Prediction of Critical Reynolds Number in Stability Curve of Liquid Jet (II)

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ABSTRACT

The prediction of the critical Reynolds number in the stability curve of liquid jet was mainly analyzed by the empirical correlations and the experimental data through the literature. The factors affecting the critical Reynolds number include Ohnesorge number, nozzle length-to-diameter ratio, ambient pressure and nozzle inlet type. The nozzle inlet type was divided into two groups according to the dependence of the critical Reynolds number on the length-to-diameter ratio of nozzle. The empirical correlations for the critical Reynolds number as a function of above factors mentioned are newly proposed.

Keywords : jet stability curve, critical Reynolds number, Ohnesorge number, nozzle length-to-diameter ratio, nozzle inlet type, ambient pressure

INTRODUCTION

This paper is the second part of a series to investigate the prediction of the critical Reynolds number in the stability curve of liquid jet. The first part of this series demonstrates the empirical correlation for the critical Reynolds number as a function of the Ohnesorge number and nozzle length-to-diameter ratio[1]. The Ohnesorge number is the dimensionless group representing the ratio of an internal viscosity force to an interfacial surface tension force($\mu/(\rho\ \sigma\ d)^{0.5}$), and is sometimes referred to as the stability number.

Most researchers have characterized liquid jet behavior by determining the relationship between jet velocity and breakup length experimentally. The change of breakup length of a liquid jet with jet velocity is called jet stability curve. It is well known from the stability curve that at low velocity after passing the dripping flow region, the breakup length increases linearly with the jet velocity. If the jet velocity is further increased, a point is reached at which the breakup length no longer increases, but reaches a maximum and then decreases. This first maximum point in the stability curve is called the critical point and is associated to the critical velocity and also the critical Reynolds number. The prediction of this critical velocity or the critical Reynolds number is, of course, of much practical interest.

The determination of the critical Reynolds number can be done both by theoretical consideration and by empirical correlations. The critical Reynolds number can be obtained from the

Weber's dispersion equation and from the modified ones by Grant and Middleman[2], Sterling and Sleicher[3], and recently Leroux et al.[4,5]. The detailed review of Weber's theory and the modified ones can be found in the literature[3,4]. The values of critical velocity provided by the above models are either underestimated or overestimated according to the experimental conditions. If the results of the calculations described above are fitted with simple analytical expressions, the need for repeated numerical calculations for each case would be eliminated. Thus main concern in this work is the determination of the critical Reynolds number through the empirical correlations. The purpose of this study is to find the factors affecting the critical Reynolds number of liquid jet through the literature and to suggest the new empirical correlations for the better prediction of the critical Reynolds number.

EMPIRICAL CORRELATIONS FOR CRITICAL REYNOLDS NUMBER

A brief review of empirical equations for the critical Reynolds number is presented. Tanasawa and Toyota[6] and Grant and Middleman[2] had suggested the empirical formula for the transition velocities from laminar to turbulent regions as a function of Ohnesorge number. On the other hand, Van de Sande and Smith[7] had proposed the empirical expression for the critical Reynolds number as a function of nozzle length-to-diameter ratio. In addition, No[8] had derived the empirical correlation between the different type of nozzle hole entrances on water jet at the atmospheric environment and the experimental data of Arai et al.[9]. A close comparison between the empirical expressions discussed above and the experimental results available in the literature allow No[8] to establish the following empirical correlations for the critical Reynolds number.

$$Re_c=218(Oh)^{-0.338}(L/d)^{0.086} \qquad (1)$$

where Re_c, Oh and L/d are the critical Reynolds number, Ohnesorge number and nozzle length-to-diameter ratio, respectively. However, the experimental data employed in Eq.(1) were limited to a liquid jet at atmospheric environment and were not included for the round-edged inlet of nozzle.

The experimental results by Leroux et al.[4,5] for a water jet under various ambient pressures indicate that the critical velocity is affected by the ambient pressures and the development of the critical velocity is not always due to the action of aerodynamic forces and may be due to the structure of liquid flow itself. From the above discussion, it can be found that the development of the critical point is related to Ohnesorge number, nozzle length-to-diameter ratio, ambient pressure and nozzle inlet type.

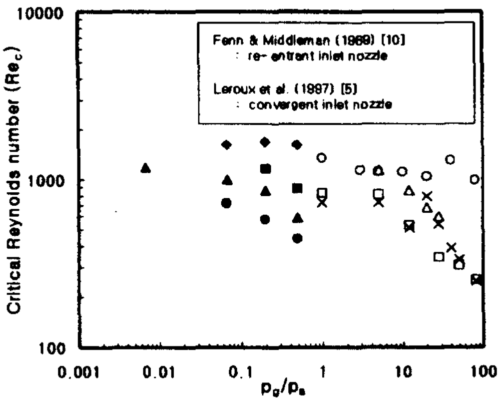

Fig.1 Influence of ambient pressure on critical Reynolds number. ; Fenn & Middleman ● L/d=52, Oh=0.372 ▲L/d=90, Oh=0.282 ■ L/d=90, Oh=0.155 ◆L/d=480, Oh=0.0386 ; Leroux et. Al. ○L/d=10, Oh=0.0048 ×L/d=100, Oh=0.025 △L/d=10, Oh=0.025 □L/d=20,

ANALYSIS OF CRITICAL REYNOLDS NUMBER

The effects of Oh and L/d on the critical Reynolds number were thoroughly discussed by No[8]. Therefore, the effect of ambient pressure and nozzle inlet type will be discussed hereafter. Figure 1 shows the effect of ambient pressure on the critical Reynolds number for the different Ohnesorge number and nozzle length-to-diameter ratio. Since the data required for the calculation of ambient Weber number were not available, the normalized pressure is adopted here. It can be found that at low Oh, the effect of ambient pressure on the Re_c is negligible. It is apparent that when the ratio of ambient pressure to atmospheric pressure(P_g/P_a) is between 1 to 5, no ambient pressure dependence is found. This is coincident with regime 1 of Leroux et al.[5]. At elevated pressure with high Oh, the critical Reynolds number decreases as the ambient pressure. It should be noted that the data from the another type of nozzle inlet are required for better empirical correlations of the critical Reynolds number.

The effect of Oh on the critical Reynolds number for the different shape of nozzle inlet is shown in Fig. 2. It is clear that the critical Reynolds number decreases linearly with Oh for all nozzle inlet type considered here. The dependence of Re_c on Oh shows the similar tendency for three inlet types of nozzle except round-edged inlet nozzle. It is clear when the comparison with same L/d=20 is made between sharp-edged and round-edged inlet nozzles.

Fig. 3 shows the influence of L/d on Re_c for the different inlet type of nozzle. It can be found that Re_c decreases linearly with L/d for convergent inlet and round-edged inlet nozzles. On the other hand, for sharp-edged inlet nozzle Re_c increases

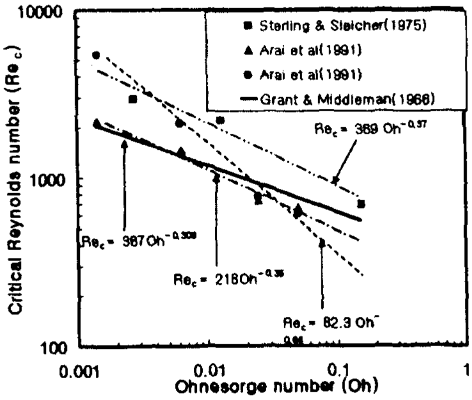

Fig .2 Effect of Oh on Re_c for different shape of nozzle inlet ; —··— convergent inlet, L/d=49, —·—· sharp-edged inlet, L/d=20, ----- round-edged inlet, L/d=20, —— re-entrant inlet

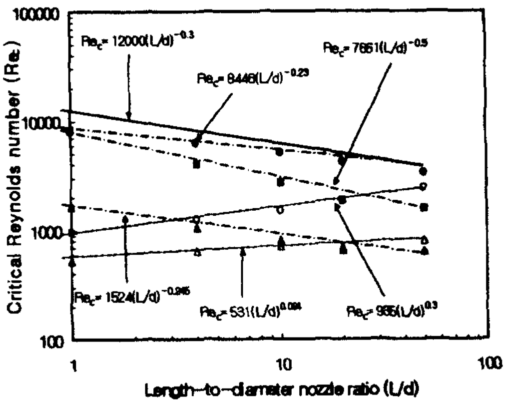

Fig. 3 Effect of L/d Re_c for different inlet type of nozzle ; ----sharp-edged inlet , ○Oh=0.001738 (Shimizu,1991), △Oh=0.049397 (Arai et.al., 1991) ; —·—· round-edged inlet ●Oh=0.001738 (Shimizu,1991) ▲Oh=0.049397 (Arai et. al., 1991) ■Oh=0.005459 (Shimzu,1991) ; —— convergent inlet (Van de Sande & Smith,1976)

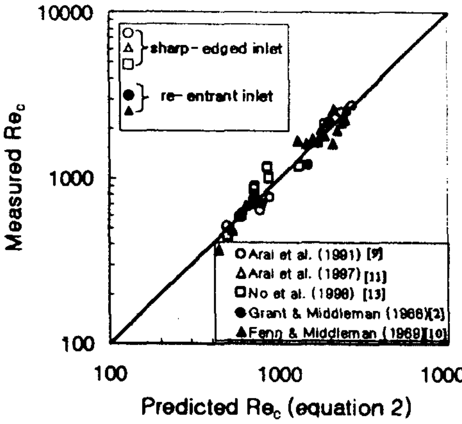

Fig .4 Comparison of measured Re_c with predicted one by Eq.(2)

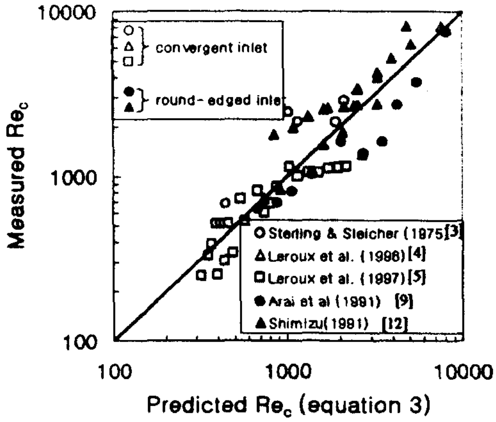

Fig .5 Comparison of measured Re_c with predicted one by Eq.(3)

linearly as L/d increases. This implies that, for sharp-edged inlet nozzle, jets produced from long nozzle which are markedly more stable than those produced from shorter nozzles. This is considered to be due to the relaxation of the velocity profile of a jet ejected under fully developed conditions. The dependence of Re_c on L/d for other inlet types of nozzles is inverted. The relaxation effect of the velocity profile due to the increase of Oh, viscosity for each inlet type of nozzle can be also found. There exist the value of L/d which represents the same Re_c between sharp-edged and round-edged inlet nozzle for the same Ohnesorge number.

Based on the discussion above, the empirical correlation for the critical Reynolds number affected by Ohnesorge number, nozzle length-to-diameter ratio, ambient pressure and nozzle inlet type can be established. To obtain an empirical correlation for Re_c, a power equation was assumed to be $Re_c = a\,(Oh)^b\,(L/d)^c\,(P_g/P_a)^d$. The effect of nozzle inlet type could not be explicitly included in the correlation because of no clear variables representing the characteristics of nozzle inlet type. The nozzle inlet type considered here divided into two groups, i.e. group I and group II due to the results of Fig.3. The nozzles of group I includes the sharp-edged and re-entrant inlet type. The round-edged and convergent inlet types are belonged to group II. The empirical correlations were obtained with

For nozzles of group I

$$Re_c = 175\,(Oh)^{-0.33}(L/d)^{0.15}(P_g/P_a)^{-0.18} \qquad (2)$$

For nozzles of group II

$$Re_c = 635(Oh)^{-0.39}(L/d)^{-0.28}\,(P_g/P_a)^{-0.19} \qquad (3)$$

It is clear from Eqs.(2) and (3) that the dependence of Re_c on Oh and ambient pressure is similar for all nozzle inlet type considered. The dependence of Re_c on L/d, however, shows the opposite tendency for two groups of nozzle. The agreements with the predicted Re_c by Eqs.(2) and (3) and the experimental data from the literature are shown in Figs. 4 and 5 respectively. Although the proposed empirical correlations can have no claim to universality, due to the lack of comparative data and failure to explicitly include effects due to nozzle inlet shape, they have reasonable agreement with the experimental data. Meanwhile, the correlation coefficients, r, between the measured and predicted Re_c for Eq.(2) and (3) are 0.96 and 0.89, respectively, by assuming two variables are random. Also, the correlation between two variables is significant with 35 and 57 degree of random for Eq.(2) and (3), respectively, at the significant level (α) of 1%.(i.e. critical value 0.418 for Eq.(2) and 0.334 for Eq.(3))[14]

CONCLUSIONS

The empirical correlation for the prediction of critical Reynolds number of liquid jet emerging from nozzle into a quisecent air was analyzed by the experimental data and expressions available in the literature. It is found that the factors affecting the critical Reynolds number include Ohnesorge number, nozzle length-to-diameter ratio, ambient pressure and nozzle inlet type. The empirical correlations were newly proposed for two groups of nozzle inlet type due to the dependence of critical Reynolds number on the nozzle length-to-diameter ratio. Although the propsed correlations can have no claim to universality, it has reasonable agreement with the experimental data. A systematic evaluation of the role played by the various factors should be conducted to present a better correlation for predicting the critical Reynolds number.

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NOMENCLATURE

d : nozzle or orifice diameter, [m]
L : nozzle length, [m]
Oh : Ohnesorge number
P : pressure, [Pa]
Re : Reynolds number
μ : dynamic viscosity, [kg/m ·s]
ρ : density, [kg/m^3]
σ : surface tension, [kg/s^2]

Subscribt

a : ambient
c : critical
g : gas

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