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어브레시브 워터젯에 의한 Drilling 의 3 차원 모델링 연구

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Development of 3-D Modeling for Abrasive Waterjet Drilling Process

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ABSTRACT

어브레시브 워터젯을 이용한 Drilling 시 깊이에 대한 예측은 가장 중요한 변수중의 하나다. 이 논문에서는 구멍 깊이의 예측 및 구멍 형상을 연구하기 위하여 3 차원 해석 모델이 제안되었다. 해석 모델은 크게 두 가지로 구성되었다. 하나는 비선형 반복 방정식에서 생성된 입자의 운동식이며, 다른 하나는 수많은 입자에 의한 충돌시 가공능력을 규정지우는 Constitutive Equation 으로 구성되었다. 이 모델은 구멍 가공이 진행됨에 따라 발생하는 감쇠 효과를 고려하였다.실험적인 고찰이 해석모델의 유용성을 검증하기 위하여 이루어졌으며, 근사한 결과를 보였다.

Key Words : Depth of drilling(드릴 깊이), Drilled hole shape(드릴 구멍 형상), Julia set(쥴리아 셋), Constitutive equation, Damping effect(감쇠 효과), Erosion(침식), 3-D drilling model(3 차원 드릴 모델)

1. INTRODUCTION

The introduction of the abrasive waterjet(AWJ) cutting as a machining method has opened a new way of machining difficult-to-machine materials. Originally, the AWJ machining technique was used only for linear cutting and shape cutting of materials such as titanium, super alloy, glass, composites and advanced ceramics. However, today this technology is used in such machining applications as turning, rilling of small diameter holes and milling. To produce a hole of controlled depth is a basic problem in AWJ machining. The depth of the AWJ penetration is a complex function of a number of factors, including pressure, flow rate, grain size/material, angle of impact, etc.

Since AWJ machining is generally understood to be rapid erosion process, investigation of the erosion mechanisms becomes the key to understand this machining process. Numerous studies on this subject have been conducted by researchers from many disciplines including physics, material science, mechanics, manufacturing, etc.

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The earliest study in relation to erosion by dry abrasive particles was conducted by Finnie for both ductile and brittle materials. Finnie[1] investigated the amount of surface material eroded by solid particles. He also discussed fluid flow conditions and analyzed the mechanisms of material removal for ductile and brittle materials. It was found that the influence of particle velocity and angle in the erosion of a brittle material was quite different from that of a ductile material. Bitter [2, 3] derived erosion equations as a function of mass and velocity of the impinging particles, impingement angle, and mechanical and physical properties, both of erosive particles and eroded body through a two paper sequence. Tilly et al.[4] investigated the influence of impinging velocity for target materials of diverse metals, plastics and ceramics. Erosion was related to the simple power law with 2.0 to 2.3 velocity exponent for these materials. Hutchings[5] has studied the impact in order to investigate the type of impact which occurs in cases of erosion by solid particles. Another model for erosion was suggested by Sheldon and Kanhere[6]. Their derivation consists of an energy balance between the kinetic energy of the particle and work based on indentation theory, which predicted a velocity exponent of three. Finnie et al.[7] developed the mechanisms which have been proposed for the erosion of ductile metals by solid particles. It was concluded that a ductile cutting mode applies when the velocity of the eroding particle makes an angle of less than about 45° with the target surface. Hashish[8] developed analytical model for determining the depth of cut for cutting and deformation wear based on Finnie's erosion model.

All the models discussed above dealt with ways to ascertain the erosion rates of a single abrasive particle. In order to apply them to predict abrasive erosive wear during AWJ machining processes, they have to be extended to the case of multiple impacts by three phase jet flow and wear aspects of the problems. Capello and Gropetti[9] developed a semi-empirical model based on energetic principle to predict erosion during AWJ machining. Although the model could predict the transient geometry of the cutting front, it utilized different parametric constants for different process conditions and hence was not a generic one, despite using a large number of constants.

Hashish[10] derived material removal rate equation for erosion at shallow angles. The damping effect due to the jet back flow was considered in this equation. However, no experimental work has been done till date on the verification of the hole drilling process. The first analytical investigation into the mechanisms of AWJ drilling process was performed by Raju and Ramulu[11]. They have reported a semi-empirical transient numerical model for prediction of the depth of AWJ drilling. The model was developed to obtain an approximate mean velocity distribution in the cavity based on principle of energy conservation. However, the experimental results were not closely machining the model predictions especially at extremely low or high drilling depths.

2. ANALYTICAL INVESTIGATION OF DRILLING PROCESS

Extensive researches in diverse disciplines[12-16] have been performed for erosive wear mechanisms. However, these studies have the limitation that they estimate the erosion rate of a single particle. In order to simulate three-dimensional abrasive waterjet drilling process, a new approach is proposed which considers erosion due to millions of particles. The proposed model will first identify the chaotic motion of abrasive particles that are involved in the AWJ drilling process and then identify the appropriate mathematical descriptions to describe that process.

Recently, fractals have attracted great attention in academic fields[17] and the concept "fractal" has already proved its use in many applied fields. The fractals are characterized by built-in self-similarity in which a figure, the motif, keeps repeating itself. The properties of this curve are extensively discussed by Mandelbrot[16].

Among the several fractal point sets, point sets generated from Julia point sets[16,18] is applied to perform kinematic and kinetic function in the simulation of AWJ machining processes.

The another important feature of this model is that erosion history of every individual particle is accurately recorded and evaluated corresponding to position variables and time instant during a three-dimensional AWJ machining process. Furthermore, the formation of a final three-dimensional shape is attributed to the erosion accumulation of all the effective particles.

After conforming a uniform abrasive particle distribution along the cross-section of the jet, constitutive equations will be derived and the memory element technique will be adopted to study the interaction of the jet with the target material.

2.1. POINT SETS

The main idea of this section is to determine the point sets which can represent the motion of abrasive particles. Each particle will be represented by a point defined by its coordinates. Subject to physical and geometrical constraints on the circular cross section of three phase particle-laden jet flow, the point sets should possess following general features:

(1) The particle behavior is disorderly.

(2) Distribution of particles is approximately axi-symmetrical after the number of particles passing through the cross section exceeds a certain value.

(3) The normal velocity of a particle varies with its position.

It was discovered[19] that a subset of a modified Julia set can be utilized to simulate the discrete particle-laden jet flow. The point sets J can be written by a complex iterative equation,

$$Z_{N+1} = \sqrt{Z_N^2 (Z_N - C_1)} + C_2 \qquad (1)$$

where $Z_N = \xi_N + \eta_N i$ is a complex variable, and C_1 and C_2 are either complex or real constants. The geometric configuration and general features motion will be discussed.

2.2. VELOCITY PROFILE OF POINT SETS

In order to model three-dimensional abrasive waterjet drilling process, it is essential to find the velocity profile of point sets, f_p along the normal direction of the cross section.

Based on the mass conservation principle, the equations for determining the velocity profile are introduced in the form of a flow rate ratio[20],

$$R_f = \frac{N_{xy}}{N_0} = \left(\int_{t_1}^{t_2} dt \iint_{S_{xy}} V_z\right) \Big/ \left(\int_{t_1}^{t_2} dt \iint_{S_0} V_z\, ds\right) \qquad (2)$$

where $V_z = v_z / v_{\max}$ $(0 < V_z \leq 1)$ is the dimensionless normal velocity ratio of a principle at the point (x, y) and instantaneous time t, v_z is the average normal velocity of particles and v_{max} is maximum jet velocity. S_{xy} is an arbitrarily shaped subarea of the cross section S_0 of the particle-laden jet flow. N_{xy} is the number of particles passing through S_{xy} in the time interval t_2-t_1 and N_0 is the number of particles passing through S_0 during t_2-t_1. The physical meaning of R_f is the ratio of local volume flow rate to the total volume flow rate.

For axisymmetric steady state flow, V_z can be expressed from Eq. (2) in the polar coordinates

$$V_z(r) = \left(\int_0^b V_z\, rdr\right) \frac{1}{r} \frac{\partial R_f}{\partial r} \qquad (3)$$

where b is the radius of the nozzle. According to Eqs.(2) and(3), a particle distribution on the cross section corresponds to a velocity field through $R_f = N_{xy} / N_0$. Once the point sets are given, the velocity is also determined in terms of R_f. It turns out that the central task for constructing a particle-laden jet flow is to find such point sets that possess the desired velocity field.

The particle distribution of point sets, f_p on the cross section is weak of the angle. Therefore, it can be treated as the representation of particle motion on the cross section of steady, axisymmetrical particle-laden jet flow. Two key parameters R_f and V_z of point sets, f_p can be determined numerically by use of equations (2) and (3). After computational investigations, it is confirmed that

$$R_{fp} = 1 - (1-r)^{\frac{11}{10}} \left(1 + \frac{11}{10} r\right) \qquad (4)$$

$$V_{zp} = (1-r)^{\frac{1}{10}} \qquad (5)$$

are accurate for the replacement of R_f and V_z of point sets, f_p when N > 5000.

The velocity profile of point sets, f_p, in general, is not consistent with the velocity profile appearing in the abrasive waterjet drilling process. To solve this problem, a mapping technique[20] has been adopted to transform point sets, f_p into new point sets with any desired velocity profile. The analytical result of Vz for one phase flow is used in order to minimize computational burden for three phase steady state flow. Therefore, for laminar flow the dimensionless velocity V_z^l is written as[21]

$$V_z^l(r) = 1 - r^2 (0 \le r \le 1) \tag{6}$$

and for turbulent flow the average velocity profile V_z^t is described by the seventh-root law[21]

$$V_z^t(r) = (1 - r)^{\frac{1}{7}} (0 \le r \le 1) \tag{7}$$

Substitution of equations (6) and (7) into (2) leads to the flow rate ratios

$$R_f^l = r^2(2 - r^2) \tag{8}$$

$$R_f^t = 1 - (1 - r)^{\frac{8}{7}}(1 + \frac{8}{7} r) \tag{9}$$

2.3. CONSTITUTIVE EQUATIONS

The relationship between the average depth of penetration caused by the impact of a single particle and other machining parameters is defined as the constitutive equation of particle-target material interaction during erosion process. Many research have been conducted in the past decades to build this type of relationship to accurately evaluate the erosion rate for machining processes. Extensive analytical and experimental investigations show that the average depth of penetration h_a on the dry surface of a ductile material can be expressed by

$$h_a = \lambda_1 V^{\lambda_2} \tag{10}$$

where V is the velocity of the impacting particle, λ_1 and λ_2 are constants relevant to target material properties, impact angle, and contact geometry of the particle and target material[1-4, 7, 22, 23].

Eq. (10) is based on metal cutting dynamics and the Hertz impact contact theory [1, 2, 24, 25] which lead to the relationship between the average material removal volume and the velocity of a particle. The constitutive equation (10) is widely accepted by the researchers in this field as a classical relation.

In deriving the relationship between the depth of penetration and relevant parameters, one critical concern is that its mathematical statement must be concise in addition to be accurate, so as to reduce computation time. The surface of a target material to be drilled is divided into a finite element network. During the erosion process, each element acts as a memory-cell to record the histories of those particles which pass through it and hit the target material. When the j-th particle trapped by the i-th cell at instant t_j strikes the surface, the constitutive equation for the average penetration depth is generalized to

$$\delta h_j = \lambda_1(h_{j-1}) [V_z(r_i, \theta_i)]^{\lambda_2(h_{j-1})} \tag{11}$$

Here h_{j-1} (j > 1) is depth of penetration by j-1 particles prior to the j-th particle in the i-th cell centered by (r_i , θ_i). It is noted that $h_j = h_{j-1} + \delta h_j$ holds.

It is important to point out that, like the constitutive equations in continuum mechanics[26], $\lambda_1(h_{j-1})$ and $\lambda_2(h_{j-1})$ in Eq. (11) are not arbitrary functions of h_{j-1} but constrained strictly by various physical principles. After analytical and experimental analyses, the suitable forms of $\lambda_1(h_{j-1})$ and $\lambda_2(h_{j-1})$ are chosen as

$$\lambda_1 = \frac{l_3}{h_{j-1}^{l_1} + l_2}, \lambda_2 = l_4 h_{j-1} + 2, (l_1 \ge 1, l_2, l_3 > 0, l_4 > -1 \tag{12}$$

where l_i (i=1-4) are experimental constants.

Substituting Eq. (12) in Eq. (11), the final form of the constitutive equation is obtained as,

$$\delta h_j = \frac{l_3}{h_{j-1}^{l_1} + l_2} [V_z(r_i, \theta_i)]^{(l_4 h_{j-1} + 2)} \tag{13}$$

This equation is consistent with the classical one when $l_1 = l_4 = 0$. In Eq. (13), the physical meaning of l_3 / l_2 can be clarified by setting $j = 1$ at the center of a hole, namely, $h_1 = \delta h_1 = l_3 / l_2$ owing to $h_0 = 0$ and $V_z = 1$. It may be noted that l_1 is a parameter that reflects damping effects, and l_4 is a parameter that controls the shape of a hole and it is independent of the maximum depth of penetration.

It is discovered[19] through drilling glass, titanium and polycarbonate materials that following relations hold for an accurate analysis of drilling.

$$l_1 = 2.0, l_2 = 1.0, l_3 = 0.2 \tag{14}$$

3. RESULTS AND DISCUSSION

3.1. VERIFICATION OF ANALYTICAL MODEL

Analytical particle source of the jet flow with specific coordinates and velocity profile was developed in detail. Based on Eq. (1), the point sets, f_p, which has 7312 point sets is generated and illustrated in Fig. 1. Therefore, the points of point sets, f_p, are considered as the representation of particle source on the circular cross section of particle-laden jet flow.

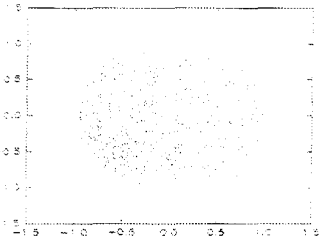


Fig. 1 Particle Distribution on the Cross Section

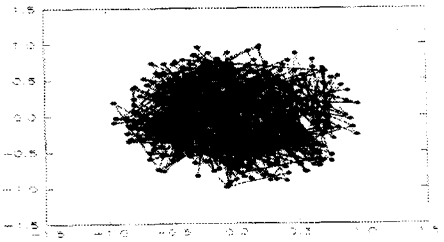


Fig. 2 Particle Distribution of Point Sets

The next step is to examine the properties of point sets. The first 300 points of the 7123 points are plotted in Fig. 2 which shows the chaotic behavior of the point appearance of point sets. Particles are irregularly scattered over the unit circle domain ($0 \leq r \leq 1$). The patterns about the chaotic feature also suggest that the global particle distribution is uniform.

As described in earlier, positions of particle are known with chaotic property over the erosion region and hence each element can trap the particles passing through it. When the number of particles that the cell received reaches the designated value, constitutive Eq. (13) can be used to evaluate the erosion rate. Substitution of the equations in (6) and (7) into (13) leads to two equations, respectively.

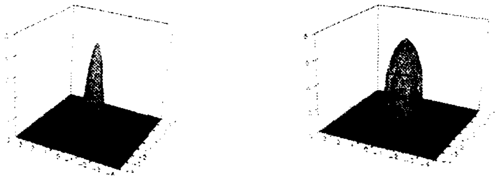
$$\delta h_j = \frac{l_3}{h_{j-1}^2 + 1}(1 - r_i^2)^{(0.2h_{j-1}+2)} \tag{15}$$

for laminar jet flow
and for turbulent jet flow.

$$\delta h_j = \frac{l_3}{h_{j-1}^2 + 1}(1 - r_i)^{\frac{1}{7}(0.2h_{j-1}+2)} \tag{16}$$

where $h_{j-1} = h_{j-1}(\ i)$ is the function of the point sets with respect to specific element. It follows from Eqs. (15) and (16) that the erosion rate decreases with increase in the depth of drilling and the shape of the hole is also changed since dimensionless velocity is associated with exponent.

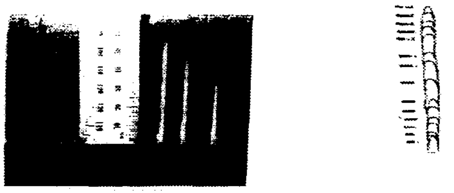
After the penetration of all the cells are found, the three-dimensional drilled hole shape was formed. The final depth of drilling is the accumulation of erosion of all the particles. A detailed computer program is written in FORTRAN to perform the necessary numerical computation as per Eqs. (2), (6), (7), (15), and (16). Two examples are illustrated in Fig. 3(a) and 3(b)



(a) (b)

Fig. 3 (a) Three-Dimensional Hole Shape by Laminar Flow (b) Three-Dimensional Hole Shpae by Turbulent Flow for three-dimensional characteristics of two holes. One is in case of laminar flow and other is in case of turbulent flow. Both the analytical and experimental results show that l_t = 2.0 (Eq.13) holds for ductile or brittle materials. Experimental shape of a hole is matched for l_4 = 0.2 (Eq.13). Obviously, the velocity profile has a high influence on the geometry of holes.

The polycarbonate with 25.4 mm x 25.4 mm x 103 mm dimension was drilled with 200 MPa pump pressure. This abrasive waterjet flow is considered as turbulent flow[27]. The erosion rate is calculated by Eq. (16). The three holes are drilled for different drilling time and are shown in Fig. 4(a). Material properties of experimental work are listed in Table 1. Process parameters of AWJ drilling for each material are given in Table 2. Similar trend was found by Hashish[28] and shown in Fig. 4(b). This figure shows traces of a waterjet/workpiece interface at different intervals (expressed in frame number). Corresponding analytically derived pattern is displayed in Fig. 5. Fig. 4(a) and Fig. 5 match very closely indicating that the model is capable of predicting the shape of the drilled hole fairly accurately.



(a) (b)

Fig. 4 (a) Shape of Drilled Holes By Experimental work (b) Hole drilling Shape with Abrasive Waterjet[28]

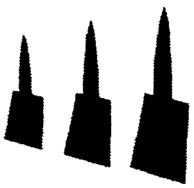


Fig. 5 Shape of Holes Produced by Analytical Turbulent Flow

The comparison between analytical and experimental analysis of the depth of drilling for different materials shows in Figs. 6(a) and (b). The model predictions are in very good agreement with the experimental work. Even though the model underpredicts at relatively low drilling time for the polycarbonate materials, these results demonstrate the accuracy of the model for predicting the depth of drilling during AWJ drilling.

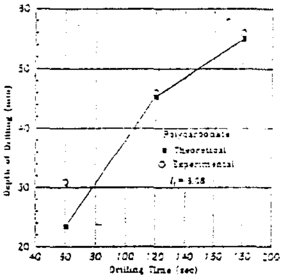


Fig. 6 (a) Comparison between Analytical and Experimental Work for Polycabonate

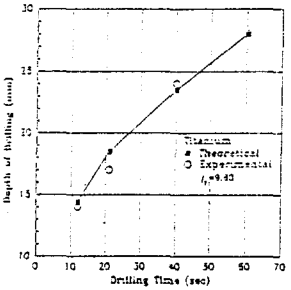


Fig. 6 (b) Comparison between Analytical and Experimental Work for Titanium

4. CONCLUSIONS

In this study, a novel approach for investigating three-dimensional AWJ drilling is proposed, which is capable of exploring new functions of this process as a non-traditional machining tool. The following conclusions can be derived from this investigation.

An erosion model has been developed for prediction of drilling shape and depth of drilling during

AWJ drilling process. The model first constructs particle-laden jet flow by fractal point sets which proves effective approach in simulating three-dimensional drilling process. Constitutive equations of the erosion as a function of other machining parameters were derived. To find such a fundamental relationship, the mathematical structure of a classical formula is assumed to be invariable. However, its parameters are generalized to be functions of the drilling depth. It should be emphasize that the shape of the drilled hole is formed by the erosive result of millions of particles with different kinetic behaviors in a small region. When a constitutive equation is developed for single particle, the erosion of model is completed when contributions of all particles are taken into account. Thus the final erosive result is the accumulation of contributions made by particles in all elements. It is proven from the comparison between analytical work and experiment that the analytical model has good accuracy.

Table 1. Used mechanical properties

Material	Density (g/cm^3)	Tensile Strength(Mpa)	Compressive Strength(Mpa)	Young's Modulus(Mpa)
Glass	2.50	N/A	N/A	69,000.0
Titanium	4.54	1010.0	1075.0	120,000.0
Polycarbonate	1.20	65.5	86.2	2,413.0

Table 2. Process parameters of each material

Constant Parameters			
AWJ Orifice Material			Sapphire
AWJ Orifice Diameter			0.46mm
Condition of Abrasive			Dry
Abrasive Particle Shape			Angular(Random)
Mixing Nozzle Diameter			1.27mm
Mixing Nozzle Length			88.9mm
Angle of Jet			90°
Method of Feed			Suction
Variable Parameters	Glass	Titanium Alloy	Polycarbonate
Abrasive Material	Aluminum Oxide	Aluminum Oxide	Garnet
Abrasive Mesh Size	100	100	120
Abrasive Flow Rate	5.4 g/s	5.4 g/s	0.6 g/s
Pump Pressure	50 Mpa	50 Mpa	50, 100, 150, 200 Mpa
Stand-off Distance	25.0 mm	25.0 mm	6.0 mm
Drilling Time	30 – 90 sec	12 – 60 sec	60 – 180 sec

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