여 기구의 운동에 영향을 미치는 설계변수들에 어떻게 공차를 효율적으로 지정하고 분배하는 가를 연구하였다. 민감도 해석에서는 각 설계변수의 민감도 계수를 유도하였으며 설계변수에 공차를 지정하였을 때 발생할 수 있는 기구 운동의 최대 응답오차를 수식화 하였으며 역으로 기구 운동에 있어서 허용 응답오차를 설정하였을 때 각 설계 변수의 공차를 분배하는 과정을 유도하였다. 실제로 공차 분배 과정을 공업용 본봉 재봉기의 이송조절기구에 적용하여 허용 응답오차를 고려한 공차 설정을 합리적으로 수행할 수 있었다.

본 논문에서 제시한 공차 분배 과정은 재봉기 또는 로봇과 같이 기구운동과 밀접한 관련이 있는 기계의 설계에 유용할 것이다.

Key Words : sensitivity(민감도), sewing machine(재봉기), tolerance allocation(공차배분)

1. Introduction

Tolerances, clearances, and static and dynamic deflections in mechanisms may lead to deviate from the desired motion of the output. Especially, tolerances and clearances have been treated as major causes of the deviations of the outputs.

Hartenberg and Denavit[1], and Knapper[2] analyzed mechanical errors which were produced by tolerances. They reported a linear relationship between mechanical errors and tolerances by a deterministic method. Corderman and Mabie[3] studied mechanical output errors considering tolerances, clearances and link lengths and introduced several charts to predict effects of these three factors. Garrett and Hall[4] analyzed mechanical errors produced by tolerances and clearances by a statistical approach. After Garrett and Hall, some literatures[5-7] have analyzed mechanical errors and tried to distribute tolerances and clearances by statistical approaches.

* 한양대학교 정밀기계공학과(한국생산기술연구원)
** 한국생산기술연구원
*** 한양대학교 정밀기계공학과

Ting and Long[8] presented a theory to determine the sensitivity of tolerances to the performance quality of mechanisms and to determine the dimensional tolerance distribution, however, did not deeply consider each error components' characteristic. Lin and Chen[9] expressed the influences of error components due to joint clearances and link dimensions, however, the generalized error analysis methodology are too complex to be easy to apply a real mechanism. This paper focuses only on tolerances because dynamic interactions of machine parts from clearances can be ignored in an example mechanism studied. This paper works on how to get sensitivity coefficient matrix, to get a maximum response error get a maximum response error of dependent variables and to allocate tolerances in consideration of each design variables' characteristic and a limit of response error of a dependent variable through a sensitivity analysis.

We choose the feeding control mechanism of an industrial lock stitch sewing machine which is a good example to study a machine kinematics. The tolerance determination method represented in this paper might be very useful to precision machine designs whose functions are much related to kinematics.

2. Sensitivity Analysis For Tolerance

The actual size (x_j) on a machine part is never able to be made the same size as the basic size (x_j^*). Thus, the actual size of a design variable always involves a deviation from the basic size. Also, this deviation of each design variable makes response errors on dependent variables as shown in *Equation (3)*.

Assuming that dependent variables $\underline{y}: D^m \rightarrow R^n$ have derivatives of all orders everywhere in $\underline{x} \in D^m$, dependent variables $\underline{y}$ can be expanded by a Taylor expansion near $\underline{y}(\underline{x}^*)$ as

$$y_i(\underline{x}) = y_i(\underline{x}^*) + \sum_{j=1}^{m} S_{i,j}(x_j - x_j^*) + R_i \qquad (1)$$

where $S_{i,j} = \partial y_i / \partial x_j$, R_i is the remainder term.

In *Equation (1)*, if $\underline{x}$ is sufficiently close to $\underline{x}^*$, $R_i \ll y_i(\underline{x}^*) + \sum_{j=1}^{m} S_{i,j}(\underline{x}^*)(x_j - x_j^*)$. Hence, a change in y_i at $\underline{x}^*$ (denoted as δy_i) is rearranged as

$$\delta y_i \approx \sum_{j=1}^{m} S_{i,j}(\underline{x}^*)\delta x_j \qquad (2)$$

where δx_j is a small change in x_j^* $(\delta x_j = x_j - x_j^*)$, $\delta y_i = y_i(\underline{x}) - y_i(\underline{x}^*)$, and $S_{i,j}(\underline{x}^*)$ is a sensitivity coefficient matrix which expresses the sensitivity of i^{th} dependent variable to a small change in j^{th} design variable.

2.1. Maximum response error

When each design variable $\underline{x}$ has a deviation $\delta \underline{x}$, the response error of a dependent variable can be derived from *Equation (2)* as

$$\begin{aligned} |\delta y_i| &\approx \left| \sum_{j=1}^{m} S_{i,j}(\underline{x}^*)(x_j - x_j^*) \right| \\ &\leq \sum_{j=1}^{m} |S_{i,j}(\underline{x}^*)||(x_j - x_j^*)| \end{aligned} \qquad (3)$$

where $|x_j - x_j^*|$ is the deviation's magnitude of j^{th} design variable.

From *Equation (3)*, it becomes clear that the maximum response error (MRE) of a dependent variable can be made when the deviation's magnitude of each design variable is maximum. That is,

$$\begin{aligned} |ME_i| \approx\ & |S_{i,1}(\underline{x}^*)||x_1 - x_1^*|_{max} + \cdots \\ & + |S_{i,m}(\underline{x}^*)||x_m - x_m^*|_{max} \end{aligned} \qquad (4)$$

where $|x_j - x_j^*|_{max}$ is the deviation's maximum magnitude of j^{th} design variable, and $|ME_i|$ is the MRE of i^{th} dependent variable.

2.2. Procedure for allocation of tolerances

If response errors on dependent variables were successfully derived from maximum deviations of design variables in §2.1, it might be practicable to reversely allocate tolerances to satisfy with a limit of response error (LRE) of the dependent variable, which is set as an allowable error by a designer.

In allocating procedure, assuming that each dependent variable has the same maximum deviation in order to simplify this procedure to allocate tolerances, this same maximum deviation can be set as an assumed allowable deviation (α) of design variables, and can be derived as

$$|LE_i| \approx |S_{i,1}(\underline{x}^*)|\alpha + \cdot\cdot\cdot + |S_{i,m}(\underline{x}^*)|\alpha$$
$$\therefore \alpha \approx \frac{|LE_i|}{\sum_{j=1}^{m}|S_{i,j}(\underline{x}^*)|} \tag{5}$$

However, as each design variable has its own characteristics, it is needed to adjust maximum deviation of each design variable to satisfy with its own characteristics. In adjusting process, the maximum response error $(|ME_i|)$ is less than or equal to the limit of response error $(|LE_i|)$ as

$$|ME_i| \leq |LE_i| \tag{6}$$

Then, the maximum deviation of the h^{th} design variable can be obtained from changing maximum deviations of other design variables arranging *Equations (5)* and *(6)* as

$$|x_h - x_h^*|_{max} \leq \alpha + \sum_{k=1}^{h-1}\frac{|S_{i,k}(\underline{x}^*)|}{|S_{i,h}(\underline{x}^*)|}(\alpha - |x_k - x_k^*|_{max})$$
$$+ \sum_{l=h+1}^{m}\frac{|S_{i,l}(\underline{x}^*)|}{|S_{i,h}(\underline{x}^*)|}(\alpha - |x_l - x_l^*|_{max}) \tag{7}$$

After adjusting maximum deviations of design variables in turn through *Equation (7)*, the size tolerance (tolerance zone) can be decided within the maximum deviation.

3. Example Study

The feeding control mechanism of an industrial lock stitch sewing machine is introduced in order to examine this tolerancing study.

The feeding control mechanism is composed of a sliding contact mechanism and a four bar linkage as shown in *Figure 1.* The reference system is the Cartesian coordinate system, which has unit coordinate vectors $\hat{i}$ and $\hat{j}$ directed along the X and Y coordinate axes respectively. The origin of a stationary Cartesian X-Y reference frame is on the rotational axis of the feed regulator (FR), which is the absolute origin.

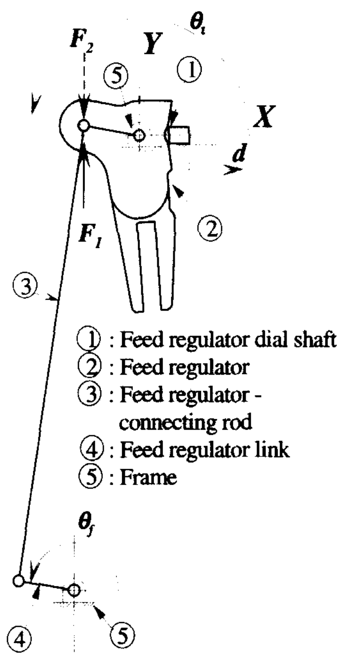

Fig. 1 Feeding control mechanism

The sliding contact mechanism is composed of the feed regulator dial shaft (FRDS, driver) motion and the FR (follower) motion. The inputs of the sliding contact mechanism are the feeding distance d of the FRDS and the force direction F on the FR, and the output of the sliding contact mechanism is the rotational angle θ_i of the FR. Here, F which is one of two force directions (F_1, F_2) controls feeding directions of a fabric and d controls stitch spacings on a fabric in sewing process. In the sliding contact mechanism, as contacting surfaces of the FR and contacting surfaces of the FRDS are symmetric respectively, two rotational angles(θ_s, θ_s') of the FR's symmetric axis made by F_l(↑) or F_2(↓) have the same magnitude and the opposite direction each other as shown in *Figure 2*. Thus, the study could be focused on the rotational angle θ_s only while F_l is acting on.

The four bar linkage is composed of the FR (driver), the feed regulator connecting rod (coupler), and the feed regulator link (FRL, follower). The input of the four bar linkage is the rotational angle θ_i of the FR, and the output of the four bar linkage is the rotational angle θ_f of the FRL.

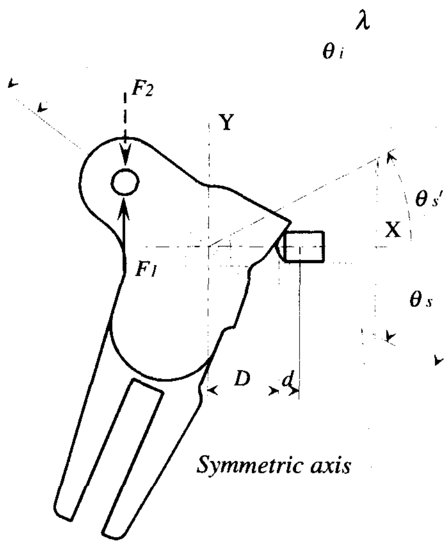

Fig. 2 Sliding contact mechanism

3.1. Kinematic analysis

In the sliding contact mechanism, there is a contacting surface (***C3_u***) on the FRDS and are two contacting surfaces (***C1_u***, ***L1_u***) on the FR.

While F_l is being on the FR and d is increasing gradually from 0.0, ***C3_u*** on the FRDS contacts with ***C1_u***, and then with ***L1_u*** after passing arcs as shown in *Figure 3* and *4*. Here, d represents the motion of ***C3_u***'s center.

In general, while a contact range mostly is been between ***C3_u*** and ***L1_u***, a sewing is being proceeded. Thus, this study try to derive the sensitivity coefficients of design variables and to allocate tolerances for sewing process on above contact range.

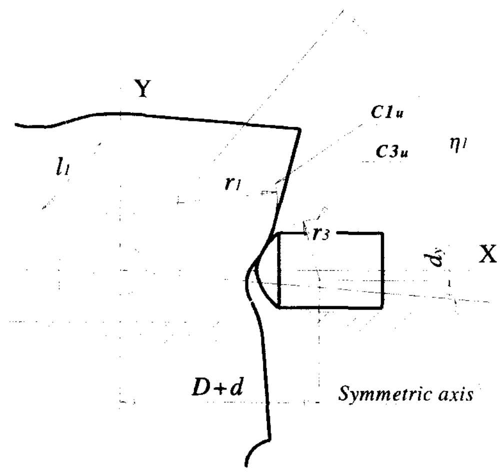

Fig. 3 when ***C3_u*** is tangent to ***C1u***

While F_l is acting on the FR and ***C3_u*** and ***L1_u*** are being tangent as shown in *Figure 4*, there are two constraint equations are derived from a closed loop $\left(\vec{G}_a + \vec{G}_b - \vec{G}_c - \vec{G}_d = 0\right)$ as

$$\begin{aligned} g_1 &\equiv (r_2 + r_3)\cos(\theta_{c2}) - t_1 \sin(\theta_{c2}) - l_2 \cos(\theta_2) \\ &= 0 \\ g_2 &\equiv (r_2 + r_3)\sin(\theta_{c2}) + t_1 \cos(\theta_{c2}) - l_2 \sin(\theta_2) \\ &= 0 \end{aligned} \tag{8}$$

where r_2 is the radius of ***C2_u***, $l_2 = \sqrt{(D+d)^2 + d_y^{\,2}}$, $\theta_2 = \tan^{-1}\left(\dfrac{d_y}{D+d}\right)$ and D is the distance between the absolute origin and the center of ***C3_u*** when the FRDS is most close to the absolute origin as follows

$$D = l_1 \cos\eta_1 + \{(r_1 + r_3)^2 - (l_1 \sin\eta_1 - d_y)^2\}^{1/2} \tag{9}$$

where l_1 is the distance between the absolute origin and the center of $C1_u$, r_1 is the radius of $C1_u$, r_3 is the radius of $C3_u$, d_y is the normal distance between the X coordinate axis and the center of $C3_u$ and η_1 is a constant that means an angle between the symmetric axis of the FR and the center of $C1_u$ as shown in *Figure* 3.

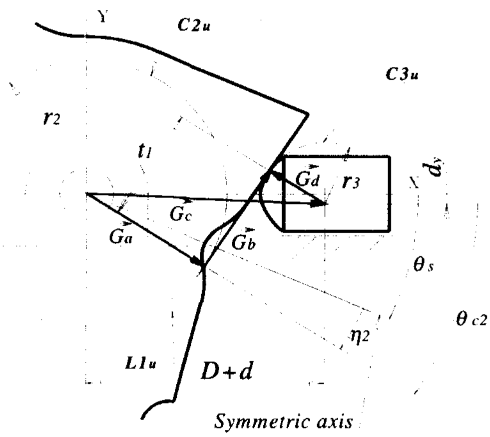

Fig. 4 when $C3_u$ is tangent to $L1_u$

In *Figure 4*, a circle $C2_u$ is tangent to $L1_u$ and the center of $C2_u$ is on the absolute origin.

Figure 5 shows the initial state of the four bar linkage. there are two constraint equations are derived from a closed loop $(\vec{H}_a + \vec{H}_b - \vec{H}_c - \vec{H}_d = 0)$ as

$$\begin{aligned} g_3 &\equiv k_1\cos(\theta_i) + k_2\cos(\xi_1) - k_3\cos(\xi_2) - k_4\cos(\theta_f) \\ &= 0 \\ g_4 &\equiv k_1\sin(\theta_i) + k_2\sin(\xi_1) - k_3\sin(\xi_2) - k_4\sin(\theta_f) \\ &= 0 \end{aligned} \quad (10)$$

where k_1 is the length of the driver(FR), k_2 is the length of the coupler(Feed regulator connecting rod), k_4 is the length of the follower(FRL), ξ_1 is the rotational angle of the coupler, and ξ_2 and k_3 are data between two joints on the frame.

In *Equation 10*, the rotational angle θ_i is derived from *Figure 2* and *4* as

$$\theta_i = \lambda + \theta_{c2} - \eta_2 \quad (11)$$

where $\eta_2 = \eta_1 + \cos^{-1}\left(\frac{r_1 - r_2}{l_1}\right) - \pi$, which is a constant that means an angle between the symmetric axis of the FR and the tangent point between $L1_u$ and $C2_u$ as shown in *Figure 4*.

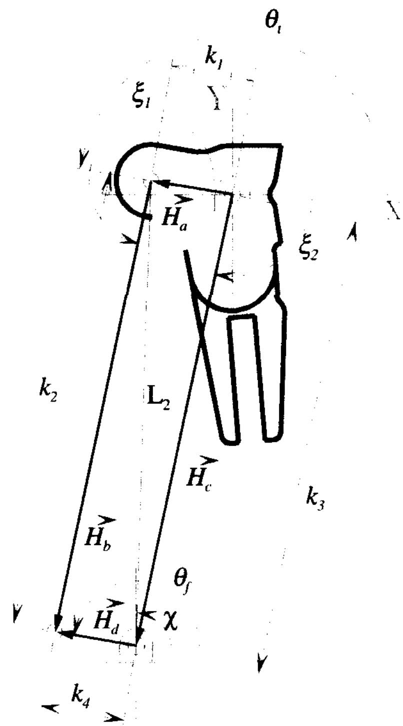

Fig. 5 Four bar linkage

3.2. Sensitivity coefficient matrix

The feeding control mechanism has twelve constants $(l_1, r_1, r_2, r_3, d_y, k_1, k_2, k_3, k_4, \eta_1, \lambda, \xi_2)$ as shown in *Figure 2* to *Figure 5*. In this example study, design variables are limited to distance dimension variables $(l_1, r_1, r_2, r_3, d_y, k_1, k_2, k_3, k_4)$ among twelve constants to simplify the procedure for allocation of tolerances. Angular dimension variables $(\eta_1, \lambda, \varepsilon_2)$ are not suitable for a good example study.

Thus, there are nine design variables

$\underline{x}=[l_1, r_1, r_2, r_3, d_y, k_1, k_2, k_3, k_4]^T$ which are independent each other, four dependent variables $\underline{y}=[\theta_{c2}, t_1, \theta_f, \xi_1]^T$ and four constraint equations as shown in *Equations (8)* and *(10)* in the feeding control mechanism. These constraint equations can be rearranged as

$$\underline{g}(\underline{x}, \underline{y}) = 0 \qquad (12)$$

where $\underline{g}=[g_1, g_2, g_3, g_4]^T$.

Equation (12) can be differentiated with the design variables $\underline{x}$ by using the chain rule, and a sensitivity coefficient matrix is derived as

$$[\,S_{p,q}] = -[\,V_{m,p}]^{-1}[\,H_{m,q}] \qquad (13)$$

where $S_{p,q} = \partial y_p / \partial x_q$, $V_{m,p} = \partial g_m / \partial y_p$, $H_{m,q} = \partial g_m / \partial x_q$, and $[V_{m,n}]$ is a nonsingular matrix.

Table 1 Nominal sizes of an example model

l_1	r_1	r_2	r_3	d_y	k_1
6.42 *mm*	7.0 *mm*	9.72 *mm*	4.5 *mm*	-0.74 *mm*	20.0 *mm*
k_2	k_3	k_4	η_1	λ	ξ_2
152.8 *mm*	152.76 *mm*	20.0 *mm*	0.972 *rad*	2.976 *rad*	4.561 *rad*

4. Results And Discussions

The nominal sizes of the feeding control mechanism from an example model are shown in *Table 1* corresponding to *Figure 2* to *Figure 5*.

4.1. Sensitivity coefficient

The sensitivity coefficients (SCs) of four dependent variables $\underline{y}=[\theta_{c2}, t_1, \theta_f, \xi_1]^T$ can be obtained from *Equation (13)*. However, as the output variable of the feeding control mechanism is θ_f among four dependent variables, this study deals with only the SCs of θ_f (denoted as $S_{3,j}$). *Table 2* represents the SCs of θ_f for each d while d is increasing from *2.0 mm* to *3.0 mm* by *0.5 mm* step and F_1 is acting on the FR.

From *Table 2*, the SCs related to l_1, k_2 and k_3 are two or seven times as large as those related to r_1, r_2, r_3, d_y, k_1 and k_4. Namely, when the same deviation magnitudes β are applied to the design variables and the portions $|S_{3,j}(\underline{x}^*)|\beta$ contributed to the maximum response error (MRE) of θ_f are investigated as shown in *Equation (4)*, $|S_{3,j}(\underline{x}^*)|\beta$ of l_1, k_2 and k_3 are two or seven times as large as those of r_1, r_2, r_3, d_y, k_1 and k_4. Thus, l_1, k_2 and k_3 are the more sensitive than other design variables.

Variations of the SCs related to r_1, r_2, r_3 and d_y are larger than those related to l_1, k_1, k_2, k_3 and k_4. Thus, in the MRE, the contributed portions $|S_{3,j}(\underline{x}^*)|\beta$ of r_1, r_2, r_3 and d_y vary larger than those of l_1, k_1, k_2, k_3 and k_4 while d is increasing from *2.0 mm* to *3.0 mm* and the same deviation magnitudes β are applied to the design variables. For example, the $|S_{3,2}(\underline{x}^*)|\beta$ of

Table 2 Sensitivity coefficient [*rad/mm*] for θ_f

Acting force	d (*mm*)	$S_{3,1}$ $\left(=\frac{\partial\theta_f}{\partial l_1}\right)$	$S_{3,2}$ $\left(=\frac{\partial\theta_f}{\partial r_1}\right)$	$S_{3,3}$ $\left(=\frac{\partial\theta_f}{\partial r_2}\right)$	$S_{3,4}$ $\left(=\frac{\partial\theta_f}{\partial r_3}\right)$	$S_{3,5}$ $\left(=\frac{\partial\theta_f}{\partial d_y}\right)$	$S_{3,6}$ $\left(=\frac{\partial\theta_f}{\partial k_1}\right)$	$S_{3,7}$ $\left(=\frac{\partial\theta_f}{\partial k_2}\right)$	$S_{3,8}$ $\left(=\frac{\partial\theta_f}{\partial k_3}\right)$	$S_{3,9}$ $\left(=\frac{\partial\theta_f}{\partial k_4}\right)$
F_1	2.0	0.065	-0.006	-0.004	-0.010	-0.021	-0.015	0.052	-0.052	0.015
	2.5	0.066	0.029	-0.032	-0.003	-0.007	-0.019	0.053	-0.053	0.019
	3.0	0.067	0.051	-0.050	0.001	0.002	-0.022	0.055	-0.055	0.022

r_1 vary rapidly, on the other hand, $|S_{3,6}(\underline{x}^*)|\beta$ of k_1 vary slowly where β =*0.1 mm (arbitrarily given)* as shown in *Figure 6*. Thus, in order to decrease the effects of large variations of the SCs, it is needed to minimize maximum deviations of r_1, r_2, r_3 and d_v for better tolerance controls in a machine design.

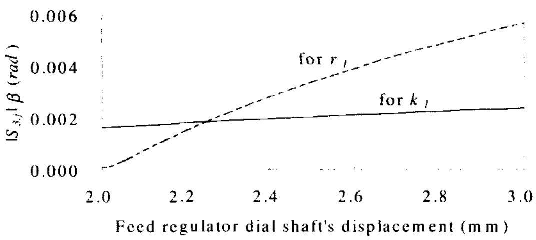

Fig. 6 $|S_{3,j}(\underline{x}^*)|\beta$ of design variable r_1 and k_1 where β =0.1 *mm*

4.2. Procedure for allocation of tolerance

In Table 2, while d is increasing from *2.0 mm* to *3.0 mm*, the sensitivities of r_3 and d_v are decreasing, on the other hand, the sensitivities of the others are increasing. This means that it might be more efficient to proceed with the tolerance allocation at d=*3.0 mm* than at all cases of d, if it make maximum deviations of r_3 and d_v be small. Thus, a procedure for allocation of tolerances could be focused on when d=*3.0 mm.*

While d is moving from *2.0 mm* to *3.0 mm*, the rotating amount of θ_f is *0.41 rad* from *Equation (8)* and *(10)*. Thus, this study arbitrarily chooses that a limit of response error (LRE) is 10.0% deviation in the rotating amount of θ_f , namely *0.041 rad.*

From *Table 1*, the nominal sizes of design variables k_2 and k_3 are larger than the others, and we assumes that the bigger size of machine parts, the more difficult to get the precise tolerance values in manufacturing. Thus, we try to make maximum deviations of k_2 and k_3 larger, also make maximum deviations of r_1, r_2, r_3 and d_y smaller in order to decrease the effects of variations of the SCs.

The allocation of tolerances is proceeded as the following procedures, and shown through *Table 3*.

Step 1 : the assumed allowable deviation α (=0.123) is calculated by *Equation (5)*.

Step 2 : Maximum deviations of l_1(=*0.07 mm*), r_2(=*0.07 mm*), k_1(=*0.10 mm*) and k_4(=*0.10 mm*) are arbitrarily set to smaller than α, and then a larger maximum deviation of k_3(=*0.254 mm*) is obtained from *Equation (7)*. Here, Maximum deviations of l_1, r_2, k_1 and k_4 may be given from some conditions like machining and assembling.

Step 3 : Maximum deviations of r_1(=*0.07 mm*), r_3(=*0.07 mm*) and d_v(=*0.04 mm*) are arbitrarily set to smaller than α, and then a larger maximum deviation of k_2(=*0.176 mm*) is obtained from *Equation (7)*.

Step 4 : Maximum deviation of k_3(=*0.215 mm*) is made smaller, and then a larger maximum deviation

Table 3 Procedure for allocation of tolerance

Procedure for allocation of tolerance	Maximum deviation $\lvert x_g - x_g^* \rvert_{max}$									A related equation
	l_1	r_1	r_2	r_3	d_y	k_1	k_2	k_3	k_4	
Step 1	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	0.123	*Eq. (5)*
Step 2	0.070	0.123	0.070	0.123	0.123	0.100	0.123	0.254	0.100	*Eq. (7)*
Step 3	0.070	0.070	0.070	0.070	0.040	0.100	0.176	0.254	0.100	*Eq. (7)*
Step 4	0.070	0.070	0.070	0.070	0.040	0.100	0.216	0.215	0.100	*Eq. (7)*
Step 5	0.07	0.07	0.07	0.07	0.04	0.10	0.22	0.22	0.10	*Round off to 2 decimals*

of k_2(=*0.216 mm*) is obtained from *Equation (7)*. Here, Maximum deviation of k_2 is tried to be equal to that of k_3 because the manufacturing conditions are the same in this example model.

Step 5 : Size tolerances of design variables are decided within maximum deviations of design variables. Here, size tolerances are set the same values as the maximum deviations.

When tolerances of design variables obtained at Step 5 are applied to *Equation (8)* and *(10)*, the responses of θ_f are shown in *Figure 7* while *d* is moving from *2.0 mm* to *3.0 mm*. In this figure, the maximum response error (MRE) is *0.029 rad* at *d=2.0* mm and is *0.041 rad* at *d=3.0 mm*. *Figure 8* shows the value change of the MRE of θ_f. Therefore, the MRE of θ_f is always within *10.0%* of the LRE (*0.41 rad*).

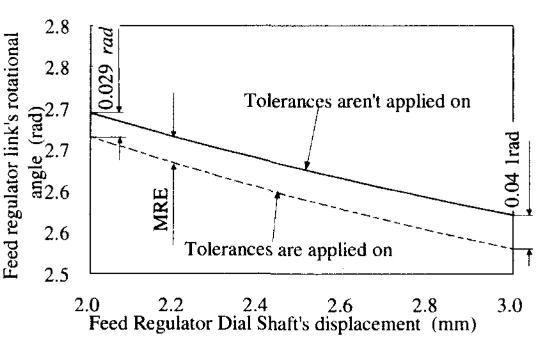

Fig. 7 The responses of θ_f when tolerances are applied

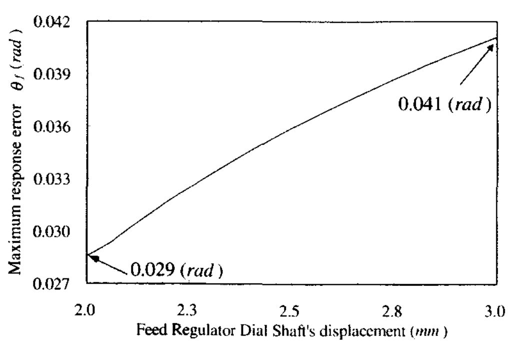

Fig. 8 The value change of the MRE of θ_f

5. Conclusions

A tolerance determination method is studied here by using a kinematics and sensitivity analysis.

Constraint equations are derived from the kinematic analysis and sensitivity coefficients of design variables. Also, maximum response errors of dependent variables are expressed when tolerances are applied and inversely tolerances of design variables allocate in consideration of each design variables' characteristic when a limit of response error of a dependent variable is set.

An allocating procedure of tolerances is introduced as follows:

1. define a limit of response error of a dependent variable,
2. calculate sensitivity coefficients of design variables,
3. calculate assumed allowable deviations of design variables,
4. arrange maximum deviations of design variables so as to satisfy with design conditions like machining and/or assembling,
5. decide size tolerances within maximum deviations of design variables.

From an example study, the sensitivity coefficients are successfully obtained, and tolerances are allocated for better design of the feeding control mechanism in an industrial lock stitch sewing machine. Finally, this tolerance determination method using kinematics and sensitivity might be very useful to precision machines whose functions are much related to kinematics like sewing machines, robots, and etc. in machine designs.

References

1. Hartenberg R. S., and Denavit J., "Kinematic Synthesis of Linkages," McGraw-Hill, New York, pp. 315-320, 1964.
2. L. F. Knappe, "A technique for analyzing mechanism tolerances," Machine Design, Vol. 25, pp. 155-157, 1963.
3. Corderman, D. G., and Mabie, H. H.,

"Development of Design Charts for Use in Predicting the Effect of Tolerances and Clearances on Mechanical Error in Four-Bar Linkages," Presented at 12th ASME Mechanisms Conference, ASME Paper 72-MECH-7, October, 1972.
4. R. E. Garrett, and Allen S. Hall, Jr., "Effect of Tolerance and Clearance in Linkage Design," Trans. ASME J. Engng Ind., Series B, Vol. 91, pp. 198-202, 1969.
5. J. Chakraborty, "Synthesis of Mechanical Error in Linkage," Mech. Mach. Theory, Vol. 10, pp. 155-165, 1975.
6. S. G. Dhande, and J. Chakraborty, "Analysis and Synthesis of Mechanical Error in Linkage-A Stochastic Approach," Trans. ASME J. Engng Ind., Series B, Vol. 95, pp. 672-676, 1973.
7. O. M. A. Sharfi, and M. R. Smith, "A Simple Method for the Allocation of Appropriate Tolerances and Clearances in Linkage Mechanisms," Mech. Mach. Theory, Vol. 18, pp. 123-129, 1983.
8. K. L. Ting and Y. Long, "Performance Quality and Tolerance Sensitivity of Mechanisms," ASME Journal of Mechanical Design, Vol. 118, pp. 144-149, 1996.
9. P. D. Lin and J. F. Chen, "Accuracy Analysis of Planar Linkages by the Matrix Method," Mech. .Mach. Theory, Vol. 27, No. 5, pp. 507-516, 1992.
10. S. J. Lee, B. J. Gilmore, and M. M. Ogot, "Dimensional Tolerance Allocation of Stochastic Dynamic Mechanical Systems Through Performance and Sensitivity Analysis," ASME Journal of Mechanical Design, Vol. 115, pp. 392-402, 1993.