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불확실한 부하저항을 받는 수중 운동체 구동부의 추적제어

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Tracking Control Design for Actuating Fin in Underwater Vehicle Under Uncertain Load Torques

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ABSTRACT

수중운동체가 수중에서 진행할 때 외부 파도에 의한 불확실한 부하 저항을 받으므로 이에 대응하는 핀(조타) 구동부의 제어 문제를 고려한다. 본 논문에서 제시하는 제어기는 본체로부터의 지정 각도를 부여받으면 이에 부응하여 핀의 각도와 각속도를 이용하여 제어기의 알고리즘을 구축하여 지정된 경로를 추적하게 한다. 또한 핀의 각속도 정보의 이용이 부득이 어려운 상황에 대처하기 위하여 핀의 각도만을 이용한 출력제어기나 추정기를 설계하여 주위 환경의 불확실성을 극복할 수 있는 제어기를 제안한다. DC 서보 모터로 구성된 핀 구동부에 대해 실제 데이터를 사용하여 제안된 제어기의 성능을 시뮬레이션을 통하여 검증한다.

Key Words : robust tracking control (견실추적제어), underwater vehicle (수중운동체), output control (출력제어), observer design(관측기설계), Lyapunov approach(리아프노프방법), uncertain system(불확실시스템), practical stability (실용적안정성)

Nomenclatures

K_1, K_2 : gear ratio
T_m, T_L: motor and load torque
θ_m, θ_1, θ_L: fin angles
u_c : control input
C_Q : torque coefficient
d_i, $i=1,2,\ldots$: torque coefficients
x : state variable
A, B, C: state, input, and output matrices
$g(\cdot)$: vector related to uncertain terms
x^d : desired state variable
$\tilde{x}$: tracking error of the motor angle
V : Lyapunov function
y : output
G : observer gain

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$\hat{x}$: estimated state

$K=[k_1\ k_2]$: feedback gain

$\overline{A}=A-BK$: state matrix

$\rho(\cdot)$: bounding function

ε : control parameter

1. Introduction

The direction of the underwater vehicle under the command signal from the main body is adjusted by the four fins attached to the rear part of the body. The fins are to be controlled in the presence of external disturbances. The external disturbances include the load torques exerted to the fin which possess nonlinearity and uncertainty. Recent papers[1-6] on the dynamics and control of the underwater vehicles have been reported. These papers mainly deal with the 6 DOF body dynamics and its control. However, in the presence of instantaneous changes of disturbances the fins should maintain the desired positions. Thus, the appropriate control of fins is crucial to overcome this environment. The wave torque which is uncertain and nonlinear is the main concern in this paper. Some control algorithms[5-6] are presented to tackle the uncertainty. These approaches are based on full state feedback.

In low speed heading or turning the external force or torque can be estimated by estimating the hydraulic force and moment. However, in high speed the force or torque changes severely due to the non-steady flow characteristics. Therefore, a bound for uncertainties is based on the experimental data or fluid dynamics analysis. Then, the bound can be implemented in control design. In this paper a robust control with full state feedback (angle and angular velocity) is first employed. Like the system in this paper, it is sometimes impossible to install all sensors needed. Therefore, the control design with feasible measurement is needed. To meet this issue an output control or an observer based control may be the key. Either the output control or the observer based control is needed for the case that the tachometer to measure the angular velocity can not be installed in the system. In this paper both the output control and the observer based control are addressed. The control schemes are based on the Lyapunov approach and the controls render the system practically stable[9]. Furthermore, the controls rely on the possible bound of uncertainty.

2. Uncertain system

Consider the following uncertain system

$$\begin{aligned}\dot{x}(t) &= f(x(t),\sigma(t),t)\\ &\quad+[B(x(t),t)+\Delta B(x(t),\sigma(t),t)]u(t),\end{aligned} \tag{1}$$

where $t\in R$, $x(t)\in R^n$ is the state, $u\in R^m$ is the control input, and $\sigma(t)\in R^o$ is the uncertain parameter vector. $f(\cdot), B(\cdot)$, and $\Delta B(\cdot)$ are respectively known or unknown vectors and matrices of appropriate dimensions. The functions $f: R^n\times R^o\times R\rightarrow R^n$, $B: R^n\times R\rightarrow R^{n\times m}$, and $\Delta B: R^n\times R\rightarrow R^{n\times m}$ are continuous.

Assumption 1. The (unknown) function $\sigma: R\rightarrow\Sigma\subset R^o$ is Lebesgue measurable with Σ prescribed and compact.

3. System Description

The fin system consists of a DC motor and reduction gears of spur gear type to increase the actuating torque as shown in Fig. 1. The motor receives the command signals from the main body and should follow these commands in the presence of uncertain load torque exerting on the fin. The system dynamics can be expressed as

$$T_m-K_1N_1=(I_m+I_{s_1})\,\ddot{\theta}_m, \tag{2}$$

$$N_2K_2-N_1=-(I_{s2}+I_{s_3})\,\ddot{\theta}_1, \tag{3}$$

$$N_2-T_L=(I_{s4}+I_L)\,\ddot{\theta}_L, \tag{4}$$

where N_1 and N_2 represent the reaction forces between the spur gears, I_m, $I_{si}(i=1\sim4)$, I_L represent the motor inertia, the gear inertia, and the fin inertia respectively. K_1 and K_2 represent the gear ratios. T_m is the motor torque and T_L is the load torque acting on the fin. θ_m, θ_1, and θ_L represent the angles as shown in Fig. 1.

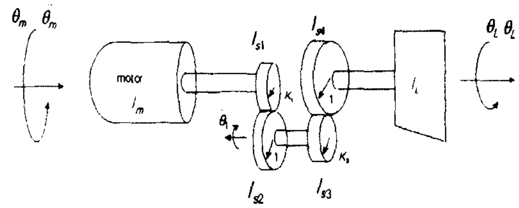


Fig. 1 Schematic diagram of actuating fin system

For the electrical part of the motor, the governing equations are written as follows.

$$L\frac{di_a}{dt}+R_c i_a+e_b=u, \quad (5)$$

$$e_b=K_e\frac{d\theta_m}{dt}, \quad (6)$$

$$T_m=K_t\ i_a, \quad (7)$$

where L and R_c are the inductance and resistance of motor armature coil, respectively. e_b is the back emf voltage which is proportional to the motor speed. K_t and K_e are the motor-torque constant and back emf constant, respectively. The following constraint equations hold

$$K_1\ \ddot{\theta}_m=\ddot{\theta}_1, \quad (8)$$

$$K_2\ddot{\theta}_1=\ddot{\theta}_L. \quad (9)$$

Arranging (3-5) and (9-10), T_m can be expressed as

$$T_m=I_{eq}\ \ddot{\theta}_m+K_1K_2T_L, \quad (10)$$

where

$$I_{eq}=I_m+I_{s1}+(I_{s2}+I_{s3})K_1^2+(I_{s4}+I_L)K_1^2\ K_2^2. \quad (11)$$

Thus, by assuming that the inductance L is negligible the dynamic equation of the actuator becomes

$$I_{eq}\ \ddot{\theta}_m+\frac{K_tK_e}{R_c}\ \dot{\theta}_m=\frac{K_tK_a}{R_c}\ u_c-K_1K_2T_L. \quad (12)$$

Note that u_c is placed before the motor amplifier, thus the control input voltage is multiplied by K_a ($u=K_a u_c$ holds). The load torque T_L is reported as a function of the fin angle, angular velocity and flow velocity with uncertain parameters such as[7]

$$T_L=C_Q(\theta_L,\dot{\theta}_L)\ (\tfrac{1}{2}\rho_f A_R U^2).] \quad (13)$$

where $C_Q(\cdot)$ denotes the torque coefficient and ρ_f denotes the flow density. Also, A_R is the fin area, and U represents the flow velocity. Note that $C_Q(\cdot)$ varies with respect to the fin displacement and its angular velocity. However, the exact value of the torque coefficient is not known. In the case that only the static load torque was considered, the governing equation of this value was reported as[7]

$$C_Q(\theta_L)=d_1\theta_L+d_2\ \theta_L{}^3, \quad (14)$$

where d_1 and d_2 are partially uncertain parameters. In this paper, the fact that the torque coefficient is not expressed explicitly as the above equation is stressed. The value may be a function of the angular velocity too. Thus, the general form of this value can be seen that

$$C_Q(\theta_L,\dot{\theta}_L)=d_1\theta_L+d_2\ \theta_L{}^2+d_3\dot{\theta}_L+d_4\ \dot{\theta}_L{}^2+d_5, \quad (15)$$

where the coefficients $(d_1\sim d_5)$ are unknown parameters but the possible bound of those is known by estimating the uncertainties or experimental work. In the later development, $C_Q(\cdot)$ in (15) is mainly utilized in control design.

4. Control Design with Full State Feedback

For the control problem to tackle the uncertainty acting on the fin due to external disturbances a robust control based on the Lyapunov approach is addressed. The control is suitable to nonlinear uncertain system and it relies on full state information. Even if the motor dynamics excluding the fin dynamics is of linear form we may not simply adopt the well known control algorithms such as H_∞ control[3] and optimal control[4] to the system taken into account. The reason is from the fact that the complex fin dynamics dominates the control system while maneuvering or turning, and the control input needs to be within the motor driving capability during actuating, so called conservativeness (occasionally occurs in H_∞ control). Here, the Lyapunov based robust control is preferred, which relies on the possible bound of uncertainty. First, a robust control with full state feedback is addressed. The other issue to exclude some state variables will be presented later.

Construct the actuator dynamics in the form of state space representation as

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & a_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ b_1 \end{bmatrix} u(t) \\ &+ \begin{bmatrix} 0 \\ c_1 T_L(x_1, x_2, \sigma) \end{bmatrix} \\ &= Ax(t) + Bu(t) + g(x_1, x_2, \sigma),\end{aligned} \tag{16}$$

where $x = [\theta_m^T \ \dot{\theta}_m^T]^T$, and a_1, b_1 and c_1 are the parameters corresponding to the motor specification. $\sigma(\cdot)$ represents the uncertain parameter. These values are expressed as

$$a_1 = -\frac{K_t K_e}{R_c I_{eq}}, \quad b_1 = \frac{K_t K_a}{R_c I_{eq}}, \quad c_1 = -\frac{K_1 K_2}{I_{eq}}. \tag{17}$$

Let the tracking error of the motor angle be $\tilde{x}$

$$\tilde{x} := x - x^d, \tag{18}$$

where x^d represents the desired motor angle. Then, the error dynamics follows from (16)

$$\begin{aligned}\dot{\tilde{x}} &= \dot{x} - \dot{x}^d \\ &= Ax + Bu + g - \dot{x}^d \\ &= A\tilde{x} + Ax^d + Bu + g - \dot{x}^d.\end{aligned} \tag{19}$$

We use a robust control u as

$$u(t) = -K\tilde{x}(t) + p(t). \tag{20}$$

Then, the error dynamics can be seen as

$$\dot{\tilde{x}} = \overline{A}\tilde{x} + Bp + h, \tag{21}$$

where

$$\begin{aligned}&h(\tilde{x}_1, \tilde{x}_2, \sigma) \\ &= \begin{bmatrix} 0 \\ a_1 x_2^d + c_1 T_L(\tilde{x}_1, \tilde{x}_2, \sigma) - \dot{x}_2^d \end{bmatrix}.\end{aligned} \tag{22}$$

Here, the gain K is chosen such that $\overline{A} = A - BK$ is Hurwitz. The uncertain term $h(\cdot)$ satisfies the matching condition[8] as

$$h(\tilde{x}_1, \tilde{x}_2, \sigma(t)) = Be(\tilde{x}_1, \tilde{x}_2, \sigma(t)). \tag{23}$$

Then, $e(\cdot)$ becomes

$$\begin{aligned}&e(\tilde{x}_1, \tilde{x}_2, \sigma(t)) \\ &= \frac{1}{b_1}(a_1 x_2^d + c_1 T_L(\tilde{x}_1, \tilde{x}_2, \sigma) - \dot{x}_2^d).\end{aligned} \tag{24}$$

Assumption 2. There exists a continuous function $\rho: R \times R \to R_+$, such that for all $(\tilde{x}_1, \tilde{x}_2) \in R \times R$

$$\max_{\sigma \in \Sigma} \|e(\cdot)\| \le \rho(\tilde{x}_1, \tilde{x}_2). \tag{25}$$

This assumption holds since $e(\cdot)$ is continuous and the parameter has its bound. Thus, the function $\rho(\cdot)$ can be a corresponding bounding function on $e(\cdot)$.

Under the Assumptions 1-2 and a constant $\varepsilon > 0$, the control is formed as

$$\begin{aligned}u(t) &= -K\tilde{x}(t) + p(t) \\ &= -\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} x_1(t) - x_1^d(t) \\ x_2(t) - x_2^d(t) \end{bmatrix} + p(t),\end{aligned} \tag{26}$$

where

$$\begin{aligned}p(t) &= -\frac{\mu(\tilde{x}, t)}{\|\mu(\tilde{x}, t)\|}\rho(x), \quad \text{if} \|\mu(\tilde{x})\| > \varepsilon \\ &= -\frac{\mu(\tilde{x}, t)}{\varepsilon}\rho(\tilde{x}), \quad \text{if} \|\mu(\tilde{x})\| \le \varepsilon,\end{aligned} \tag{27}$$

$$\mu(\cdot) = \tilde{x}^T PB \, \rho(\cdot). \tag{28}$$

Here, the positive definite P is the solution of the Lyapunov equation

$$\overline{A}^T P + P\overline{A} = -Q, \quad Q > 0. \tag{29}$$

The detail proof on control (26) is shown[8].

Remark 1. The proposed control guarantees the practical stability. Thus, the error dynamics possesses the uniform boundedness, the uniform ultimate boundedness and the uniform stability. In other words, the error can enter some bound after a certain time elapses, and remains there afterward. Also, this uniform ultimate bound size can be adjusted by a suitable choice of ε.

5. Control design with output feedback

Normally, the control design utilizes the full state information which makes the design of control algorithm easy. However, the full state feedback requires all sensors for the state variables to be installed. This causes high cost and possibility that some signals are contaminated, hence noise rejection should be taken into account in implementation. The previous work done in Section IV uses a potentiometer to detect the fin angle, and a tachometer to measure the angular velocity in real implementation. However, it is sometimes not possible to install the tachometer due to the possible noise effect and the space limitation in the system. Therefore, in this case the control design should find out other alternatives. One of these is an output control. The output control scheme which is also robust to uncertainty is introduced in this section.

For the given system we propose an output control

$$u(t) = -W\tilde{x}(t) + p_2(t), \tag{30}$$

where

$$\begin{aligned} p_2(t) &= -\gamma\ \tilde{y}(t) \\ &= -\gamma\ C\ \tilde{x}, \end{aligned} \tag{31}$$

$$W = [w_1\ \ 0]. \tag{32}$$

Here, C is the output matrix and the control parameters γ, C_1 and C_2 are determined as

$$\begin{aligned} \gamma &= \gamma_0 \\ &\geq \max_{\|\tilde{x}\|>0} \frac{\rho(\tilde{x})^2}{2(C_2 + C_1\lambda_{\min}(Q^TQ)\tilde{x}^2)} \end{aligned} \tag{33}$$

$C_1 \in (0, 1)$, $C_2 \in (0, \infty)$.

Here, the bounding $\rho(\tilde{x})$ is computed from (26). The control design employs Kalman-Yocubovitch Lemma[10] and the detail proof will be shown later. To obtain the output gain shown in (33) may be troublesome due to the complexity of (33). The feasible computation of the output control gain γ is needed beforehand if possible. This is determined according to the bounding condition of $e(\cdot)$. This will be shown later. Thus, more feasible control design can be developed.

Eventually, the system (19) is practically stable under the control (30). Furthermore, the uniform ultimate bound ball size can be arbitrary small by a suitable choice of C_2. The stability of the proposed control is proved by the Lyapunov approach[8-9]. In the following development for the proof, the dimension of $\tilde{x}$ is assigned by n. Choose a Lyapunov function candidate as

$$V = \tilde{x}^T P \tilde{x}, \tag{34}$$

where P is a positive definite matrix. The derivative of V along (20) follows from (30) and (31)

$$\begin{aligned} \dot{V} &= 2\,\tilde{x}^T P \dot{\tilde{x}} \\ &= 2\,\tilde{x}^T P\,(\overline{A}\tilde{x} + Bp + h) \\ &= \tilde{x}^T (P\overline{A} + \overline{A}^T P)\tilde{x} + 2\,\tilde{x}^T PBp \\ &\quad + 2\,\tilde{x}^T PBe \\ &= \tilde{x}^T (P\overline{A} + \overline{A}^T P)\tilde{x} + 2\,\tilde{x}^T PB(-\gamma\tilde{y}) \\ &\quad + 2\,\tilde{x}^T PBe. \end{aligned} \tag{35}$$

Define

$$H(s) = C(sI - \overline{A})^{-1}B. \tag{36}$$

Since the function $H(s)$ is SPR (strictly positive real), there exist positive definite matrices $P \in R^{n\times n}$ and $Q \in R^{n\times n}$ (Kalman-Yacubovich Lemma) such that

$$\overline{A}^T P + P\overline{A} = -Q^T Q, \tag{37}$$

$$C = B^T P. \tag{38}$$

Then, $\dot{V}$ follows from (37) and (38)

$$\begin{aligned} \dot{V} &= -\,\tilde{x}^T Q^T Q\tilde{x} - 2\gamma\,\tilde{x}^T PB(C\tilde{x}) + 2\,\tilde{x}^T PBe \\ &\leq -\lambda_{\min}(Q^TQ)\|\tilde{x}\|^2 - 2\gamma\,\tilde{x}^T PBB^T P\tilde{x} \\ &\quad + 2\|\,\tilde{x}^T PB\|\|e\| \\ &\leq -\lambda_{\min}(Q^TQ)\|\tilde{x}\|^2 - 2\gamma\|\,\tilde{x}^T PB\|^2 \\ &\quad + 2\rho(\tilde{x})\|\,\tilde{x}^T PB\| \\ &\leq -\lambda_{\min}(Q^TQ)\|\tilde{x}\|^2 + \frac{\rho(\tilde{x})^2}{2\gamma}. \end{aligned} \tag{39}$$

If we choose γ such that

$$\gamma > \frac{\rho(\tilde{x})^2}{2(C_2 + C_1\lambda_{\min}(Q^TQ)\|\tilde{x}\|^2)}. \qquad (40)$$

then, we have

$$\begin{aligned}\dot{V} &\le -\lambda_{\min}(Q^TQ)\|\tilde{x}\|^2 \\ &\quad + C_1\lambda_{\min}(Q^TQ)\|\tilde{x}\|^2 + C_2 \\ &= -(1-C_1)\lambda_{\min}(Q^TQ)\|\tilde{x}\|^2 + C_2.\end{aligned} \qquad (41)$$

If we choose $C_1 \in (0,1)$ and $C_2 \in (0,\infty)$ we conclude that $\dot{V} < 0$ for all $\|\tilde{x}\| > R$, where

$$R = \sqrt{\frac{C_2}{(1-C_1)\lambda_{\min}(Q^TQ)}}. \qquad (42)$$

Following (41) for $r > 0$, if $\|\tilde{x}\| \le r$, we can satisfy the requirements of the uniform boundedness, the uniform ultimate boundedness and the uniform stability[8].

For the brevity of the computation of the gain γ, consider special cases regarding to the bounding conditions. Firstly, in case that the uncertain term $e(\cdot)$ is bounded linearly, namely the cone-bounded

$$\|e(\tilde{x}, \sigma)\| \le m_1 + m_2\|\tilde{x}\|. \qquad (43)$$

then, we have the following value on γ.

$$\gamma = \gamma_0 \ge \frac{1}{2}\left(\frac{m_2^2}{C_1\lambda_{\min}(Q^TQ)} + \frac{m_1^2}{C_2}\right). \qquad (44)$$

Secondly, the case that the uncertainty satisfies Lipschitz condition (globally or locally) is considered.

$$\|e(\tilde{x}, \sigma)\| \le m_2\|\tilde{x}\|. \qquad (45)$$

Then, the gain γ is determined as

$$\gamma = \gamma_0 \ge \frac{m_2^2}{4C_1\lambda_{\min}(Q^TQ)}. \qquad (46)$$

For both case, the gain γ is easily computed and the value can be implemented in the output control design.

6. Observer based control design

Consider the dynamic system

$$\dot{x} = f(x, u), \qquad (47)$$
$$y = g(x, u), \qquad (48)$$

where x is the state variable and u is the control input. y is the measurable output. Since the system considered here is nonlinear, the control needs to be of a nonlinear type as follow.

$$u = \alpha(x). \qquad (49)$$

Under the control input shown in (48), the closed loop dynamics can be written as

$$\dot{x} = f(x, \gamma(x)) = F(x). \qquad (50)$$

Let the state estimation error be

$$\phi := x - \hat{x}, \qquad (51)$$

where $\hat{x}$ represents the estimated state variable. Then, the control has the following form

$$u = \alpha(\hat{x}) = \alpha(x - \phi). \qquad (52)$$

Now, we propose the estimator design as

$$\dot{\hat{x}} = f(\hat{x}, u) + G(y - g(\hat{x}, u)), \qquad (53)$$

where G is the observer gain. The method to obtain it will be shown later. Thus, the estimator error dynamics can be seen that

$$\begin{aligned}\dot{\phi} &= f(x, \alpha(\hat{x})) - f(x-\phi, \alpha(\hat{x})) \\ &\quad + G[g(x-\phi, \alpha(\hat{x})) - g(x, \alpha(\hat{x}))].\end{aligned} \qquad (54)$$

By Taylor's series all terms in (58) are arranged as follows

$$\begin{aligned}&f(x, \gamma(x-\phi)) \\ &= f(x, \alpha(x)) - \left(\frac{\partial f}{\partial \alpha}\right)\left(\frac{\partial \alpha}{\partial \phi}\right)\phi + O(\phi^2),\end{aligned} \qquad (55)$$

$$\begin{aligned}&f(x-\phi, \alpha(\hat{x})) \\ &= f(x, \alpha(\hat{x})) - \left(\frac{\partial f}{\partial x}\right)\phi + O(\phi^2),\end{aligned} \qquad (56)$$

$$\begin{aligned}&g(x-\phi, \alpha(\hat{x})) \\ &= g(x, \alpha(\hat{x})) - \left(\frac{\partial g}{\partial x}\right)\phi + O(\phi^2).\end{aligned} \qquad (57)$$

Then, the system dynamics and the error dynamics can be expressed as

$$\dot{x} = F(x) - \left(\frac{\partial f}{\partial \alpha}\right)\left(\frac{\partial \alpha}{\partial \phi}\right)\phi + O(\phi^2), \qquad (58)$$

$$\dot{\phi} = \left[\frac{\partial f}{\partial x} + G\left(\frac{\partial g}{\partial x}\right)\right]\phi + O(\phi^2). \qquad (59)$$

Here, we see that the state variable x and the estimation error ϕ are decoupled, hence if the estimation error dynamics can be locally asymptotically stable by a suitable choice of gain G, the state variable x in system (58) vanishes.

6.1 Adopting to the fin control system

As suggested in Section VI, we apply the estimator design to the fin control system. The fin dynamics model shown in (16) is also utilized here. For the tracking control system as shown in (19) the control u in (26) is employed to adopt the proposed estimator design.

We introduce the observer based on (53) as follow

$$\dot{\hat{x}} = \overline{A}\hat{x} + Bu + g(\hat{x}) + G(y - C\hat{x}). \tag{60}$$

Again, $\overline{A}$ is same with that introduced in Section IV. Then, the estimator error dynamics can be written as

$$\dot{\phi} = (\overline{A} + \frac{\partial g}{\partial \hat{x}} + GC)\phi + O(\phi^2). \tag{61}$$

Here, the observer gain G such that the eigenvalues of $(\overline{A} + \frac{\partial g}{\partial \hat{x}} + GC)$ become negative needs to be chosen. Remember that the eigenvalues of $(\overline{A} + \frac{\partial g}{\partial \hat{x}} + GC)$ depends on the state variable $\hat{x}$. The illustrated examples to obtain the appropriate observer gain G are shown later.

Experience indicates that if the observer poles are selected to be placed farther to the left in the s-plane than the desired closed loop poles a good design results. However, the placement of all observer poles far to the left in order to speed the convergence of the estimated state and the real state is not always a best strategy in case of the possible model uncertainty. Therefore, after constructing a linearized system for the system taken into consideration here, the some of the observer poles should be placed to the plant zeros and the rest of the observer poles need to be placed to the left along the classical Butterworth configuration. Also, the output matrix C is chosen such that $\overline{A}$ and C are observable.

The observer based control has the following form.

$$u(t) = -K(\hat{x} - x^d) + p(t), \tag{62}$$

where $p(t)$ as shown in (25) compensates the uncertain and nonlinear term. Furthermore, the term $p(t)$ needs to be of a function of the estimator variables $\hat{x}$ rather than the state variable x. To see how it can be changed the following procedures are introduced. Firstly, the bounding function $\rho(\hat{x})$ shown in (25) can be computed as

$$\rho(\hat{x}, t) \geq \max_{\sigma \in \Sigma} \frac{1}{b_1} \| a_1 x_2^d + c_1 T_L(\hat{x}_1, \hat{x}_2, \sigma) - \dot{x}_2^d \|. \tag{63}$$

Here, the possible bound of the load torque $T_L(\cdot)$ is used in computing the bounding function $\rho(\hat{x})$. Secondly, for the system taken into consideration determine $K = [k_1 \ k_2]$ such that $\overline{A} = A - BK$ becomes Hurwitz. If we choose an identity matrix as Q we obtain a positive definite matrix $P \in R^{2\times 2}$. Thus, the function $\mu(\hat{x})$ in control is written as

$$\begin{aligned} \mu(\hat{x}) &= (\hat{x} - x^d)^T PB\rho(\hat{x}) \\ &= \begin{bmatrix} \theta_m - \theta_m^d \\ \dot{\theta}_m - \dot{\theta}_m^d \end{bmatrix}^T P\ B\ \rho(\hat{x}). \end{aligned} \tag{64}$$

Finally, for a given $\varepsilon > 0$ the robust control $u(t)$ can be seen that

$$u(t) = -[k_1 \ k_2] \begin{bmatrix} \hat{x}_1 - x_1^d \\ \hat{x}_2 - x_2^d \end{bmatrix} + p(t), \tag{65}$$

where

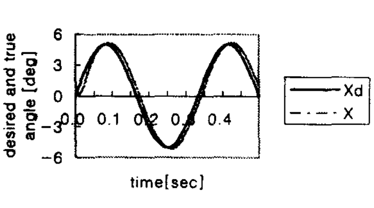
$$\begin{aligned} p(t) &= -\frac{\mu(\hat{x}, t)}{\|\mu(\hat{x}, t)\|} \rho(\hat{x}, t) \quad \text{if} \|\mu(\hat{x}, t)\| > \varepsilon \\ &= -\frac{\mu(\hat{x}, t)}{\varepsilon} \rho(\hat{x}, t) \quad \text{if} \|\mu(\hat{x}, t)\| \leq \varepsilon. \end{aligned} \tag{66}$$

As shown in (65), the control only utilizes the estimated state variable rather than the real state variable. However, for the system mentioned above θ_m which is corresponding to x_1 is already measurable. Thus, it does not need to estimate if necessary. Therefore, the control excluding $\hat{x}_1$ (i.e., reduced observer design) is more feasible. This needs a further investigation.

7. Simulations

The performance of the proposed controllers for the fin system is verified through simulations. For the system, the possible real data are adapted even though some data is not clearly known. The parameters are not given in public for the brevity of security in this article. These values are almost known. Even if these are not exact the proposed controllers tackle the unknown parts by designing robust schemes.

Based on the experimental report[7] the coefficients regarding to load torque exerting on the fin are given as $d_1 = 0.0756$ and $d_2 = -0.6710$. However, these values may not be exact but variable as the external conditions change. Therefore, other formula representing the external torque as shown in (14) are employed instead. The corresponding coefficients are also adapted. These values are roughly assigned in simulations, hence the fact that the torque is governed by this formula and coefficients is not guaranteed. However, the proposed schemes enable the system to tackle the uncertainty for the partially known structure with the possibly known bound of the uncertainty. A motor angle is chosen to track the $5\sin(2\pi * 3t)$ deg. command. Fig. 2 shows control histories of the fin system with a simple PID control. There is a pretty much steady state error. Fig. 3 shows the results of the tracking histories through the output control. Here, an output control gain $\gamma = 100$ which satisfies the condition expressed by (32) is selected. Notice that the gain can be adjusted by suitable choices of C_1, C_2 after estimating the bounding function $\rho(x)$. We choose $C_1 = 0.2$, $C_2 = 10$. It is shown that the control enhances tracking performance. Fig. 4 illustrates the tracking control performance via the estimator designed which is robust to the external disturbances.



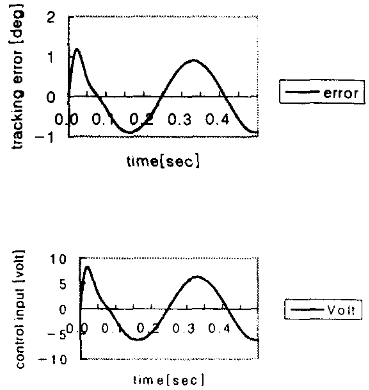


Fig. 2 Control histories of the system with a PID control

An observer gain $G = 200$ which satisfies the condition expressed by (64) is chosen. The tracking error is pretty small after a certain time elapsed. It shows that the control tackles the unknown external torque acting on the fin. The tracking errors are pretty small and the control performance is in good shape.

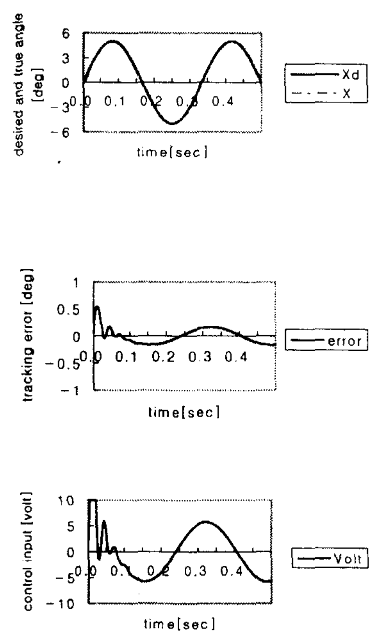


Fig. 3 Control histories of the fin system with a robust output control

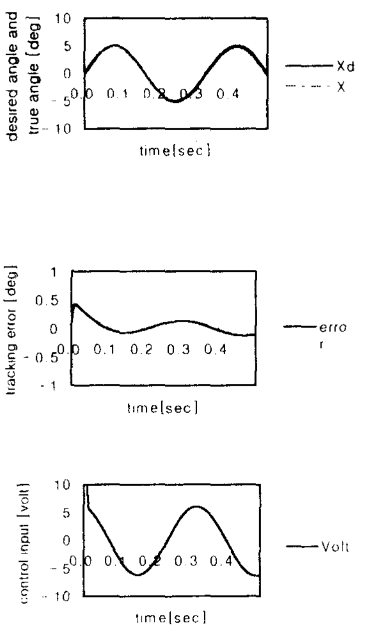


Fig. 4 Control histories of the fin system with robust control via an estimator

8. Conclusions

The classes of robust control schemes for underwater vehicle which possesses highly nonlinearity and uncertain parameters are proposed. Firstly, a robust control via full state feedback is employed. Secondly, due to the space limitation in the system an output control scheme not using a tachometer to measure the angular velocity of the fin is proposed. Under the control through a suitable choice of constants the controlled system guarantees the practical stability. Lastly, the observer based control which also excludes the necessity of using tachometer is proposed. The control also renders the system practically stable. The ultimate boundedness ball size also can be adjusted for a suitable choice of control parameters. Both of the output control and the observer based control overcome the burden to install a tachometer in the system. Since the sensor is liable to be contaminated and is troublesome in installation due to the limited space, the control scheme with no tachometer sensor is extremely recommendable.

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