A Study on Stability Evaluation of the Nail-Anchor Mixed Support System

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요 지

쏘일네일 또는 지반앵커와 같은 보강재는 지하굴착 및 사면의 안정성 확보를 위해 효율적으로 적용될 수 있음이 여러 지반공학자들에게 널리 알려져 있다. 그러나 경우에 따라서는, 쏘일네일과 지반앵커를 굴착면 상하 또는 좌우에 복합적으로 적용하여 지반굴착이 진행되는 시공사례가 종종 있어 왔다. 본 연구에서는 한계평형적 접근에 근거하여, 이와같은 상하 또는 좌우 쏘일네일-지반앵커 복합 지지시스템의 전체적인 안정성을 평가할 수 있는 해석법을 제시하였다. 이 과정에서 예상 파괴흙쐐기의 형상은 $FLAC^{2D}$ 및 $FLAC^{3D}$ 프로그램 해석결과를 토대로 결정하였다. 또한 관입전단파괴에 대한 안정성 확보를 위해 요구되는 본 쏘일네일-지반앵커 복합 지지시스템의 전면부 숏크리트 벽체의 두께를 검토하였다. 아울러 쏘일네일 구조체와 지반앵커 구조체가 서로 접하게 되는 경계영역에서는 응력집중, 상대변위 및 이로 인한 전단력 유발 등 전면벽체에 대한 추가 안정성 검토가 요구된다. 이를 위해, 경계영역에서의 상대변위를 예측키 위한 간편 유한요소해석기법을 제시하였으며, 또한 충분한 안정성 확보를 위해 숏크리트 전면벽체의 두께가 과도하게 요구되는 문제점을 적절히 해결하기 위해 수정된 지압판 시스템을 제시하였다.아울러 관련 설계변수들의 영향에 대한 다양한 분석과 더불어, 예측된 상대변위를 $FLAC^{2D}$ 프로그램 해석결과와 서로 비교하여 제시된 간편 유한요소해석기법의 적용 가능성을 평가하였다.

Abstract

The benefits of utilizing internal reinforced members, such as soil nails and ground anchors, in maintaining stable excavations and slopes have been known among geotechnical engineers to be very effective. Occasionally, however, both soil nails and ground anchors are simultaneously used in one excavation site. In the present study, a method of limit equilibrium stability analysis of the excavation zone reinforced with the vertically or horizontally mixed nail-anchor system is proposed to evaluate the global safety factor with respect to a sliding failure. The postulated failure wedges are determined based on the results of the $FLAC^{2D}$ and $FLAC^{3D}$ program analyses. This study

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also deals with a determination of the required thickness of the shotcrete facing. An excessive facing thickness may be required due to both the stress concentration and the relative displacement at the interface zone between the soil nailing system and the ground anchor system. A simple finite element method of analysis is presented to estimate the corresponding relative displacement at the interface zone between two different support systems. As an efficient resolution to reduce the facing thickness, the modified bearing plate system is also proposed. Finally with various analyses related to the effects of design parameters, the predicted displacements are compared with the results of the $FLAC^{2D}$ program analyses.

Keywords : Nail-anchor mixed system, limit equilibrium stability analysis, Simple FEM analysis, Modified bearing plate system, Relative displacement

1. Introduction

Soil nailing is a method of reinforcing in-situ soils of various types by using passive inclusions to retain excavations or stabilize slopes. Occasionally, however, both soil nails and ground anchors are simultaneously used in one excavation site. A mixed type of this support system may be divided into two cases. In the first case, soil nails and ground anchors are vertically mixed mainly due to typical subsoil conditions that foundation rocks generally exist at the lower part of the excavation zone, and in the second case, soil nails and ground anchors are horizontally mixed mainly due to space limitations in the excavation site nearby the existing structures. To appropriately deal with these cases of the nail-anchor mixed systems occasionally encountered in urban excavation sites, particular considerations must necessarily be given in pursuing the reliable stability analysis and design. However, there is currently no specific rational method of the stability analysis and design procedure with respect to the nail-anchor mixed excavation support systems.

In the present study, a method of limit equilibrium stability analysis of the excavation zone reinforced with the vertically or horizontally mixed nail-anchor system is proposed to evaluate the global safety factor with respect to a sliding failure along the postulated multi-linear or wedge-shaped surface determined on the basis of the results of the $FLAC^{2D}$ and $FLAC^{3D}$ program analyses. This study also deals with a determination of the required thickness of the shotcrete facing. An excessive facing thickness may be required due to both the stress concentration and the relative displacement at the interface zone between the soil nailing system and the ground anchor system. A simple finite element method of analysis is presented to estimate the corresponding relative displacement at the interface zone between two different support systems. As an efficient resolution to reduce the facing thickness, the modified bearing plate system is also proposed. In addition, with various analyses related to the effects of the design parameters, the predicted displacements are compared with the results of the $FLAC^{2D}$ program analyses.

2. Determination of Failure Wedges

To determine the postulated two-dimensional failure wedge expected in vertically mixed nail-anchor system, 2D FDM(Finite Difference Method) analysis is carried out using the *FLAC* ^{2D} program. In the *FLAC* ^{2D} program analysis, the Mohr-Coulomb soil model is adopted and the shotcrete facing is modeled as beam elements. Also, soil nails and ground anchors are modeled as cable elements. Pertinent parameters and grid model used in 2D FDM analysis are summarized in Table 1, Table 2 and Fig. 1. Note that values of these parameters are adopted referring to data in the actual excavation site at Dogok-Dong, Gangnam-Ku, Seoul, Korea.

Table 1. Pertinent soil parameters used in 2D FDM analysis

Soil layer	Depth(m)	Cohesion (t/m^2)	Internal friction angle (°)	Unit weight (t/m^3)	Elastic modulus (t/m^3)	Poisson's ratio
Layer 1	0～-4.5m	0.5	28	1.8	517	0.35
Layer 2	-4.5～-11.0m	0.5	30	1.9	2085	0.35
Layer 3	below -11.0m	3.0	34	2.0	3846	0.3

Table 2. Pertinent reinforcement parameters used in 2D FDM analysis

	Total installation length (m)	Installation angle (°)	Drilled-hole diameter (mm)	Re-bar/Steel tendon diameter (mm)	Tensile yield strength (t/m^2)
Soil nail	10	15	100	φ25	36000
Ground anchor	15	30	150	φ12.7×4	36000

$$*S_{bond} = grouted\ \ perimeter \times \tau = \pi \cdot D_g \cdot (\sigma \tan\phi + c) \times \frac{L_b}{(S_H + L_b)}$$

$$K_{bond} = \frac{S_{bond}}{bonded\ \ length \times \dfrac{\Delta L}{bonded\ \ length}} = \frac{S_{bond}}{L_b \times \varepsilon}$$

(D_g : hole diameter, L_b : bonded length, S_H : horizontal spacing of reinforcement)

**For unbonded zone in ground anchors, $S_{bond} = K_{bond} = 1.0$

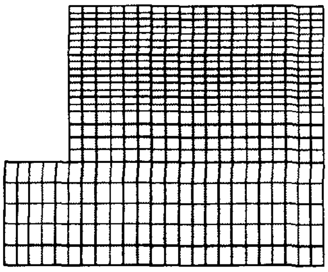


Fig 1. Grid model used in 2D FDM analysis

Contrary to the vertically mixed nail-anchor system, the postulated three-dimensional failure wedge is investigated in the case of horizontally mixed nail-anchor system. A three-dimensional explicit finite difference program called the FLAC^{3D} is used for this investigation. Unlike the FLAC^{2D} analysis, shotcrete facing is dealt with based on an elastic model in this FLAC^{3D} analysis. Pertinent soil parameters and grid model used in 3D FDM analysis are summarized in Table 3 and Fig. 2. It is noted that reinforcement parameters adopted for the case of horizontally mixed nail-anchor system are the same as those previously given in Table 2.

Table 3. Pertinent soil parameters used in 3D FDM analysis

Soil layer	Depth(m)	Cohesion (t/m^2)	Internal friction angle (°)	Unit weight (t/m^3)	Elastic modulus (t/m^2)	Poisson's ratio
Layer #1	0~-22.0m	0.5	28	1.8	20387	0.3

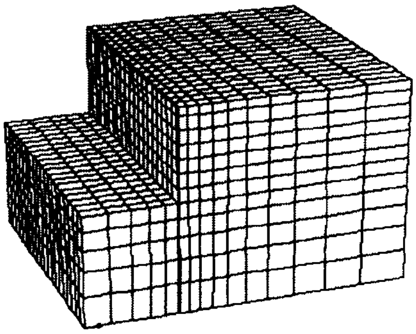


Fig 2. Grid model used in 3D FDM analysis

A typical displacement distribution obtained by 2D or 3D FDM analysis is depicted in Figs. 3 and 4, respectively. The postulated 2D or 3D failure wedge is approximately estimated by examining the shapes of displacement contours.

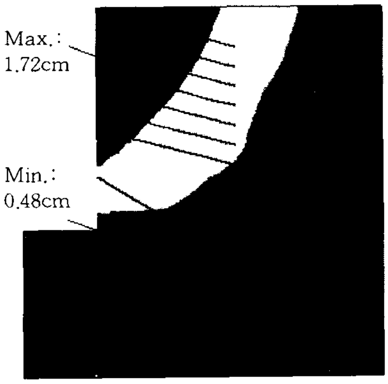
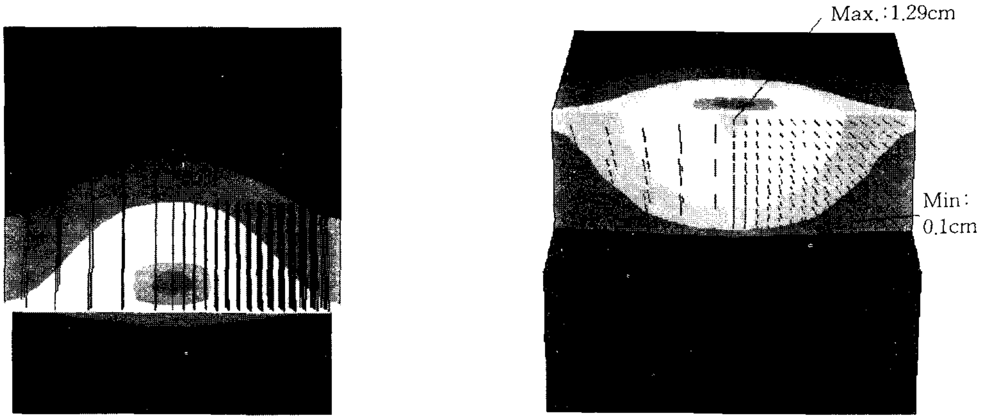


Fig 3. Results of 2D FDM analysis(vertically mixed nail-anchor system)



(a) Plane view

(b) Front view

Fig 4. Results of 3D FDM analysis(horizontally mixed nail-anchor system)

3. Limit Equilibrium Stability Analysis

3.1 Basic Approach

The postulated two-dimensional failure wedge illustrated in Fig. 5 is determined by analyzing the shape of displacement contours obtained through 2D FDM analysis.

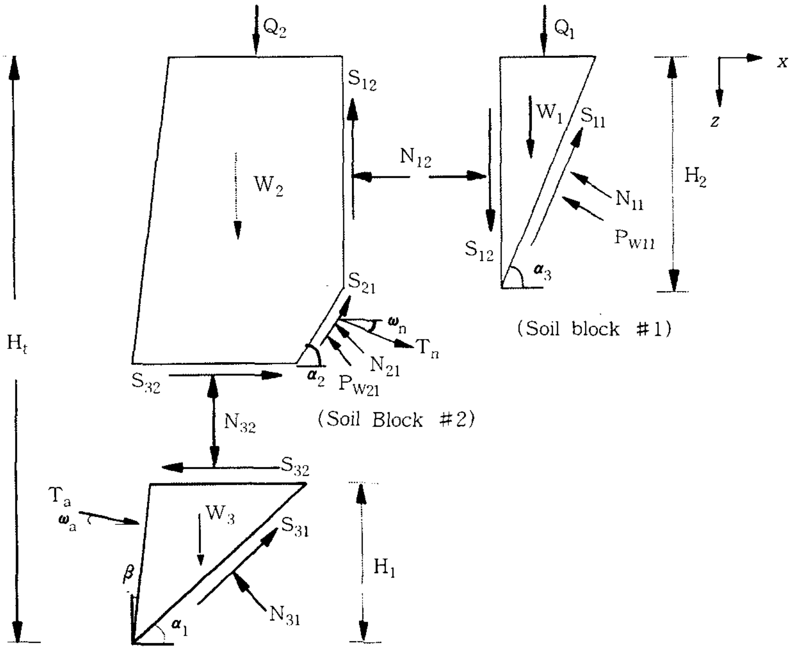


Fig 5. Forces acting on the postulated two-dimensional failure wedge

It appears that the shape of the postulated two-dimensional failure wedge for the case of the vertically mixed nail-anchor system may be reasonably assumed to consist of three planar surfaces with transitions occurring at the top soil nail and the interface between two different support systems. As illustrated in Fig. 5, the postulated two-dimensional failure wedge is divided into three soil blocks based on the 'two body translation mechanism' reported by Gassler(1988). In Fig. 5, normal forces are expressed as N and shear forces are expressed as S. Multi-layered soil conditions as well as seepage forces are also considered in the analysis to reflect various field conditions actually encountered. As indicated in Fig. 5, angles geometrically defining the postulated two-dimensional failure wedge are angle α_1 in the soil block reinforced ground anchors, angle α_2 in the soil block reinforced with soil nails, and angle α_3 in the unreinforced soil block, respectively. Note that the condition $\alpha_1 < \alpha_2 < \alpha_3$ is required when performing the analysis. The failure wedge is determined by finding a set of angles, which yields the lowest overall factor of safety.

In the case of the horizontally mixed nail-anchor system, similar basic approach is also made with the postulated three-dimensional failure wedge.

3.2 Vertically Mixed Nail-Anchor System

3.2.1 Formulation of Forces Acting on the Unreinforced Soil Block

As shown in Fig. 5, the forces acting on the unreinforced soil block #1 are determined as follows.

$$W_1 = \sum_{i=1}^{m} W_{1i}, \quad Q_1 = \frac{H_2}{\tan\alpha_3} q, \quad N_{12} = \int_o^{H_2} \sum_{i=1}^{m} n_{12i} dx, \quad S_{12} = \beta_{12} N_{12} \tag{1}$$

where, W_1 = weight of the soil block #1, q = surcharge per unit area of ground surface, Q_1 = total force caused by surcharge q, $n_{12i} = \frac{1}{2} K_{0i} \cdot \gamma_i \cdot h_i^2 + h_i \cdot (\sum_{j=1}^{i} K_{0j} \cdot \gamma_j \cdot h_{j-1} \cdot \gamma_{j-1})$, γ_i = soil unit weight of each layer, h_i = depth of each layer from the top, K_0 = coefficient of at-rest earth pressure and β_{12} = ratio between normal and tangential forces.

Based on the force equilibrium conditions in x and z directions, the corresponding normal and shear forces are determined as follows.

$$N_{11} = (W_1 + Q_1 + S_{12}) \cos \alpha_3 + N_{12} \sin \alpha_3 - P_{w3} \tag{2}$$

$$S_{11} = (W_1 + Q_1 + S_{12}) \sin \alpha_3 - N_{12} \cos \alpha_3 \tag{3}$$

Where, P_{w3} indicates seepage force possibly caused by the heavy rainfall as well as an existence of the groundwater.

To estimate the ratio β_{12}, it is assumed that the driving force expected along the postulated failure surface is equal to the resistance force developed along the same failure surface, i.e.,

$$S_{11} = N_{11}\ \tan\phi_{av}' + c_{av}' \frac{H_2}{\sin\ \alpha_3} \tag{4}$$

Where, c_{av}' = developed average soil cohesion, ϕ_{av}' = developed average soil internal friction angle. Here, the average soil properties are estimated on the basis of the method proposed by Woods(1990) as follows.

$$c_{av} = \left(\frac{1}{H_t}\right) \sum_{i=1}^{m} c_i h_i,\ \ c_{av}' = \frac{c_{av}}{FS_c},\ \ \tan\phi_{av} = \frac{\sum_{i=1}^{m} h_i^2 \cdot \gamma_i \cdot \tan\phi_i}{\sum_{i=1}^{m} h_i^2 \cdot \gamma_i},\ \ \tan\phi_{av}' = \frac{\tan\phi_{av}}{FS_\phi} \tag{5}$$

where, m = total number of soil layers, H_t = total excavation depth, FS_c = factor of safety with respect to soil cohesion, and FS_ϕ = factor of safety with respect to soil internal friction angle.

Combining with Eqs. (2) ~ (5) and solving, the ratio between the normal and tangential forces β_{12} is determined as follows.

$$\beta_{12} = \frac{B+C}{A} \tag{6}$$

Where, $A = \sin\ \alpha_3 - \cos\ \alpha_3 \cdot \tan\phi_{av}'$, $B = \{N_{12} \sin\alpha_3 + (W_1 + Q_1)\ \cos\alpha_3 - p_{w11}\}\ \tan\phi_{av}'$, and $C = c_{av}' \cdot \frac{H_2}{\sin\alpha_3} + N_{12}\ \cos\ \alpha_3 - (W_1 + Q_1)\sin\ \alpha_3$

Since the tangential force cannot be greater than the maximum shear resistance, $N_{12} \cdot \tan\phi_{av}'$, the ratio β_{12} must be smaller than $\tan\phi_{av}'$. Value of the ratio β_{12}, therefore, must be positive, i.e.,

$$0.0 \le \beta_{12} \le \tan\phi_{av}' \tag{7}$$

3.2.2 Formulation of Forces Acting on the Anchor Reinforced Soil Block #3

As shown in Fig. 5, the forces acting on the soil block #3 reinforced with ground anchors are determined as follows.

$$W_3 = \sum_{i=1}^{m} W_{3i},\ \ N_{32} = \sum_{i=1}^{m}\left(\frac{H_1}{\tan\alpha_1} - H_1 \tan\beta\right)(H_t - H_1 - h_i)\gamma_i,\ \ S_{32} = \beta_{32} N_{32} \tag{8}$$

Where, W_3 = weight of the reinforced soil block #3 and β_{32} = ratio between normal and tangential forces.

Based on the force equilibrium conditions in x and z directions, the corresponding normal and shear forces are determined as follows.

$$N_{31} = S_{32}\sin\alpha_1 + (N_{32} + W_3)\cos\alpha_1 + T_a(\cos\omega_a \sin\alpha_1 + \sin\omega_a \cos\alpha_1) \tag{9}$$

$$S_{31} = -S_{32}\cos\alpha_1 + (N_{32} + W_3)\sin\alpha_1 + T_a(\cos\omega_a \cos\alpha_1 + \sin\omega_a \sin\alpha_1) \tag{10}$$

To estimate the ratio β_{12}, it is assumed that the driving force expected along the postulated failure surface is equal to the resistance force developed along the same failure surface, i.e.,

$$S_{31} = N_{31}\tan\phi_{av}{}' + c_{av}{}' \cdot \frac{H_1}{\sin\alpha_1} \tag{11}$$

Combining with Eqs. (4) ~ (7) and solving, the ratio between the normal and tangential forces β_{12} is determined as follows.

$$\beta_{32} = \frac{B' + C'}{A'} \tag{12}$$

Where, $A' = \sin\alpha_1 \tan\phi_{av}{}' + \cos\alpha_1$, $B' = (N_{32}+W_3)\sin\alpha_1 + T_a(\cos\omega_a\cos\alpha_1 + \sin\omega_a\sin\alpha_1)$, and $C' = c_{av}{}' \cdot \frac{H_1}{\sin\alpha_1} + \{(N_{32}+W_3)\cos\alpha_1 + T_a(\cos\omega_a\sin\alpha_1 + \sin\omega_a\cos\alpha_1)\}\tan\phi_{av}{}'$

Since the tangential force cannot be greater than the maximum shear resistance, $N_{32} \cdot \tan\phi_{av}{}'$, the ratio β_{32} must be smaller than $\tan\phi_{av}{}'$. Value of the ratio β_{32}, therefore, must be positive, i.e.,

$$0.0 \le \beta_{32} \le \tan\phi_{av}{}' \tag{13}$$

3.2.3 Formulation of Forces Acting on the Nail Reinforced Soil Block #2

As shown in Fig. 5, the forces acting on the soil block #2 reinforced with soil nails are determined as follows.

$$W_2 = \sum_{i=1}^{m} W_{2i},\quad Q_2 = \{\frac{H_1}{\tan\alpha_1} + \frac{(H_t - H_1 - H_2)}{\tan\alpha_2} - H_t\tan\beta\}q \tag{14}$$

Where, W_2 = weight of the reinforced soil block #2 and Q_2 = total force caused by surcharge q.

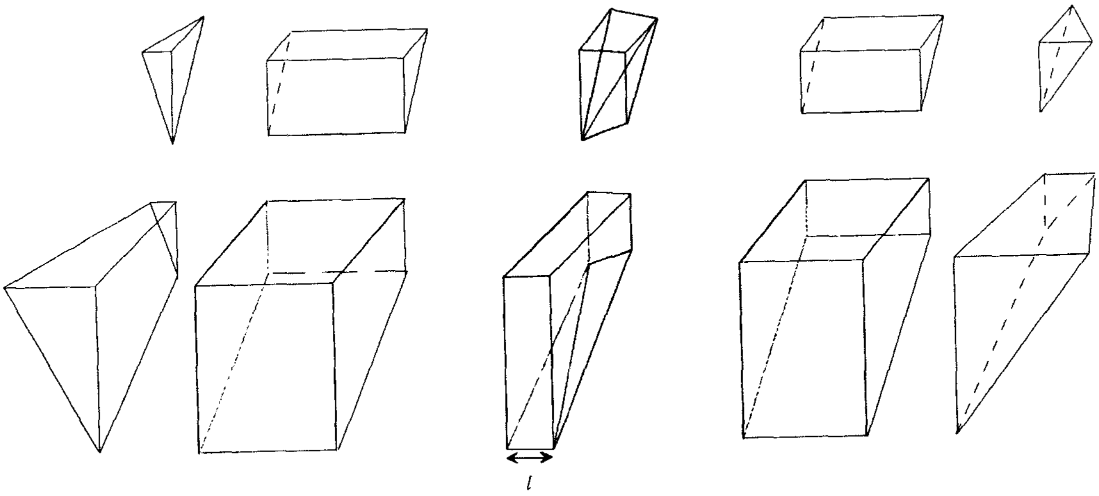
Based on the force equilibrium conditions in x and z directions, the corresponding normal and shear forces are determined as follows.

$$N_{21} = A_{21} \sin \alpha_2 + \beta_{21} \cos \alpha_2, \quad S_{21} = A_{21} \cos \alpha_2 + B_{21} \sin \alpha_2 \tag{15}$$

Where, $A_{21} = -(N_{12} + S_{32} - T_n \cos \omega_n)$ *and* $B_{21} = (W_2 + Q_2 - N_{32} - S_{12} + T_n \sin \omega_n)$

3.3 Horizontally Mixed Nail-Anchor System

By examining the shapes of displacement contours obtained from the $FLAC^{3D}$ program analysis illustrated in Fig. 4, the postulated three-dimensional failure wedge corresponding to the case of the horizontally mixed nail-anchor system is approximately estimated and is shown in Fig. 6.



(a) Ground anchor zone (b) Transition zone (c) Soil nailing zone

Fig 6. Forces acting on the postulated three-dimensional failure wedge (Soil block #1 ~ #10)

A formulation of forces acting on the three-dimensional failure wedge shown in Fig. 6 is made on the basis of the similar procedures described in Ref. 5 for the case of the irregularly shaped excavation corner with skew soil nailing system. However, detailed descriptions are omitted here due to a space limitation.

3.4 Estimation of Reinforcement Tensile Forces

The maximum soil nail tension expressed as T_n is expected to occur at the intersection point between the postulated failure surface and the corresponding nail. Value of T_n is estimated by integrating the shear stresses developed between the reinforcing soil nail and the surrounding soils, based on the mean value over the effective nail length of the normal stresses in the transformed axis which is in the plane perpendicular to the nail, i.e.,

$$T_n = \frac{\pi \cdot d_{hole} \cdot l_n \cdot \left\{ (\sigma_n - \sigma_{wr}) \cdot \tan\phi_{av}' + c_{av}' \right\}}{S_h} \leq \frac{A_{nail} \cdot f_y}{S_h} \tag{16}$$

Where, d_{hole} = drilled-hole diameter of reinforcing element, l_n = effective length of n^{th} nail, σ_n = mean value of the normal stresses, σ_{wr} = seepage pressure acting on nail, A_{nail} = cross-sectional area of reinforcing element, and f_y = tensile yield strength of nail.

The ground anchor capacity defined as a pretension force is further estimated by the following equations.

$$T_a = \frac{\pi \cdot L_b \cdot D \cdot \tau}{FS_{anchor}} \tag{17}$$

Where, T_a = ground anchor capacity, FS_{anchor} = safety factor, L_b = bond length, D = drilled-hole diameter, and τ = bond stress between cement grout and surrounding soils.

And, $T_a = \pi \cdot L_b \cdot D_s \cdot \tau_s$ (18)

Where, D_S = steel tendon diameter and τ_s = adhesion strength between cement grout and surface of steel tendon components of anchor root section.

Comparing values of T_a from Eq. (17) and Eq. (18), the smaller one is finally chosen as a pretension force that can be applied.

3.5 Evaluation of Overall Stability

Based on the equilibrium conditions, the overall stability of the mixed nail-anchor system may be analyzed. The total driving force S_D and the total developed resisting force S_F along the 2D or 3D postulated failure wedge surfaces must be equilibrium state, i.e.,

$$S_D = S_F \tag{19}$$

The overall factor of safety of the mixed nail-anchor system FS is estimated on the basis of the Taylor's criterion.

$$FS_c = FS_\phi = FS \tag{20}$$

By solving the derived equilibrium equations, the overall 2D or 3D factor of safety of the mixed nail-anchor system *FS* can be determined. An iterative solution procedure with various angles defining shapes of the postulated failure wedge is necessary to finalize the overall factor of safety.

4. Seepage Forces Acting on the Postulated Three-Dimensional Failure Wedge

The heavy rainfall as well as an existence of the groundwater may cause in-situ soils to be saturated, resulting in an unstable state of the reinforced soil mass due to a decrease of soil shear strengths and a significant increase of the pore water pressures. Expanding the Laplace's equation, Gray(1958) proposed an analytical solution method of the seepage flow. To deal appropriately with the stabilities of the vertically and horizontally mixed nail-anchor systems, equations necessary to estimate the associated seepage pressures are derived on the basis of the Gray's approach(refer to the Ref. 3). Because of a space limitation, only three-dimensional continuity equation of the seepage water corresponding to the horizontally mixed nail-anchor system is summarized below and flow conditions considered in this analysis are schematically illustrated in Fig. 7.

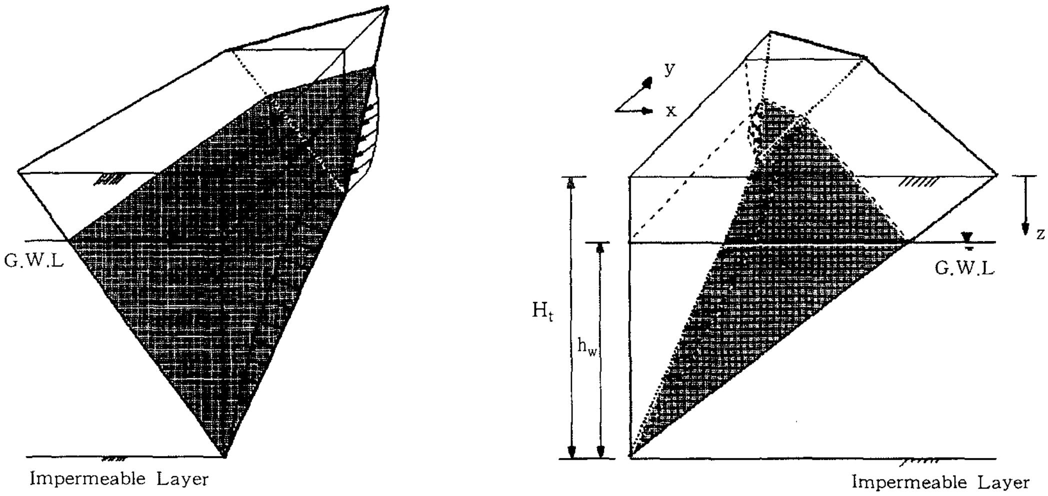


Fig 7. A schematic representation of the three-dimensional seepage flow in the horizontally mixed nail-anchor system

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0 \tag{21}$$

The general solution of Eq. (21) is expressed as follows.

$$h(x,\ y,\ z)=h_w - X(x)\cdot Y(y)\cdot Z(z) \tag{22}$$

$$where, X(x) = c_1 \cdot e^{k_1 x} + c_2 \cdot e^{-k_1 x},\ \ Y(y) = c_3 \cdot e^{k_2 Y} + c_4 \cdot \bar{e}^{k_2 y},\ \ and\ \ Z(z) = A \cdot \cos kz + B \cdot \sin kz.$$

Necessary boundary conditions to solve the Eq. (22) are described below.

$$1)\ h(\infty, y, z) = h_w,\ \ 2)\ h(x, \infty, z) = h_w,\ \ 3)\ h(x, y, 0) = h_w$$

$$4)\ \frac{\partial h}{\partial z}\Big|_{z=h_w} = 0,\ \ 5)\ h(0, y, z) = h_w - z,\ \ 6)\ h(x, 0, z) = h_w - z$$

Solving the Eq. (22) with the above boundary conditions, the particular solution is then obtained as follows.

$$\begin{aligned} h(x,y,z) = h_w \cdot \{1 - \frac{8}{\pi^2} \cdot \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)^2} \cdot \sin(Mz) \\ \cdot \exp(-M \cdot \frac{2xy}{\sqrt{x^2+y^2}})\} \end{aligned} \tag{23}$$

$$where,\ M = \frac{(2m+1) \cdot \pi}{2h_w} and\ \ m = 0 \cdot 1 \cdot 2 \cdot 3 \ldots.$$

Neglecting the velocity head, the hydraulic head h at any point along the postulated failure surfaces is simply determined by summing the fluid pressure head h_p and the elevation head h_e . Note that the fluid pressure h_p at any point is expressed as

$$h_p = h - h_e = h - (h_w - z) \tag{24}$$

Seepage pressure σ_w can then be estimated as follows.

$$\begin{aligned} \sigma_w &= \gamma_w \cdot (h - h_w + z) \\ &= \gamma_w \{z + \frac{8}{\pi^2} \cdot \sum_{m=0}^{\infty} \frac{(-1^m)}{(2m+1)^2} \cdot \sin(Mz) \cdot \exp(-M \cdot \frac{2xy}{\sqrt{x^2+y^2}})\} \end{aligned} \tag{25}$$

A sum of seepage pressures acting on the postulated failure surfaces is calculated by using the 4-point Gauss quadrature numerical technique.

5. Stability Analysis of the Shotcrete Facing Wall

Facing wall in the soil nailing system usually consists of a shotcrete, a welded wire mesh, and a square steel bearing plate. In the present study, an analytical procedure associated with a design of the shotcrete facing wall in the case of the mixed nail-anchor system is briefly described, emphasizing on an evaluation of the stability of a punching shear failure for the shotcrete facing wall in the zone reinforced with ground anchors.

Similar to a plate supported by columns, an element of the shotcrete facing wall attached at its four corners to the nails is considered as illustrated in Fig. 8

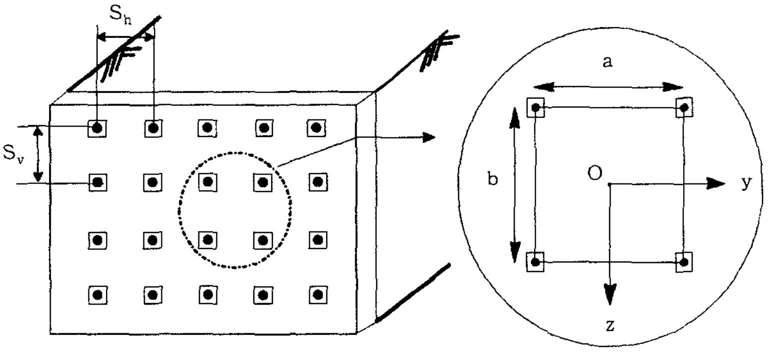


Fig 8. Details of the shotcrete facing element

It is assumed that because of the arching effect, the overburden pressure above each element is transferred to the surrounding soils and that pressure is distributed uniformly on the tributary area of each element. The bending moment and shear force induced by the soil pressures and nail forces are calculated on the basis of a plate theory with an assumption of the small deformation. The maximum horizontal displacement expected to occur at a middle point, O, of each element is also calculated on the basis of a plate theory. No consideration is given to the solutions based on the theory of membrane due to a possibility of excessive deformations. The stability calculations for the facing are then performed following the LRFD approach on the strength limit state(FHWA, 1996). In the ground anchor zone, shear strength V_{ps} of the shotcrete facing wall that can resist against the punching shear failure is evaluated on the basis of shear stresses acting across the effective area shown in Fig. 9 and the corresponding expression is given below.

$$V_{ps} = \phi \cdot 1.1\sqrt{\sigma_{ck}} \cdot \pi \cdot D_c' \cdot T \tag{26}$$

Where, ϕ = strength reduction factor, σ_{ck} = design standard compressive strength, D_c' = effective diameter, and T = thickness of shotcrete facing wall.

Derivations of the equations related to determinations of the minimum required thickness of a shotcrete facing, the necessary quantity of a wire mesh and the maximum horizontal displacement expected to occur at a middle point of each element are omitted here due to a space limitation. Detailed descriptions are given in Ref. 4.

As previously mentioned, in the cases of both the vertically and the horizontally mixed nail-anchor systems, the punching shear failure is of a particular consideration due to the anchor

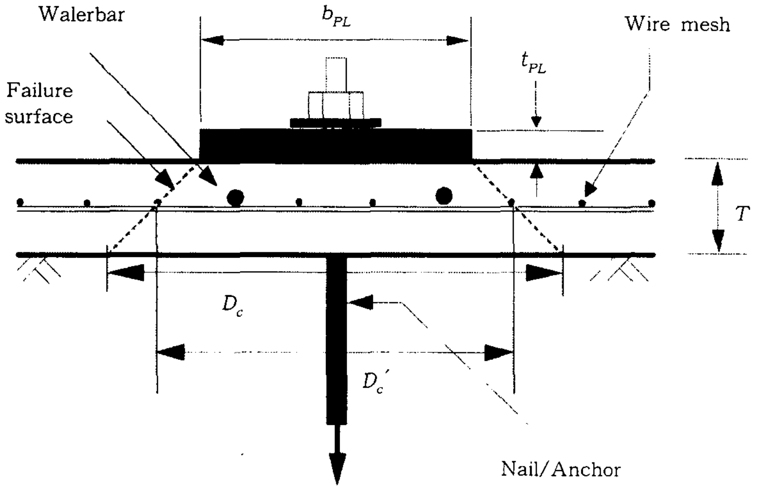


Fig 9. Schematics of the punching shear failure

pretension forces. This may result in an increase of the shotcrete facing thickness, which can cause some difficulties in constructing the facing wall. As an efficient countermeasure to solve this problem appropriately, the modified bearing plate system connected by four bridges shown in Fig. 10 is further proposed in the present study. This system focuses on a dispersion of the anchor pretension force, ensuring that the stability against the punching shear failure is fully satisfied. Then, Eq. (26) applied to a general bearing plate schematically shown in Fig. 10 may be altered to Eq. (27) expressed below to deal with the modified bearing plate system.

$$V_{ps}' = \phi \cdot 1.1\sqrt{\sigma_{ck}} \cdot \pi \cdot D_c' \cdot T \cdot 4 \quad (27)$$

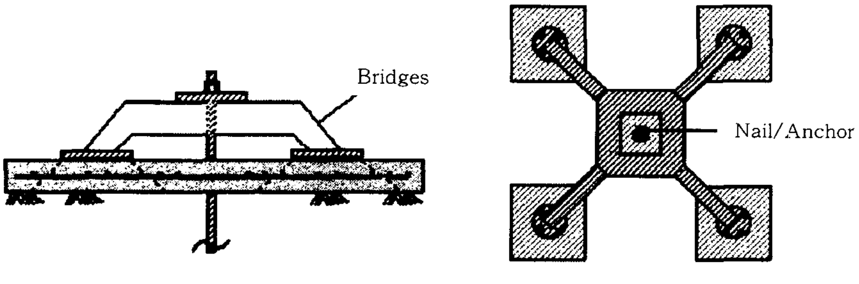


Fig 10. Schematics of the modified bearing plate system

6. Simple Finite Element Method of Analysis

Stress concentration may possibly occur at the interface zone between the soil nailing system and the ground anchor system due to typical behavior characteristics of the mixed nail-anchor system. Therefore, an additional consideration against local shear failure of the shotcrete facing due to the relative displacement is required to fully ensure the stability of the mixed nail-anchor system. To predict relative displacement a simple finite element method of analysis is formulated and the procedure is briefly described below.

In-situ ground is assumed to be an elastic continuum. In-situ ground reinforced with soil nails is also dealt with based on the composite unit cell concept. As schematically shown in Fig. 11, in-situ ground is modeled as a composition of 4-node quadrilateral elements.

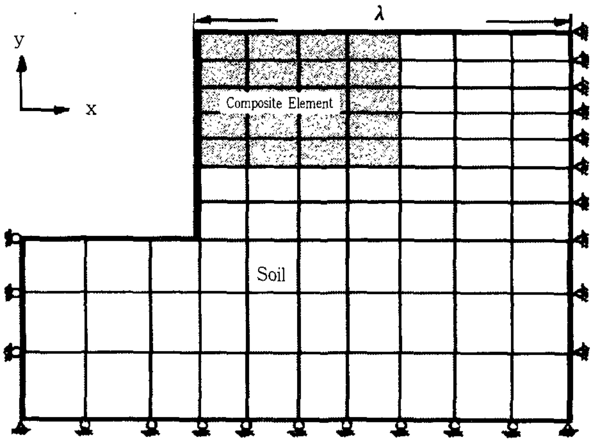


Fig 11. Simple FEM analysis model

A minimum horizontal distance λ(refer to Fig. 11) of the simple finite element method of analysis zone may also be defined on the basis of the length beyond which deformations are clearly insignificant. Empirical method to determine such a minimum horizontal distance λ is described in French National Research Project Clouterre(1991) and the proposed expression is as follows.

$$\lambda = H \cdot (1 - \tan\beta) \cdot x \tag{28}$$

Where, H = total excavation depth, β = wall inclination angle, and $\varkappa$ = constant value representing the condition of foundation soils(in general, $\varkappa = 0.8 \sim 1.5$).

Assuming that the in-situ ground is under plane strain condition, the stiffness of reinforced soil zone with closely spaced nails is modeled on the basis of a composite unit cell concept. The volume ratio η associated with a composite unit cell concept is then evaluated as follows.

$$\eta = V_r \cdot E_r / V_c \cdot E_s \qquad (29)$$

Where, V_γ = volume of reinforcement, E_γ = elastic modulus of reinforcement, V_C = volume of composite materials, and E_S = elastic modulus of soils.

Simply assembling the element stiffness matrices formed by using composite properties, the over-all stiffness matrix may then be set up. The shotcrete facing is also considered as a continuum connected by elastic beam elements(Vaziri, 1996).

The external node force consists of the pretension force due to the ground anchor, the body force and the earth pressure. The expression of body force R_B is described as follows.

$$R_B = \sum_{i,j} \cdot \ \alpha_{i,j} \cdot t_{i,j} \cdot H_{i,j}^T \cdot f_{i,j}^B \cdot \det J_{i,j} \qquad (30)$$

Where, $\alpha_{i,j}$ = associated weight in the Gaussian quadrature method, $t_{i,j}$ = thickness of element, $f_{i,j}^B = \begin{bmatrix} 0 \\ \gamma \end{bmatrix}$, $\det J_{i,j}$ = determinant of the Jacobian matrix, and

$$H_{ij} = \frac{1}{4}\left[\begin{matrix} (1-r_i)(1-s_j) & 0 & (1+r_i)(1-s_j) & 0 \\ 0 & (1-r_i)(1-s_j) & 0 & (1+r_i)(1-s_j) \\ (1-r_i)(1-s_j) & (1-r_i)(1-s_j) & (1+r_i)(1-s_j) & (1+r_i)(1-s_j) \end{matrix}\right.$$

$$\left.\begin{matrix} (1+r_i)(1+s_j) & 0 & (1-r_i)(1+s_j) & 0 \\ 0 & (1+r_i)(1+s_j) & 0 & (1-r_i)(1+s_j) \\ (1+r_i)(1+s_j) & (1+r_i)(1+s_j) & (1-r_i)(1+s_j) & (1-r_i)(1+s_j) \end{matrix}\right]$$

For the full sets of nodes, and using matrix notation, the fundamental expression in the present simple finite element method of analysis becomes

$$[P] = [K] \cdot [\delta] \qquad (31)$$

Where, [P] = external force acting on each node, [K] = over-all stiffness matrix, and $[\delta]$ = displacement of each node.

7. Additional Stability Analysis of the Facing Wall due to Relative Displacement

As previously mentioned, additional stability against local shear failure of the shotcrete facing due to the relative displacement at the interface zone between the soil nailing system and the ground anchor system must be fully ensured. The associated procedure is summarized below.

Shear force V_{re} expected to induce at the shotcrete facing in the interface zone is estimated as follows.

$$V_{re} = \frac{12EI}{l^3}\Delta \tag{32}$$

where, EI = bending stiffness, Δ = relative displacement, and l = interface zone(transition zone) distance between the soil nailing system and the ground anchor system.

Also maximum shear force V_{SF} that can resist by the shotcrete facing with predetermined thickness is evaluated as follows.

$$V_{SF} = \phi \cdot 0.53\sqrt{\sigma_{ck}} \cdot B \cdot T \tag{33}$$

Where, ϕ = strength reduction factor; B = width of shotcrete facing, and T = facing thickness.

An iterative procedure with successive comparisons of V_{re} and V_{SF} is necessary to finalize the thickness of the shotcrete facing satisfying the following condition.

$$V_{SF} \geq V_{re} \tag{34}$$

8. Parametric Analyses

By using the proposed 2D or 3D limit equilibrium method of stability analysis, the effects of geometric angles defining the failure wedge, reinforcement types and seepage forces on the stability of the excavation zones are examined. For each type of reinforcements, parametric analyses are also performed to investigate the effects and significances of various pertinent parameters such as nail length and nail installation angle. The pretension force assumed to apply to each ground anchor is 35ton.

8.1 Vertically Mixed Nail-Anchor System

Properties of the multi-layered soils and the reinforcements used in this parametric analysis are described in Table 1 and 2, respectively. A schematic sectional representation of the vertically mixed nail-anchor system analyzed in this parametric study is also given in Fig. 12.

For the unreinforced case of the excavation zone(refer to Fig. 12), a relationship between the factor of safety FS and the angle $\alpha_u(\alpha_u = \alpha_1 = \alpha_2 = \alpha_3)$ geometrically defining the two-dimensional failure wedge is analyzed and the result is described in Fig. 13. From the result of Fig. 13, the minimum factor of safety FS_{min} is estimated as 0.662, and the geometric angle α_u defining the failure wedge corresponding to this state of stability yields approximately as 72 °.

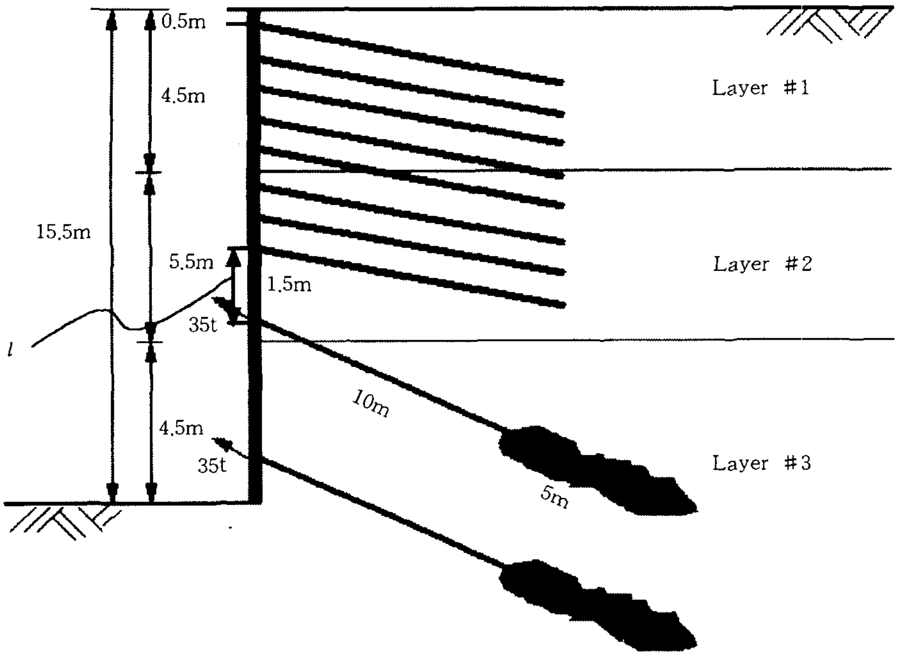


Fig 12. A schematic sectional representation of the vertically mixed nail-anchor system

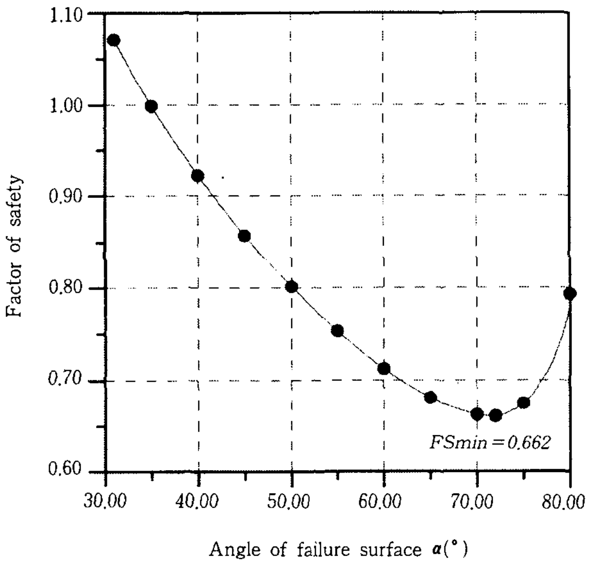
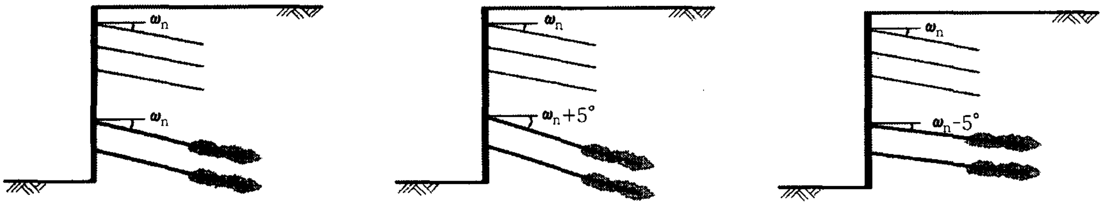


Fig 13. Safety factors with corresponding angles geometrically defining the unreinforced failure wedge

According to the previous analysis of the unreinforced excavation zone, the minimum safety factor $FS_{\min}$ is 0.662 and then, reinforcements are necessary to secure the stability.

By using the proposed 2D limit equilibrium-based method of stability analysis, safety factors of each of three different types of the vertically mixed nail-anchor system denoted as Case 1, Case 2 and Case 3 in Fig. 14 are evaluated by varying the nail length L and the nail installation angle ω_n.



(a) *Case 1* (b) *Case 2* (c) *Case 3*

Fig 14. Three different types of the vertically mixed nail-anchor system

By examining the evaluated safety factors illustrated in Figs. 15 and 16, it is observed that *Case 3* is most effective among three types of the vertically mixed nail-anchor system, whereas *Case 2* yields the lowest safety factors as compared to both *Cases 1 and 2*. By synthetically analyzing the results in Figs. 15 and 16, it is further observed that the range of nail installation angle efficient in stability improvement lies in general between 5° ~ 7° for all the three different types. It is also analyzed from Figs. 16 and 17 that the minimum safety factors FS_{min} decrease by about 5 ~ 10% due to seepage effects.

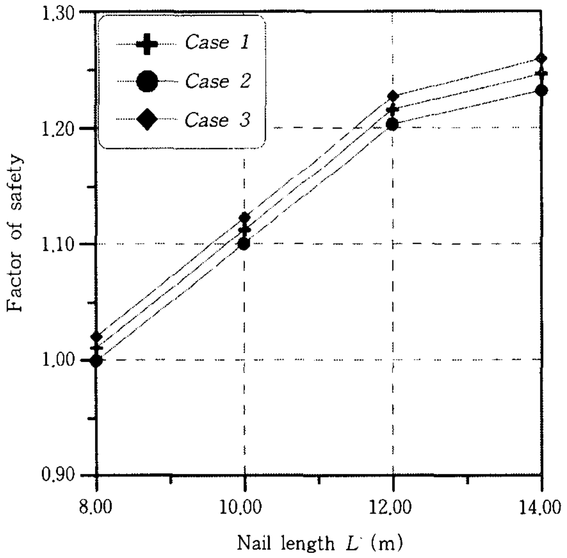


Fig 15. Safety factors with various values of the nail length

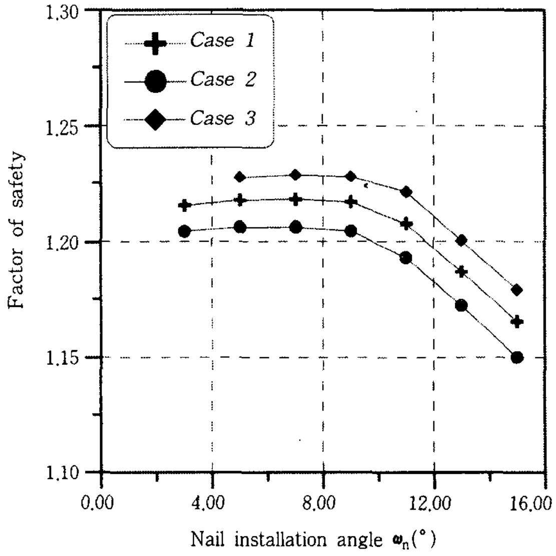


Fig 16. Safety factors with various values of the nail installation angle

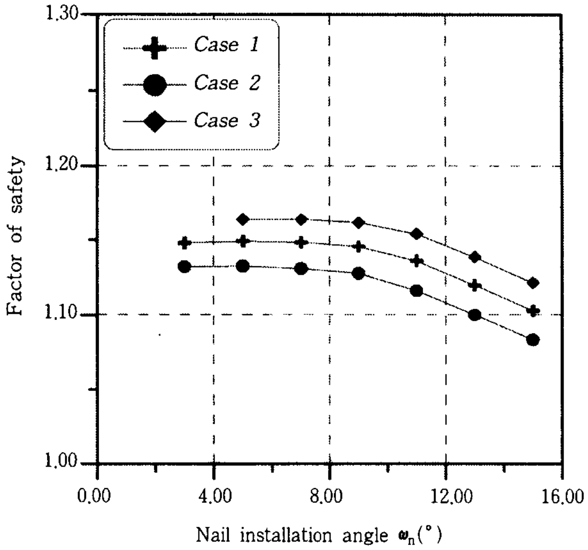
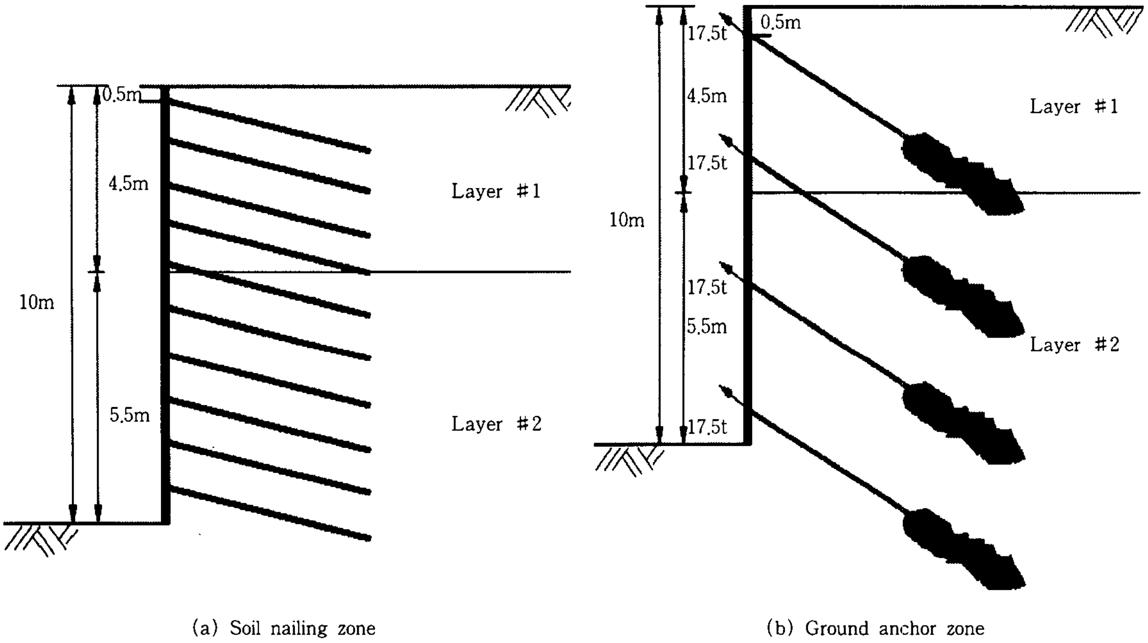


Fig 17. Safety factors with various values of the nail installation angle(seepage case)

8.2 Horizontally Mixed Nail-Anchor System

Properties of the multi-layered soils and the reinforcements used in this parametric analysis are the same as those given in Tables 1 and 2. A schematic sectional representation of the horizontally mixed nail-anchor system analyzed is also given in Fig. 18. Also the transition zone distance *l* (refer to Figs. 6 and 12) is assumed to be 1.0m.



(a) Soil nailing zone

(b) Ground anchor zone

Fig 18. A schematic sectional representation of the vertically mixed nail-anchor system

For the unreinforced case of the excavation zone(refer to Fig. 18), a relationship between the factor of safety *FS* and the angle $\alpha_u(\alpha_u = \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4)$ geometrically defining the three-dimensional failure wedge is analyzed and the result is described in Fig. 19. From the result of Fig. 19, the minimum factor of safety $FS_{\min}$ is estimated as 0.621, and the geometric angle defining the failure wedge corresponding to this state of stability yields approximately as 75°.

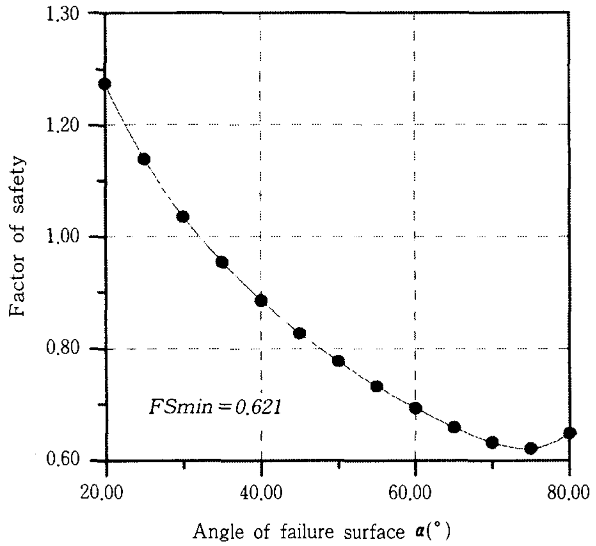
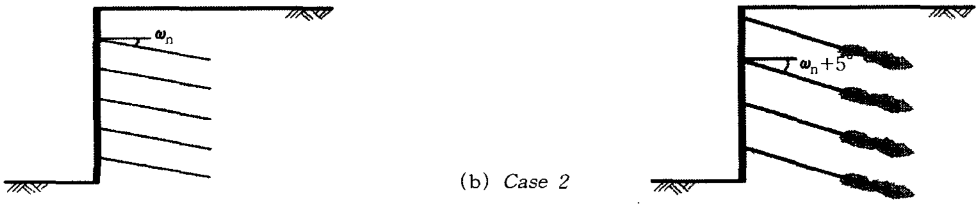


Fig 19. Safety factors with corresponding angles geometrically defining the unreinforced failure wedge

According to the previous analysis of the unreinforced excavation zone, the minimum safety factor $FS_{\min}$ is 0.621 and then, reinforcements are necessary to secure the stability.

By using the proposed 3D limit equilibrium-based method of stability analysis, safety factors of

(a) Case 1



(b) Case 2

(c) *Case 3*

Fig 20. Three different types of the horizontally mixed nail-anchor system

each of three different types of the horizontally mixed nail-anchor system denoted as Case 1, Case 2 and Case 3 in Fig. 20 are evaluated by varying the nail length L and the nail installation angle ω_n.

By examining the evaluated safety factors illustrated in Figs. 21 and 22, it is observed that *Case 3* is most effective among three types of the vertically mixed nail-anchor system, whereas *Case 2* yields the lowest safety factors as compared to both *Cases 1 and 2.* By synthetically analyzing the results in Figs. 21 and 22, it is further observed that the range of nail installation angle efficient in stability improvement lies in general between 5° ~ 9°for all the three different types.

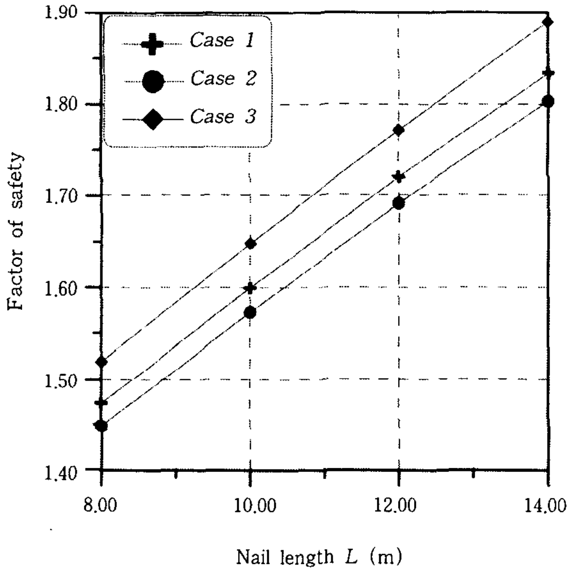


Fig 21. Safety factors with various values of the nail length

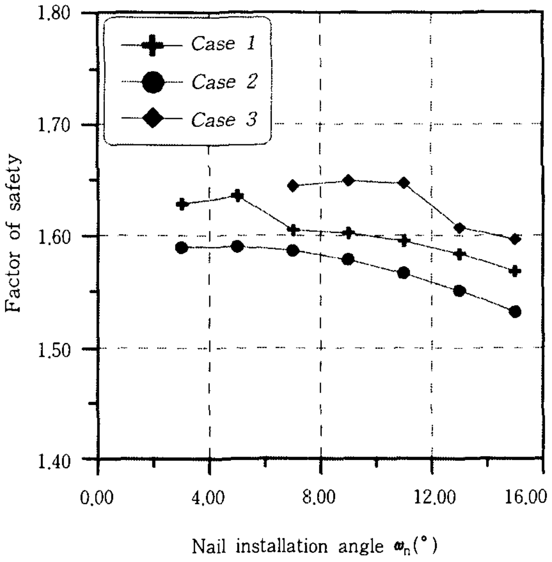


Fig 22. Safety factors with various values of the nail installation angle

8.3 Analysis of the Required Shotcrete Facing Thickness

Based on the same properties and geometric conditions described in the previous section, required thickness of the shotcrete facing for the vertically or the horizontally mixed nail-anchor system is estimated by using procedures explained in detail in chapter 5. The type of *Case 2* in both the vertically and horizontally mixed nail-anchor systems is dealt with in the present analysis. Also the anchor installation angle is assumed to be 10° and the transion zone distance *l* (refer to Figs. 6 and 12) is assumed to be 1.0m. Further, note that seepage pressures are not considered in the present analysis. Required thickness of the shotcrete facing in the ground anchor zone predominantly controls local stability of the facing against a punching shear failure. Therefore, a determination of the shotcrete facing thickness required in the ground anchor zone of a particular consideration in the present analysis. Results of the estimated facing thicknesses with both various horizontal anchor spacings and two different types of bearing plates are given in Figs. 23 and 24. A steel bearing plate denoted as general type in Figs. 23 and 24 has a dimension of 15cm x 15cm and the modified bearing plate system is previously described in Fig. 10.

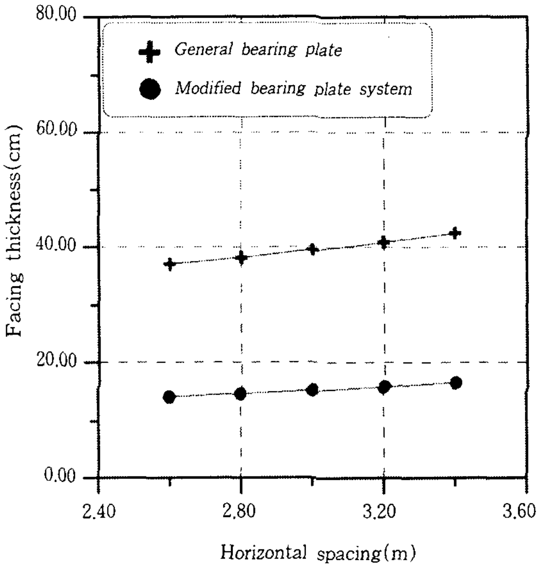


Fig 23. Required shotcrete facing thicknesses with different ground anchor spacings in horizontal direction(Vertically mixed nail-anchor system)

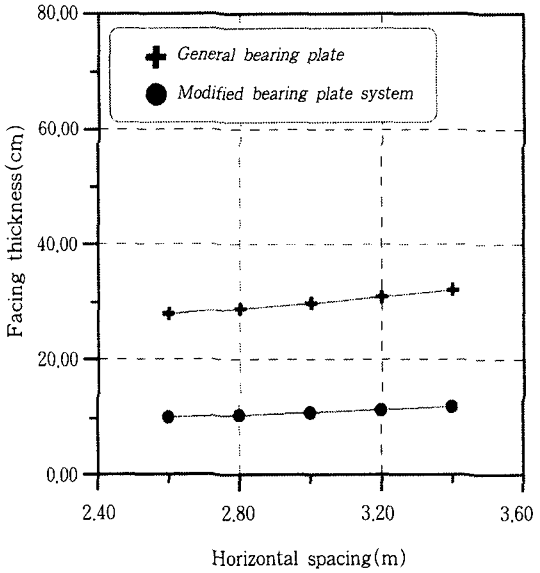


Fig 24. Required shotcrete facing thicknesses with different ground anchor spacings in horizontal direction(Horizontally mixed nail-anchor system)

Through the general analysis of the results of both cases of the vertically and horizontally mixed nail-anchor systems in Figs. 23 and 24, it is realized that the usage of the modified bearing plate system could possibly reduce the required shotcrete facing thickness at relatively large percentage(%) of 60 ~ 65 as compared to the general type of bearing plate.

9. Analysis of Deflection Behaviors Using the Proposed Simple FEM

Except the general stability analyses and a determination of the required shotcrete facing thickness of the nail-anchor mixed system previously described, reasonable predictions of the excavation wall deflections are very important issue especially to ensure local facing stability associated with relative displacements at the interface zones between the soil nail support system and the ground anchor support system. To properly analyze such deflection behaviors of the nail-anchor mixed system, truly three-dimensional analysis may have to be used. However, a three-dimensional analysis is generally considered difficult to approach mainly due to the large computer storage and enormous computation time required. As an alternative, a simple finite element method of analysis is presented in this study to predict the shotcrete facing displacements at various depths of a wall. In a simple finite element method of analysis previously presented in chapter 6, in-situ ground is assumed to be an elastic continuum. In-situ ground reinforced with soil nails is also dealt with based on the composite unit cell concept. Detailed values of relevant soil properties and cross-sectional conditions used in the present simple finite element method of analysis are the same as those given in Table 1, Table 2, Fig. 12 and Fig. 18. Also the installation angles of soil nails and ground anchors are 15° and 30°, respectively. The shotcrete facing thickness is assumed to be 10cm. Further, note that seepage pressures are not considered in the present analysis. $FLAC^{2D}$ program is further used as a tool to estimate the expected facing horizontally displacements under the same properties and geometric conditions, and the estimated results are compared with those predicted by the present simple finite element method of analysis.

By analyzing the comparisons of the facing horizontally displacements with depth shown in Figs. 25, 26 and 27, it is found that the predicted facing displacements by the present simple finite element method of analysis generally exceed those estimated by the $FLAC^{2D}$ program analyses. Detailed range of differences in magnitude of the facing displacements is 0.11~14.66mm(vertically mixed nail-anchor system, Fig. 25), 0.28~9.89mm(ground anchor system zone in the horizontally mixed nail-anchor system, Fig. 26), and 5.53~15.75mm(soil nailing system zone in the horizontally mixed nail-anchor system, Fig. 27), respectively. It can be however observed that the variations with depth of the facing displacements illustrated in Figs. 25 and 27 match well with a general trend expected in nailed soil walls. Top-down excavation process usually leads to larger displacements at the upper part of the nailed soil wall and smaller displacements in the lower part of this wall as compared to the reinforced earth wall. Fig. 26 further shows that the tendency of the variation of the facing displacements with depth is quite consistent with a general feature typically expected in the case of the ground anchor support system.

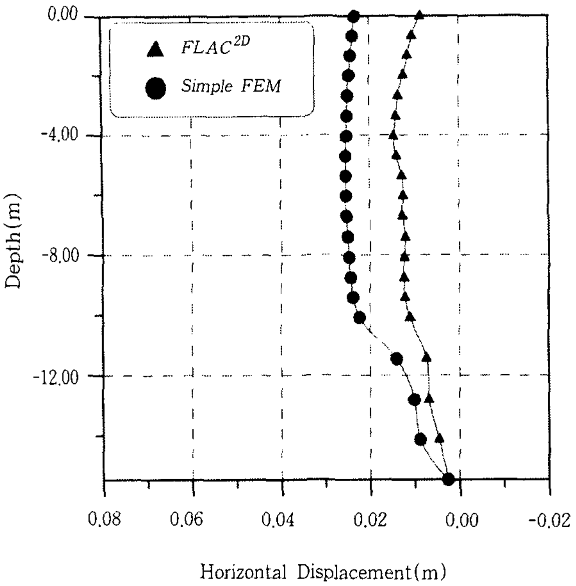


Fig 25. Facing displacements comparison(vertically mixed nail-anchor system)

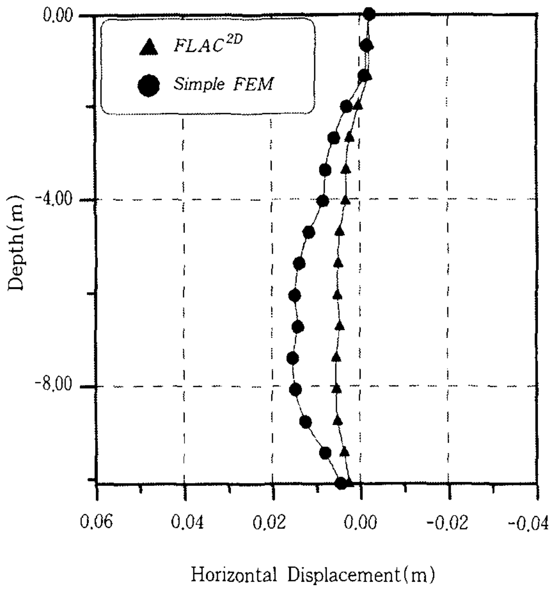


Fig 26. Facing displacements comparison(ground anchor zone in horizontally mixed nail-anchor system)

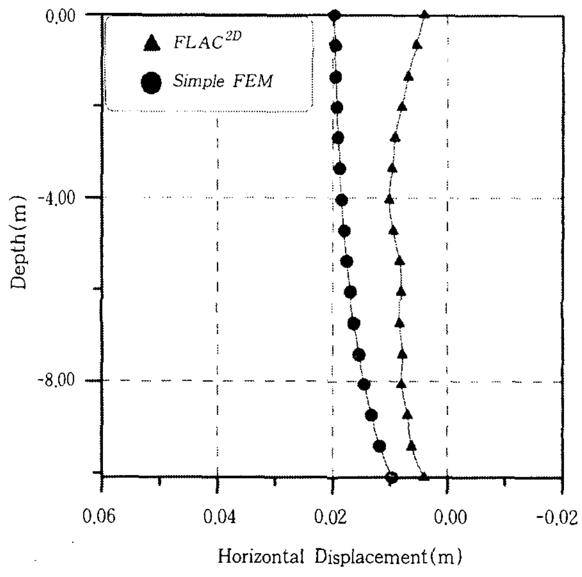
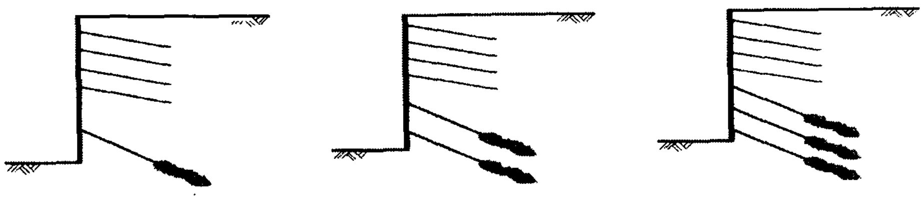


Fig 27. Facing displacements comparison(soil nailing zone in horizontally mixed nail-anchor system)

10. Analysis of the Facing Stability Associated with Relative Displacement

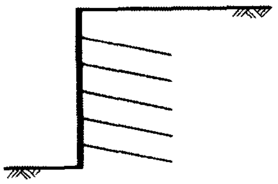
The procedure outlined in the previous chapter 5 provides an estimation of the shotcrete facing thickness required to ensure local stability against the punching shear failure. Once the required facing thickness is determined, subsequently based on the procedure briefly described in chapter 7 an evaluation of either a minimum vertical distance between the lowest nail and the uppermost ground anchor in the case of the vertically mixed nail-anchor system or a minimum horizontal distance between the soil nailing zone and the ground anchor zone in the case of the horizontally mixed nail-anchor system is carried out to ascertain the stability against the shear failure due to a relative displacement. In the case of the vertically mixed nail-anchor system, a relative displacement is defined as a difference between the horizontal facing displacements expected to occur at the lowest nail and the uppermost ground anchor. This minimum vertical or horizontal distance is called 'transition zone limit distance' hereafter. Detailed values of relevant soil properties and cross-sectional conditions used in the present analysis are basically the same as those given in Table 1, Table 2, Fig. 12 and Fig. 18. Also the installation angles of soil nails and ground anchors are 15° and 10°, respectively. Further, note that seepage pressures are not considered in the present analysis.

In addition, three different cases of the ground anchor system shown in Figs. 28 and 29 respectively, are further dealt with in the present analysis.

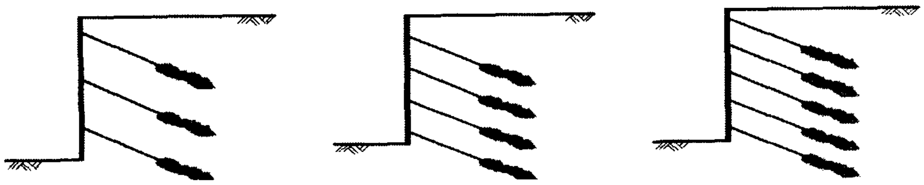


(a) *Case 1* (b) *Case 2* (c) *Case 3*

Fig 28. Cases of the vertically mixed nail-anchor system



(a) Soil nailing zone

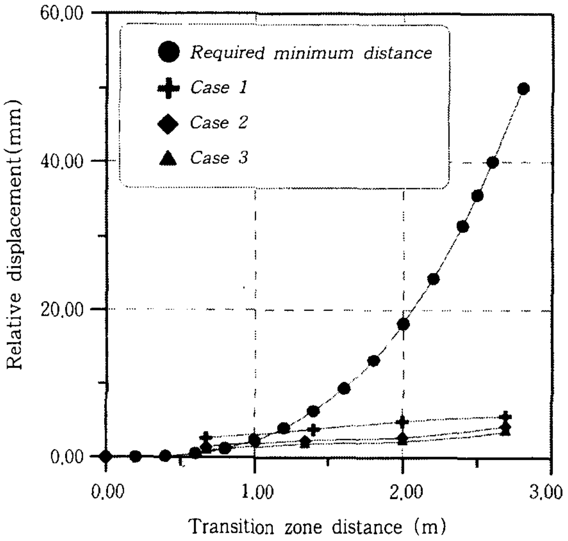


(a) *Case 1* (b) *Case 2* (c) *Case 3*

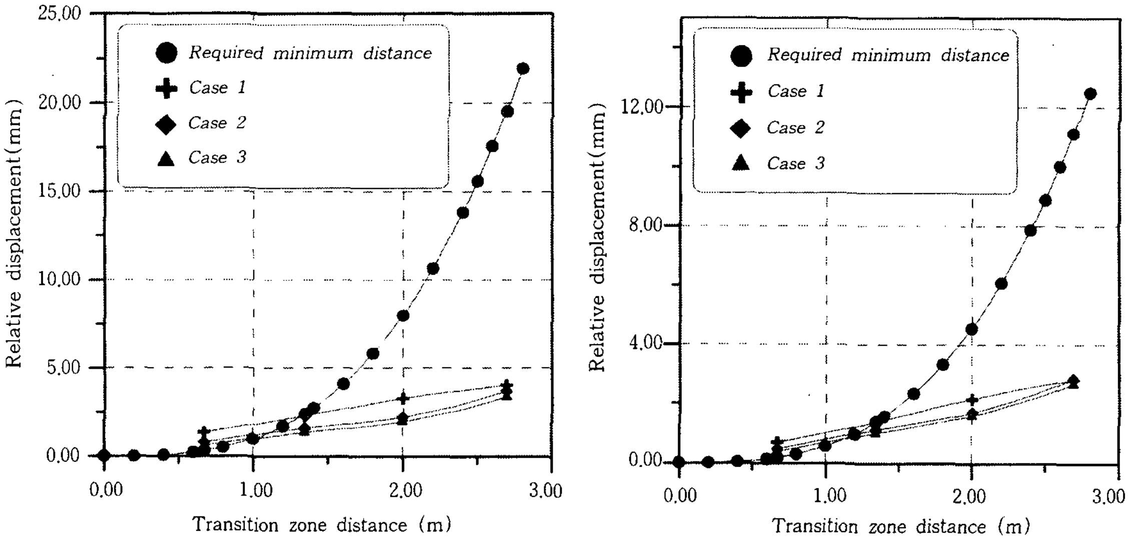
(b) Ground anchor zone

Fig 29. Cases of the horizontally mixed nail-anchor system

It is also noted that total pretension force assumed to apply to each case of the ground anchor system is limited to 70ton and is distributed equally to each ground anchor. Seepage pressures are not taken into account. For each case of the ground anchor system schematically illustrated in Figs. 28 and 29, relative displacements at various cases having different transition zone distances(0.67, 1.35, 2.0 and 2.70m) are calculated by using the proposed simple finite element method of analysis. Calculated results of each case are shown in Figs. 30 and 31, and also curved lines connected through dots in Figs. 30 and 31 represent the corresponding minimum distances previously called the 'transition zone limit distance' associated with different values of relative displacements and transition zone distances. These minimum distances are evaluated on the basis of the procedure in chapter 7.



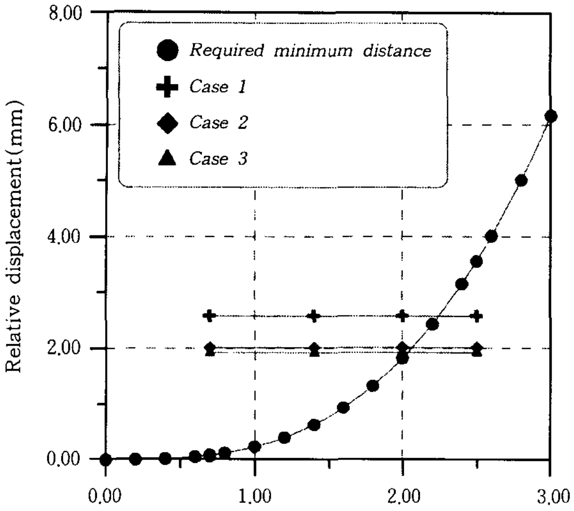
(a) facing thickness = 10cm



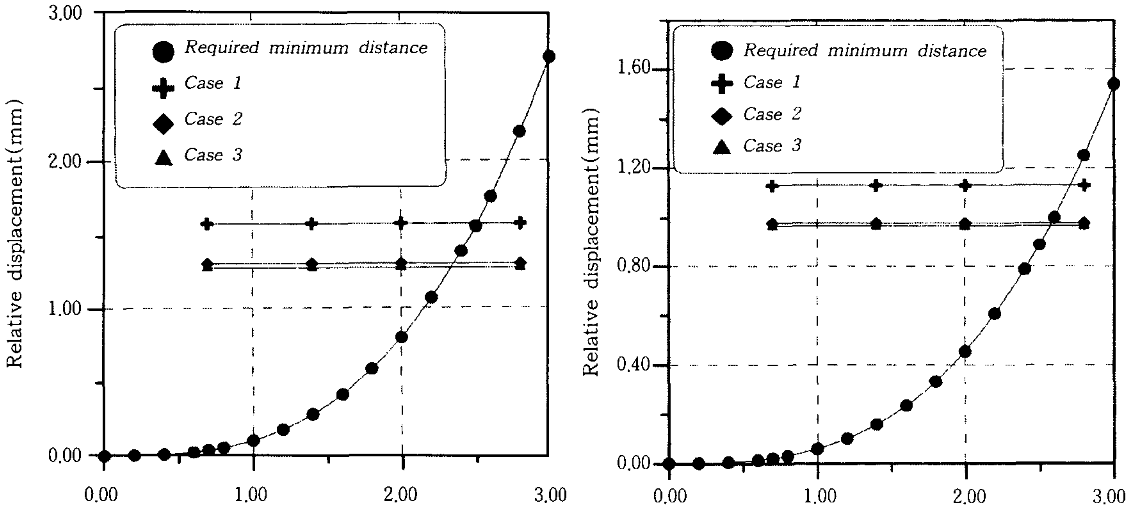
(b) facing thickness = 15cm

(c) facing thickness = 20cm

Fig 30. Relative displacement-transition zone distance relationship(vertically mixed nail-anchor system)



(a) facing thickness=10cm



(a) facing thickness=15cm

(b) facing thickness=20cm

Fig 31. Relative displacement-transition zone distance relationship(horizontally mixed nail-anchor system)

Through the synthetical analysis of the results in Figs. 30 and 31, it may be realized that generally the predicted relative displacements are not quite large and the 'transition zone limit distance' increases with an increase of the shotcrete facing thickness for both cases of the vertically and horizontally mixed nail-anchor system. Detailed examinations indicate that the 'transition zone limit distance' is approximately above 0.8m for the case of the vertically mixed nail-anchor system and is approximately above 2.0m for the case of the horizontally mixed nail-anchor system. When compared to the Case *1*, the predictions of both the relative displacement and the 'transition zone limit distance' are quite the same in the *Cases 2 and 3*.

11. Summary and Conclusion

In the present study, a method of limit equilibrium stability analysis of the excavation zone reinforced with the vertically or the horizontally mixed nail-anchor system is proposed. The postulated multi-linear or wedge-shaped surface is determined on the basis of the results of the $FLAC^{2D}$ and $FLAC^{3D}$ program analyses. This study also deals with a determination of the required thickness of the shotcrete facing. An excessive facing thickness may be required due to both the stress concentration and the relative displacement at the interface zone between the soil nailing system and the ground anchor system. By adopting the composite unit cell concept, a simple finite element method of analysis is presented to examine the facing deflection behaviors with corresponding relative displacement at the interface zone between two different support systems. As an efficient resolution to reduce the facing thickness, the modified bearing plate system is also proposed. By using the proposed methods of limit equilibrium stability analyses, parametric analyses are performed to investigate effects of the nail installation angle and the nail length for three different types of the nail-anchor mixed system. Also by mainly focusing on a verification of the suitability of the proposed simple finite element method of analysis, the predicted facing displacements are compared with the results of the $FLAC^{2D}$ program analyses. In general the simple FEM analysis provides resonable predictions and shows that the tendency of the variation of the facing displacements with depth is quite consistent with a typical feature expected in the case of soil nailing or ground anchor support system.

Furthermore, an evaluation of either a minimum vertical distance between the lowest nail and the uppermost ground anchor in the case of the vertically mixed nail-anchor system or a minimum horizontal distance between the soil nailing zone and the ground anchor zone in the case of the horizontally mixed nail-anchor system required to ascertain the stability against the shear failure due to a relative displacement is carried out, together with analyses for both various facing thickness and different types of reinforcements.

Continuous research is required involving both laboratory model studies and full-scale measurements on actual structures to provide a rational basis for further development and verification of the proposed design procedure.

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