Journal of the Korean Nuclear Society
Volume 31, Number 6, pp.586~594, December 1999

Study on the Effective Stiffness of Base Isolation System for Reducing Acceleration and Displacement Responses

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(Received July 3, 1999)

Abstract

To limit both the large displacement and acceleration response of the structure efficiently, the relationships between acceleration and displacement responses of the structure under several earthquakes are investigated for various horizontal stiffness of the base isolation system to determine the effective stiffness of the base isolation system in this paper.

An example structure is a five-storey steel frame building as the primary structure and the secondary structures are assumed to be located on the fifth floor of the primary structure. Input motions used in the structural analysis are El Centro 1940, Taft 1952, Mexico 1985, San Fernando 1971 Pacoima Dam, and artificially generated earthquakes. The relationships of the absolute peak acceleration and the displacement at the top of the structure are calculated for various natural periods of base isolators under various earthquakes. The peak acceleration response of the fifth floor in the base isolated structure is significantly reduced by a factor of 2.1 through 6.25. Also, the relative displacement response of the floor to the base of the superstructure is very small. The results of this study can be utilized to determine the effective stiffness of the base isolation system.

Key Words : effective stiffness, base isolation system, acceleration, displacement, response, relationship

I. Introduction

In general, the horizontal components of earthquake motions mainly damage a structure rather than the vertical component.(1) Therefore, if the ground is allowed to move relatively under the base of a structure, then the damage can be greatly reduced. The isolation-system installed at the base of the structure reduces the acceleration response significantly,[2-4] while the low horizontal stiffness of the isolation system induces the large relative displacement of the structure to the ground. In particular, the large relative displacement may be the major engineering problems in the adjacent building, connecting pipes, and complex structural systems. Thus, to limit both the large displacement and the acceleration response of the structure efficiently, the relationships between acceleration and displacement responses of the structure under several earthquakes are investigated for various horizontal stiffness of the base isolation system in this paper.

This paper studies the control method of the displacement and acceleration response through the seismic analysis for the primary structure as well as the secondary structure as equipments considering the damping effect and the coupling effect of the subsystem under various input motions such as El Centro 1940, Taft 1952, Mexico 1985, San Fernando 1971 Pacoima Dam, and artificially generated earthquakes.

The research results can be used in the determination of the characteristics of the base isolation system in this country, and also contribute to the seismic design technology. The developed technology increases the safety of the safety-related structures and equipments such as the nuclear power plant and the spent fuel storage facilities, etc.

2. Analysis Method

A constant average acceleration method which is one of the direct integration methods is used for numerical integration of equations of motion in the elastic limit. Interaction effect between the primary and the secondary structures is also considered in the analysis.

2.1. Equation of Motion for a Base-isolated Structure

The displacement at each floor of a base-isolated structure can be expressed by

$$u_i = x_i + x_b \tag{1}$$

where u_i : relative displacement of the i-th floor to the ground

x_i : relative displacement of the i-th floor to the base slab

x_b : relative displacement of base slab to the ground

The equation of motion for the base-isolated structure can be written as equations (2) and (3)

$$[M](\{\ddot{u}\} + \ddot{x}_g\{I\}) + [C]\{\dot{X}\} + [K]\{X\} = \{0\} \tag{2}$$

$$m_b(\ddot{x}_b + \ddot{x}_g) + c_b\dot{x}_b + k_b x_b = -\sum_{i=1}^{n} m_i(\ddot{u}_i + \ddot{x}_g) \tag{3}$$

where [M], [C] and [K] are the mass, damping and stiffness matrices, respectively;$\{\ddot{X}\}$, $\{\dot{X}\}$ and $\{X\}$ are the acceleration, velocity and displacement vectors of the superstructure and the secondary structure; and $\ddot{x}_g$ is the ground acceleration. m_b is the mass of the base slab, cb and kb are the damping and stiffness of the base-isolation system, respectively; $\ddot{x}_b$ and $\dot{x}_b$ are the acceleration and velocity of the base slab, respectively, and mi is the i-th floor mass.

The damping matrix is expressed using the Rayleigh damping[5] as follows,

$$[C] = \alpha[M] + \beta[K] \tag{4}$$

and the coefficientsrm α and β are calculated by using properties of the first and the second structur

$$\alpha = \frac{2\omega_1\omega_2(\zeta_1\omega_2 - \zeta_2\omega_1)}{\omega_2^2 - \omega_1^2} \tag{5}$$

$$\beta = \frac{2(\zeta_2\omega_2 - \zeta_1\omega_1)}{\omega_2^2 - \omega_1^2} \tag{6}$$

where ω_1, ω_2 : natural frequencies for the first and the second modes

ζ_1, ζ_2 : modal damping values for the first and the second modes

Introducing equation (1) into equation (2) yields equation (7) in the matrix form and summation of all equations in this matrix gives equation (8).

$$\begin{aligned}[M](\{\ddot{u}\} + \ddot{x}_g\{I\}) + [C](\{\dot{u}\} - \dot{x}_b\{I\}) \\ + [K](\{u\} - x_b\{I\}) = \{0\}\end{aligned} \tag{7}$$

$$\begin{aligned}\sum_{i=1}^{n} m_i(\ddot{u}_i + \ddot{x}_g) + \{I\}^T[C]\{I\}\{\dot{u}\} - \{I\}^T[C]\{I\}\dot{x}_b \\ + \{I\}^T[K]\{u\} - \{I\}^T[K]\{I\}x_b = 0\end{aligned} \tag{8}$$

Using equation (8), equation (3) can be written as follows :

$$\begin{aligned}m_b(\ddot{x}_b + \ddot{x}_g) + (c_b + \{I\}^T[C]\{I\})\dot{x}_b - \{I\}^T[C]\{\dot{u}\} \\ - \{I\}^T[K]\{u\} + k_b x_b + \{I\}^T[K]\{I\}x_b = 0\end{aligned} \tag{9}$$

The equations of motion for the base-isolated structure subjected to the ground motion can be represented by the matrix form as follows from equations (7) and (9):

$$\begin{aligned}\begin{bmatrix}[M] & 0 \\ 0 & m_b\end{bmatrix}\{\ddot{v}\} + \begin{bmatrix}[C] & -[C]\{I\} \\ -\{I\}^T[C] & c_b + \{I\}^T[C]\{I\}\end{bmatrix}\{\dot{v}\} \\ + \begin{bmatrix}[K] & -[K]\{I\} \\ -\{I\}^T[K] & k_b + \{I\}^T[K]\{I\}\end{bmatrix}\{v\} = -\ddot{x}_g\begin{Bmatrix}\{M\} \\ m_b\end{Bmatrix}\end{aligned} \tag{10}$$

where $\{v\} = \begin{Bmatrix}\{u\} \\ x_b\end{Bmatrix}$

$\{M\}$: vector composed of the diagonal elements in the mass matrix, [M] for the superstructure

2.2. Equation of Motion for the Base-isolated Structure with Secondary Structure

Considering the interaction effect of the primary and the secondary structure, the equation of motion for the base-isolated structure with the secondary structure can be modified from equation (10) as equation (11)[6]

$$\begin{aligned}\begin{bmatrix}[M] & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_e\end{bmatrix}\{\ddot{w}\} + \begin{bmatrix}[C]+[c_{ei}] & -[C]\{I\} & -[c_{ei}]\{I\} \\ -\{I\}^T[C] & c_b + \{I\}^T[C]\{I\} & 0 \\ -\{I\}^T[c_{ei}] & 0 & c_e\end{bmatrix}\{\dot{w}\} \\ + \begin{bmatrix}[K]+[k_{ei}] & -[K]\{I\} & -[k_{ei}]\{I\} \\ -\{I\}^T[K] & k_b + \{I\}^T[K]\{I\} & 0 \\ -\{I\}^T[k_{ei}] & 0 & k_e\end{bmatrix}\{w\} = -\ddot{x}_g\begin{Bmatrix}\{M\} \\ m_b \\ m_e\end{Bmatrix}\end{aligned} \tag{11}$$

where $\{w\} = \begin{Bmatrix}\{u\} \\ x_b \\ x_e\end{Bmatrix}$

c_e, k_e : damping and stiffness of the support of the secondary structure, respectively, and m_e is the mass of the secondary structure

$[c_{ei}]$, $[k_{ei}]$: diagonal matrices of which the i-th diagonal element is c_e and k_e respectively and the others are zeros in which i denotes the supporting floor of the secondary structure

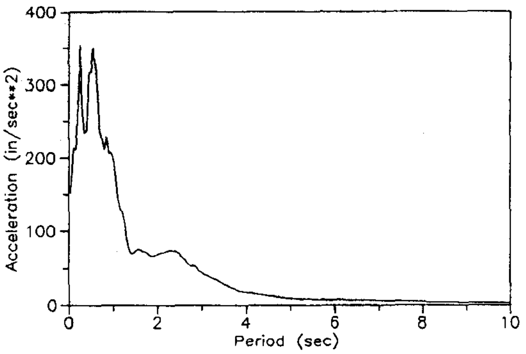
Table 1. Characteristics of Input Ground Motions

Earthquake and location	Component	Peak Acceleration (g)
Imperial Valley, 5/18/1940 El Centro	S00E	0.348
Kern County, 7/21/1952 Taft Lincoln School Tunnel	S69E	0.179
San Fernando, 2/9/1971 Pacoima Dam	S16E	1.17
Mexico City, 9/19/1985 Central De Abastos, Frigorifico	S00E	0.084
Artificial Time History	S90W	0.226

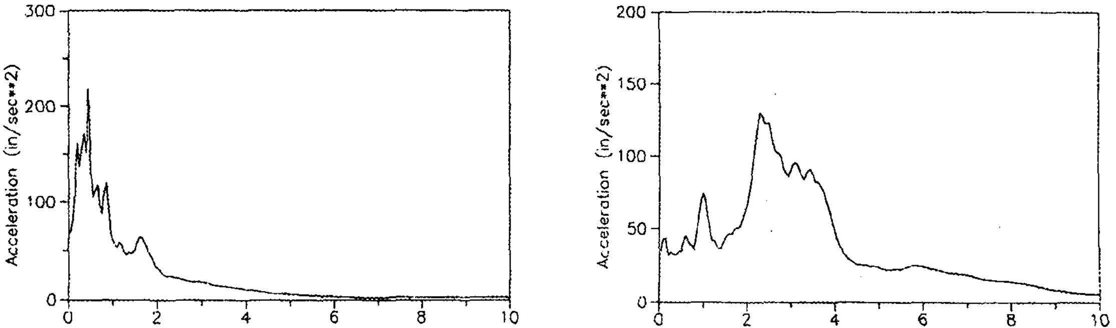
3. Determination of Horizontal Stiffness

The horizontal stiffness of isolation systems is investigated under various input motions such as El Centro 1940, Taft 1952, Mexico 1985, San Fernando 1971 Pacoima Dam, and artificially generated earthquakes for the isolation systems installed at the base of the primary structure and at the support of the secondary structure[6] to determine the effective values. The laminated rubber system selected in this study is the most practical in various base isolation system.[2-3] The artificial time history is generated which is consistent with the design response spectra with the maximum horizontal ground acceleration of 0.20g. The spectral values calculated from the artificial time history meet design spectra enveloping requirements as specified in the US Standard Review Plan, Section 3.7.1.[7] These earthquake records have a variety of peak accelerations, ranging from 0.084g to 1.17g, and frequency content as shown in the Table 1[6] and the acceleration response spectra for the input ground motions are shown in Figure 1.

The multistorey structure is taken to be a five

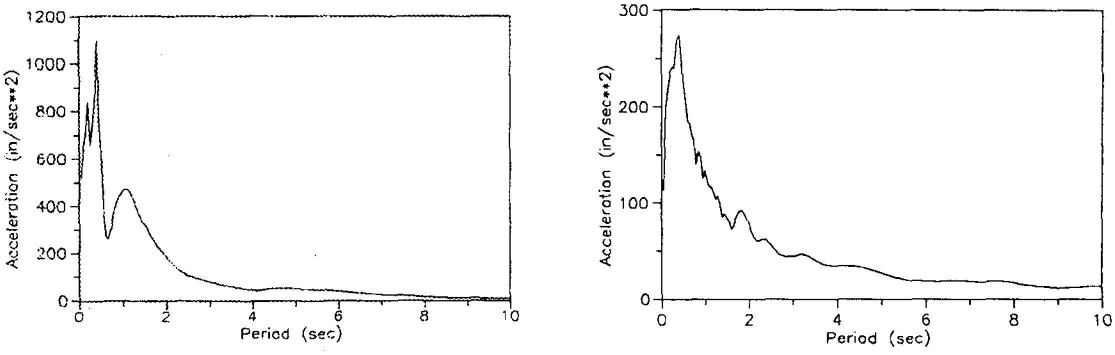


(a) El Centro 1940 Earthquake



(b) Taft 1952 Earthquake

(c) Mexico 1985 Earthquake



(d) San Fernando 1971 Pacoima Dam Record

(e) Artificial Time History

Fig. 1. Acceleration Response Spectra of Input Earthquakes (damping ratio : 5%)

storey steel frame building of width 6.10 m (240 in), each storey is 3.05 m (120 in) high. The column section is W14 × 159 and the beam section is W36 × 160 of the superstructure. The mass of each floor and base is 8756 kg (0.05 kips-s^2/in). The damping ratio is assumed to be 5% and 10% of the critical damping value for the superstructure and the base-isolation system, respectively. The mass of the secondary structure mounted on the top floor of the multistorey structure is 875.6 kg(0.005 kips-s^2/in) as 1/10 of the floor mass. The natural frequency of the fixed support secondary structure is assumed as the fundamental frequency of the fixed-base structure and the damping ratio is 5% of the critical damping value.

3.1. Determination of the Effective Stiffness of Base Isolation Systems

The isolation system installed at the base of a structure reduces the acceleration response significantly, while the low horizontal stiffness of the isolation system induces the large displacement of the structure. Thus, to limit the large displacement response of the structure efficiently, the relationships between acceleration and displacement responses of the structure under several earthquakes are investigated for various horizontal stiffness of the base-isolation system ranging from 42.03 N/m(2.4 kips/in) corresponding to the long period of 2.2 seconds to 998.18 N/m(57.0 kips/in) equivalent to the short period of 0.5 second in the base-isolated structure.

The relationships of the absolute peak acceleration and displacement at the top of the structure are shown in Figures 2 - 6 for various natural periods of a base isolation system. Those figures are composed of the points which represent the maximum displacements and accelerations of the structure for specific periods. Figure 2 under the El Centro earthquake indicates that the displacement responses are increased significantly for the periods longer than 1.3 second but the acceleration responses are almost the same in the period range of 1.3 to 2.2 seconds. While the acceleration responses are highly increased and the displacement is slightly decreased for the periods shorter than 1.3 second. Therefore, the effective horizontal stiffness of the base isolation system for the El Centro record is 116.63 N/m(6.66 kips/in) equivalent to the period of 1.3 second. Similarly, the relationships of the maximum acceleration and displacement for the Taft earthquake significantly varies with the change of the periods as shown in Figure 3. The displacements are highly increased for the periods longer than 1.25 second, but the accelerations are slightly decreased. For the periods less than 1.25 second, the accelerations are highly increased and the displacements are slightly decreased. But the effective horizontal stiffness is similar to that for the El Centro earthquake. As shown in Figure 4 for the Mexico earthquake, the acceleration and displacement increase linearly for the periods longer than 1.33 second and the accelerations are significantly changed from the period of 0.8 second to 1.33 second. It can be noted that the responses are significantly increased for the lower stiffness of the base isolation system in the case of the Mexico earthquake. Since most of the base-isolators are designed with the natural periods of about 2.0 seconds[8], the Mexico earthquake should be treated as a special case. Figure 5 shows the relationships of the maximum acceleration and displacement responses for the San Fernando 1971 Pacoima Dam record. The shape of the relationship is similar to those for the El Centro and Taft records. But the effective horizontal stiffness for reducing the responses is larger than the others. Figure 6 shows the relationship of responses under the artificial time history.

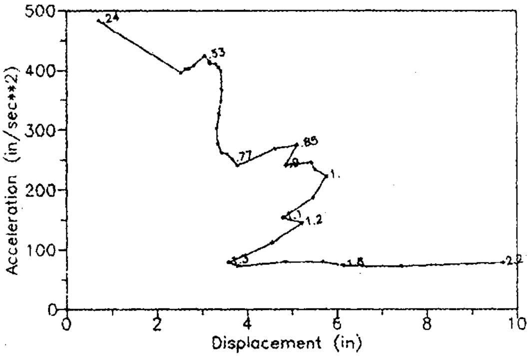


Fig. 2. Relationship of Maximum Displacement and Acceleration for Specific Period(sec.) of the Fifth Floor on Base-Isolated Structure (El Centro 1940 Earthquake)

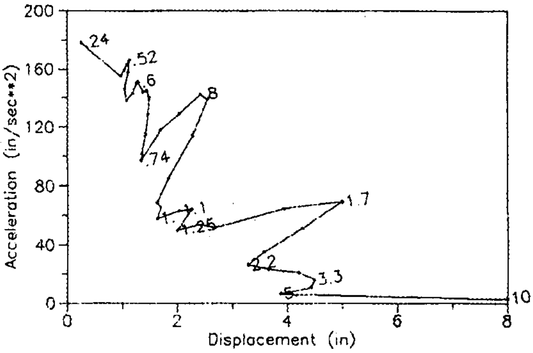


Fig. 3. Relationship of Maximum Displacement and Acceleration for Specific Period(sec.) of the Fifth Floor on Base-Isolated Structure (Taft 1952 Earthquake)

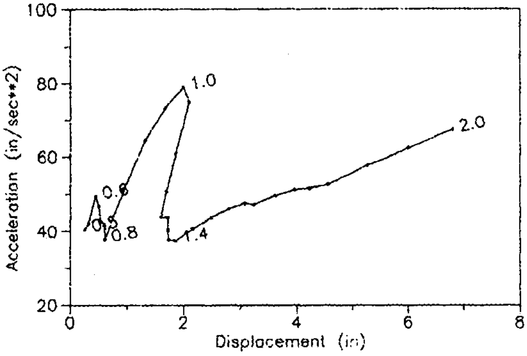


Fig. 4. Relationship of Maximum Displacement and Acceleration for Specific Period(sec.) of the Fifth Floor on Base-Isolated Structure (Mexico 1985 Earthquake)

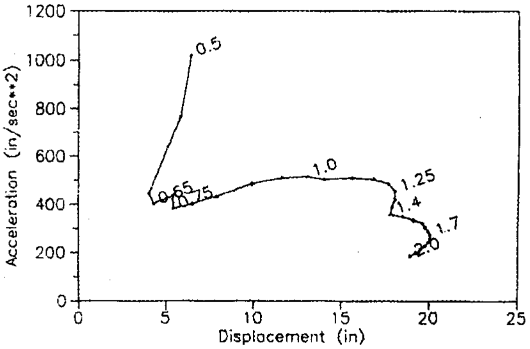


Fig. 5. Relationship of Maximum Displacement and Acceleration for Specific Period(sec.) of the Fifth Floor on Base-Isolated Structure (San Fernando 1971 Pacoima Dam Record)

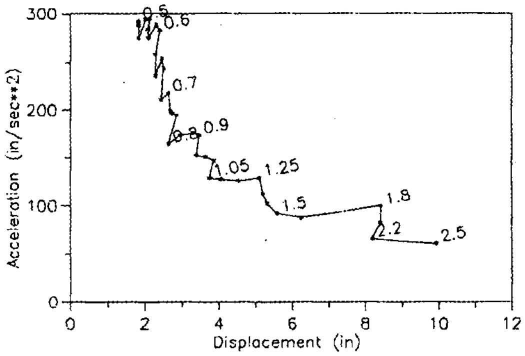


Fig. 6. Relationship of Maximum Displacement and Acceleration for Specific Period(sec.) of the Fifth Floor on Base-Isolated Structure (Artificial Time History)

3.2. Responses of Base-Isolated Structure

The efficient stiffness of the base-isolation system for seismic analysis is taken as 116.63 N/m (6.66 kips/in) corresponding to the fundamental period of 1.33 second for El Centro earthquake as a representative earthquake in this study. The natural periods of the fixed-base structure and the base-isolated structure are given in Table 2[6], which shows that the fundamental periods are 0.244s and 1.33s, respectively.

The maximum acceleration response for the fifth

Table 2. Natural Periods of Vibration for Fixed-base Structure

mode structure	1st	2nd	3rd	4th	5th	6th
fixed-base	0.244	0.080	0.046	0.033	0.027	
base-isolated	1.333	0.137	0.063	0.041	0.031	0.027

Table 3. Maximum Acceleration Responses of Primary Structure

structure earthquake	fixed-base structure	base-isolated structure	reduction factor
El Centro	1.25g	0.20g	6.25
Taft	0.46g	0.14g	3.31
Mexico	0.095g	0.10g	0.95
Pacoima Dam	2.23g	1.06g	2.10
Artificial	0.76g	0.29g	2.60

floor of the base-isolated structure is significantly reduced by a factor of 2.1 through 6.25 comparing with that of the fixed-base structure except for the Mexico earthquake as shown in the Table 3.

3.3. Determination of the Effective Stiffness of Support-isolation Systems

The procedures to determine the horizontal stiffness of the support-isolation system for the secondary structure is similar to that for the base isolator. That is, the relationships of acceleration and displacement are calculated from the period of 2.0 seconds corresponding to the horizontal stiffness of 0.876 N/m(0.05 kips/in) to the period of 0.5 second equivalent to 13.83 N/m(0.79 kips/in). As shown in Figure 7 for the El Centro earthquake, the displacements increase for the

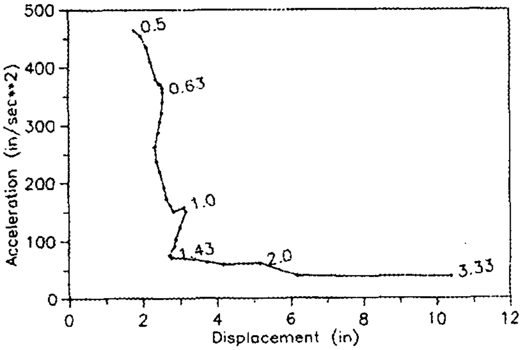


Fig. 7. Relationship of Maximum Displacement and Acceleration for Specific Period (second) of Secondary Structure(El Centro 1940 Earthquake)

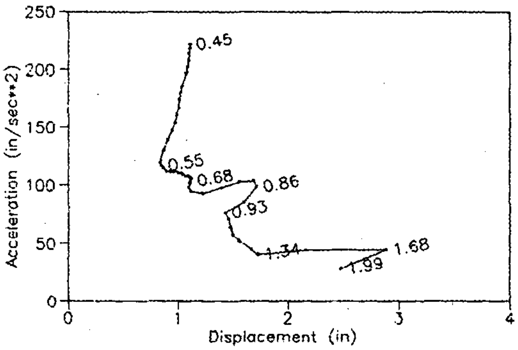


Fig. 8. Relationship of Maximum Displacement and Acceleration for Specific Period (second) of Secondary Structure(Taft 1952 Earthquake)

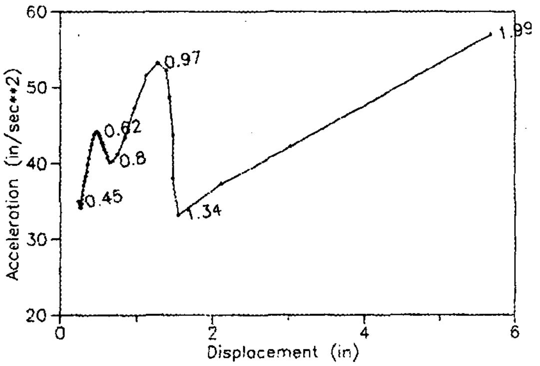


Fig. 9. Relationship of Maximum Displacement and Acceleration for Specific Period (second) of Secondary Structure(1985 Earthquake)

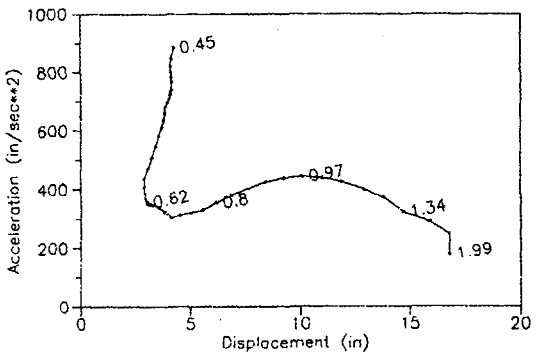


Fig. 10. Relationship of Maximum Displacement and Acceleration for Specific Period (second) of Secondary Structure(San Fernando 1971 Pacoima Dam Record Dam Record)

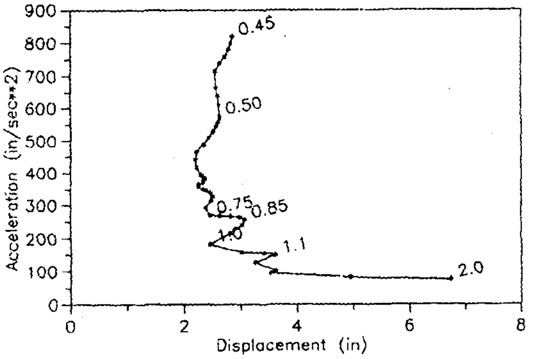


Fig. 11. Relationship of Maximum Displacement and Acceleration for Specific Period (second) of Secondary Structure(Artificial Time History)

periods longer than 1.43 second, whileas the accelerations are almost invariable. For the periods shorter than 1.43 second, the accelerations are highly increased and the displacements are slightly decreased. Similarly, the relationship for the Taft earthquake as shown in Figure 8 indicates that the responses are significantly changed at the period of 1.34 second. The displacement responses increase for the periods longer than 1.34 second, whileas the accelerations are unchanged. The acceleration responses significantly increase for the period less than 1.43 second. Figure 9 shows the relationship of responses for the Mexico earthquake, and the trend of responses is similar to that of Figure 4. The relationship of both responses for the San Fernando 1971 Pacoima dam record is shown in Figure 10 and the responses are significantly changed at around the period of 0.62 second similar to that of Figure 5. Figure 11 shows the relationship of responses under the artificial time history. The shape of relationship is almost the same as that of the El Centro earthquake and both responses are small simultaneously around the period of 1.2 seconds.

3.4. Response of Secondary Structure

Although the relationships of acceleration and displacement for the support-isolation system are different for variety of the magnitudes, frequency contents, and duration of earthquakes, the horizontal stiffness of a support-isolation system is taken as 1.75 N/m(0.1 kips/in) corresponding to 1.4 second which is the period for simultaneously small acceleration and displacement responses for the El Centro earthquake in this study. The responses of the selected support-isolated secondary structure have been evaluated for the input ground motions with the different characteristics.(6)

The support-isolation systems are useful to reduce the response of the secondary structure for earthquake motions. Since the response of the support-isolated secondary structure is smaller than that of the support-fixed secondary structure of the base-isolated structure, the support-isolation system of the secondary structure is very efficient to reduce the response in a complex building.[6]

4. Conclusions

To limit both the displacement and acceleration responses of the structure efficiently, the relationships between acceleration and displacement responses of the structure under several earthquakes such as the El Centro, the Taft, the Mexico, the San Fernando 1971 Pacoima Dam, and the artificially generated earthquakes are investigated for various horizontal stiffness of a base isolation system to determine the effective stiffness of the base isolation system in this paper. The results show that the isolation system is very effective at reducing the seismic response of the secondary structure as well as the primary structure for various earthquakes such as the El Centro, the Taft, the San Fernando 1971 Pacoima Dam, and the artificially generated earthquakes. But the isolation system is not effective for reducing the seismic response under the Mexico earthquake which contains long periods. The effective horizontal stiffness of base-isolated structure is taken as 116.63 N/m(6.66 kips/in) corresponding to 1.33 second which is the fundamental period for simultaneously small acceleration and displacement responses for the El Centro earthquake. Also, the effective horizontal stiffness of support-isolated structure is equivalent to the period of 1.4 second which is the natural period for simultaneously small acceleration and displacement responses.

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